

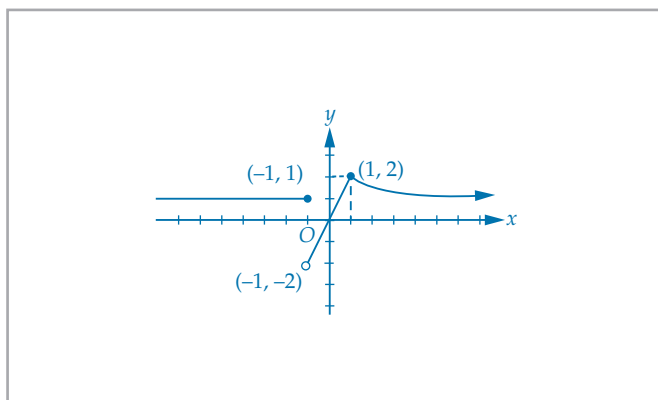
Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 1

- a Sketch the graph of $y = \begin{cases} 1, & x \leq -1 \\ 2x, & -1 < x < 1 \\ \frac{1}{x} + 1, & x \geq 1 \end{cases}$

labelling the coordinates of any axial intercepts and endpoints.



2 marks

- b Evaluate $f(-2)$, $f(0)$ and $f(2)$.

$$\begin{aligned} f(-2) &= 1 \\ f(0) &= 0 \\ f(2) &= \frac{3}{2} \end{aligned}$$

2 marks

- c Find the x value(s) for which $f(x) = 2$

$$f(x) = 2 \text{ only when } x = 1$$

1 mark

(Total: 5 marks)

QUESTION 2

- a A cubic function of the form $y = ax(x - b)(x - c)$ has x -intercepts at $x = 2$ and $x = 3$, and passes through the point $(1, 1)$. Write down the equation of the function.

Incorporating the x -intercept information into

$$y = ax(x - b)(x - c), \text{ we get}$$

$$y = ax(x - 2)(x - 3)$$

Substituting the point $(1, 1)$ in the above equation, we have

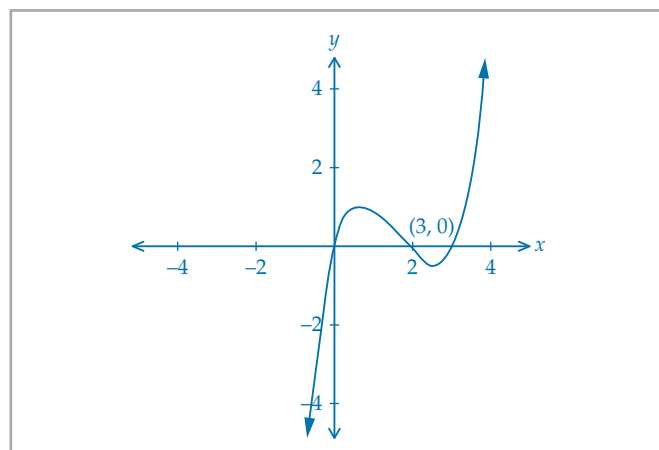
$$1 = a(-1)(-2)$$

$$\therefore a = \frac{1}{2}$$

$$\text{The equation is } y = \frac{1}{2}x(x - 2)(x - 3).$$

2 marks

- b Hence sketch the function, labelling axial intercepts.



2 marks

(Total: 4 marks)

QUESTION 3

An increasing exponential function of the form $y = ae^x + b$ has a horizontal asymptote at $y = 2$ and passes through the origin. Write down the equation of the function.

$$y = ae^x + 2 \text{ because of the horizontal asymptote at } y = 2.$$

Substituting $(0, 0)$ gives

$$0 = ae^0 + 2$$

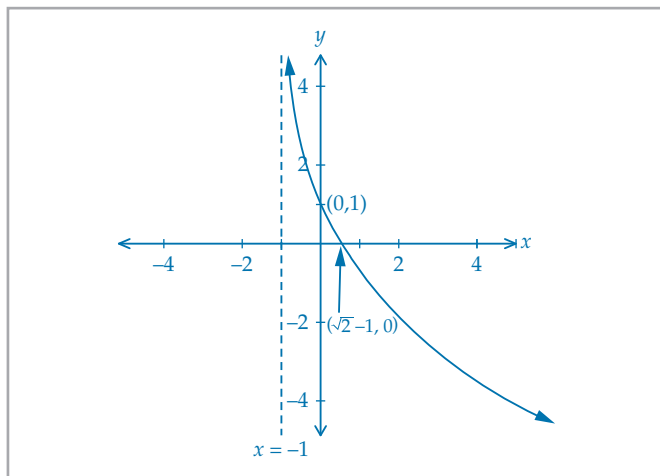
$$\therefore a = -2$$

$$\text{The equation is } y = -2e^x + 2$$

2 marks

QUESTION 4

- a Sketch the graph of $y = -2 \log_2 (x + 1) + 1$, giving the equations of any asymptotes and labelling any axial intercepts.



2 marks

- b State the domain and range of the graph.

Domain = $(-1, \infty)$

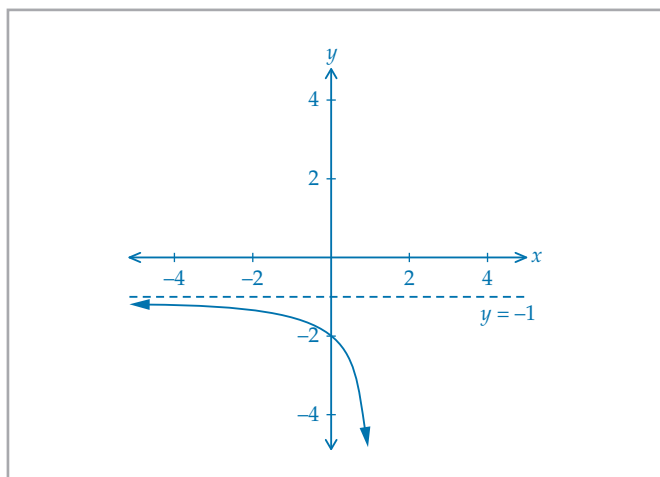
Range = R

2 marks

(Total: 4 marks)

QUESTION 5

- a Sketch the graph of $y = -e^{2x} - 1$, labelling any intercepts with the axes and giving the equations of any asymptotes.



2 marks

- b State the domain and range of the graph.

Domain = R

Range = $(-\infty, -1)$

2 marks

(Total: 4 marks)

QUESTION 6

Let $P(x) = x^3 - 2x^2 - 4x + 8$. Use the factor theorem to solve the equation $P(x) = 0$

$$\text{Try } P(2) = 8 - 8 - 8 + 8 = 0$$

This means that $(x - 2)$ is a factor.

Use 2×2 grouping (or long division) to find a quadratic factor.

$$x^3 - 2x^2 - 4x + 8 = x^2(x - 2) - 4(x - 2)$$

$$= (x - 2)(x^2 - 4)$$

$$= (x - 2)(x - 2)(x + 2)$$

$$= (x - 2)^2(x + 2)$$

$$P(x) = 0 \text{ at } x = 2, x = -2$$

2 marks

QUESTION 7

Find the remainder when $Q(x) = x^3 + x^2 + 5x + 1$ is divided by $x + 2$.

$$Q(x) = x^3 + x^2 + 5x + 1$$

$$Q(-2) = (-2)^3 + (-2)^2 + 5(-2) + 1$$

$$= -8 + 4 - 10 + 1$$

$$= -13$$

$$\text{Remainder} = -13$$

2 marks

QUESTION 8

Find the values of constants a and b if the polynomial $P(x) = x^3 + x^2 + ax + b$ is divisible by the factor $x - 2$, but when divided by $x + 3$ the remainder is 5.

$$P(x) = x^3 + x^2 + ax + b$$

$$P(2) = 2^3 + 2^2 + 2a + b$$

$$\Rightarrow 12 + 2a + b = 0$$

$$\text{Also } P(-3) = (-3)^3 + (-3)^2 - 3a + b$$

$$\Rightarrow -18 - 3a + b = 5$$

Solve simultaneously

$$2a + b = -12 \text{ and } -3a + b = 23$$

$$\text{This gives } a = -7, b = 2$$

2 marks