

Section A: Multiple-choice questions

Specific instructions to students

- A correct answer scores 1 mark, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one letter is shaded on the answer sheet.
- Choose the alternative that most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 ☐ A ☐ B ☐ C ☐ D

 **USE PENCIL ONLY**

- Use pencil only.
- Working space is provided under those questions that require working out.

QUESTION 1

Simplified,

$$\log_{10}(100) + \log_{10}\left(\frac{1}{100}\right) - 3 \log_{10}(\sqrt{x}) - 2 \log_{10}(2)$$

can be expressed as

A $-\frac{3}{2} \log_{10}(4x)$

B $\log_{10}\left(4x^{\frac{3}{2}}\right)$

C $\log_{10}(4) + x^{\frac{3}{2}}$

D $\log_{10}\left(-4x^{\frac{3}{2}}\right)$

E $-\log_{10}\left(4x^{\frac{3}{2}}\right)$

$$\begin{aligned} & \log_{10}(100) + \log_{10}\left(\frac{1}{100}\right) - 3 \log_{10}(\sqrt{x}) - 2 \log_{10}(2) \\ &= \log_{10}(10^2) + \log_{10}(10^{-2}) - 3 \log_{10}(\sqrt{x}) - 2 \log_{10}(2) \\ &= 2 \log_{10}(10) - 2 \log_{10}(10) - \log_{10}\left(x^{\frac{3}{2}}\right) - \log_{10}(4) \\ &= 2 - 2 - \log_{10}\left(x^{\frac{3}{2}}\right) - \log_{10}(4) \\ &= -\log_{10}\left(4x^{\frac{3}{2}}\right) \end{aligned}$$

QUESTION 2

Solve for x in the equation $2^x - 3 \times 2^{-x} = 2$

A $x = \frac{\log_2(3)}{\log_2(2)}$

B $x = \frac{\log_e(2)}{\log_e(3)}$

C $x = \log_3(2)$

D $x = 2$

E $x = 3$

solve((2^x) - 3 (2^{-x}) = 2, x)

{x = $\frac{\ln(3)}{\ln(2)}$ }

$$2^x - 3 \times 2^{-x} = 2$$

$$2^x - 3 \times \frac{1}{2^x} = 2$$

$$2^{2x} - 3 = 2 \times 2^x$$

$$2^{2x} - 2 \times 2^x - 3 = 0$$

$$(2^x - 3)(2^x + 1) = 0$$

$$2^x = 3 \text{ and } 2^x = -1 \quad (\text{no solution})$$

$$x = \log_2(3)$$

$$\text{Using change of base, } \log_2(3) = \frac{\log_e(3)}{\log_e(2)} = \frac{\log_2(3)}{\log_2(2)}$$

QUESTION 3

Consider $f(x) = 2 + \log_e(3 - x)$ and $g(x) = 3 - e^{x-2}$. The rule and domain for the function $g(f(x))$, respectively, are

A $y = 2 + \log_e(3 - x), x \in (-\infty, 3)$

B $y = 3 - e^{x-2}, x \in (-\infty, 3)$

C $y = 3 - e^{x-2}, x \in \mathbb{R}$

D $y = x, x \in \mathbb{R}$

E $y = x, x \in (-\infty, 3)$

$$f(x) = 2 + \log_e(3 - x) \text{ and } g(x) = 3 - e^{x-2}.$$

Test $g(f(x))$.

Test $\text{ran}(f) \subseteq \text{dom}(g)$.

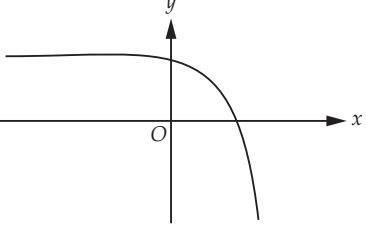
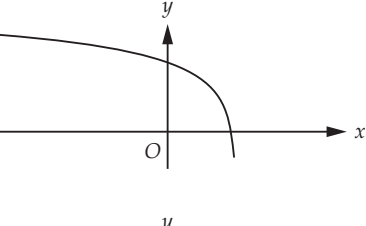
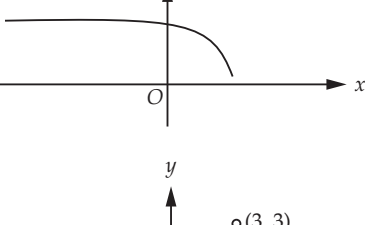
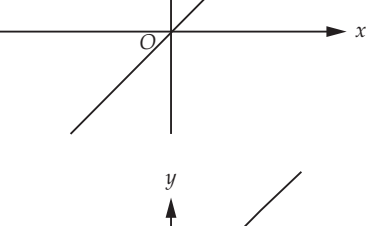
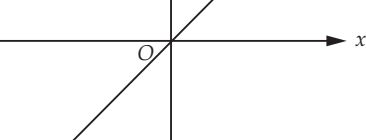
$$\mathbb{R} \subseteq \mathbb{R}$$

$f(x) = 2 + \log_e(3 - x)$ and $g(x) = 3 - e^{x-2}$ are inverses of each other so $g(f(x)) = x$.

$$\text{dom } g(f(x)) = \text{dom } f(x) = (-\infty, 3)$$

QUESTION 4

If $f(x) = 2 + \log_e(3 - x)$ and $g(x) = 3 - e^{x-2}$, then the graph for the function $g(f(x))$ looks like

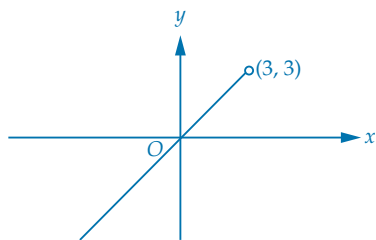
- A** 
- B** 
- C** 
- D** 
- E** 

$$f(x) = 2 + \log_e(3 - x) \text{ and } g(x) = 3 - e^{x-2}.$$

$$\text{Rule } g(f(x)) = x.$$

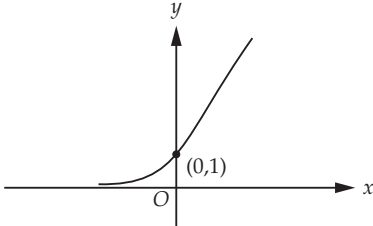
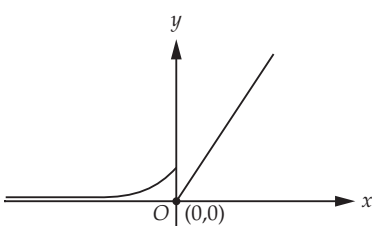
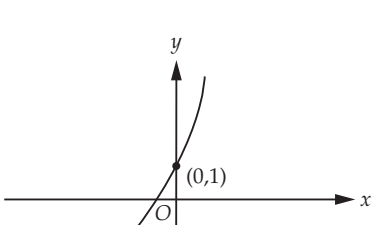
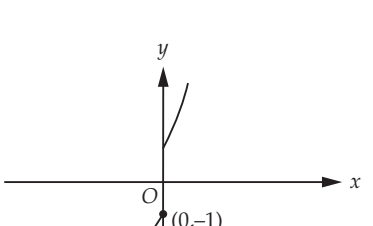
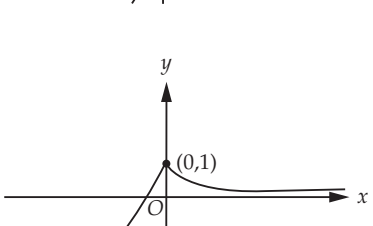
$$\text{dom } g(f(x)) = \text{dom } f(x) = (-\infty, 3)$$

$g(f(x))$ looks like

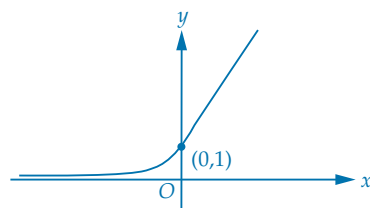


QUESTION 5

For $f(x) = \begin{cases} x+1, & x \geq 0 \\ e^x, & x < 0 \end{cases}$ the graph looks like

- A** 
- B** 
- C** 
- D** 
- E** 

$$f(x) = \begin{cases} x+1, & x \geq 0 \\ e^x, & x < 0 \end{cases} \text{ looks like}$$



QUESTION 6

The maximal domain and range, respectively, of the graph of $y = x^{\frac{5}{2}} + 1$ is

- A $\mathbb{R}, \mathbb{R}$
 B $\mathbb{R}, (0, \infty)$
 C $[0, \infty), [-1, \infty)$
 D $[0, \infty), [1, \infty)$
 E $(0, \infty), [0, \infty)$

$$y = x^{\frac{5}{2}} + 1$$

domain: $[0, \infty)$, range: $[1, \infty)$

QUESTION 7

$P(x) = ax^3 + bx^2 + x - 10$ is a cubic polynomial. Factorised into the product of linear factors, $P(x)$ can be expressed as $P(x) = (2x + 5)(x - 1)(x + 2)$. The values of a and b are

- A $a = 5, b = -2$
 B $a = 1, b = -10$
 C $a = 2, b = 7$
 D $a = 7, b = -2$
 E $a = 2, b = -7$

$$P(x) = ax^3 + bx^2 + x - 10 = (2x + 5)(x - 1)(x + 2).$$

Expanded, $(2x + 5)(x - 1)(x + 2) = 2x^3 + 7x^2 + x - 10$
 $\therefore a = 2, b = 7$

QUESTION 8

A graph is reflected over the y -axis, then dilated by 2 units parallel to the y -axis. The transformation matrix, T , which describes this is

- A $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$
 B $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$
 C $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$
 D $\begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}$
 E $\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$

A graph is reflected over the y -axis, then dilated by 2 units parallel to the y -axis.

$$T = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

QUESTION 9

The graph of $y = \frac{1}{(x-1)^2}$ is reflected over the y -axis, then dilated by 2 units parallel to the y -axis. The image after this transformation is

- A $y = \frac{-1}{(x-1)^2}$
 B $y = \frac{1}{(x-1)^2}$
 C $y = \frac{2}{(x+1)^2}$
 D $y = \frac{1}{(x+1)^2}$
 E $y = \frac{-2}{(x-1)^2}$

The graph of $y = \frac{1}{(x-1)^2}$ is reflected over the y -axis, then dilated by 2 units parallel to the y -axis. Image is

$$y = \frac{2}{(-x-1)^2} = \frac{2}{(x+1)^2}$$
QUESTION 10

If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ is applied to the function $y = \sqrt{x}$, the image function is

- A $y + 4 = \sqrt{x+2}$
 B $y = \sqrt{x-2} + 4$
 C $y = \sqrt{x-2} - 4$
 D $y = 4\sqrt{x-2}$
 E $y = 4\sqrt{x+2}$

If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ is applied to the

function $y = \sqrt{x}$,

$y = f(x - b)$ means that $y = f(x)$ has been translated ' b ' units to the right.

$y = f(x) + c$ means that $y = f(x)$ has been translated ' c ' units up.

Translation of b units right and c units up

$$\begin{bmatrix} x+2 \\ y+4 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \text{ is applied to the function } y = \sqrt{x},$$

$$\text{so } x = x_1 - 2 \text{ and } y = y_1 - 4$$

$$\text{So } y = \sqrt{x} \text{ becomes } y_1 - 4 = \sqrt{x_1 - 2}$$

$$\therefore y = \sqrt{x-2} + 4$$

QUESTION 11

The transformations required to change $y = \frac{1}{\sqrt{x}}$ to $y = 3 - \frac{2}{\sqrt{x+1}}$ are:

- A dilation by a factor of 2 from the x -axis, translation of 1 unit to the right, reflection over the y -axis, translation of 3 units down
- B dilation by a factor of 2 from the y -axis, translation of 1 unit to the right, translation of 3 units up
- C dilation by a factor of 2 from the x -axis, translation of 1 unit to the left, reflection over the x -axis and translation of 3 units up
- D dilation by a factor of 2 from the x -axis, reflection over the y -axis, translation of 1 unit to the left, translation of 3 units up
- E dilation by a factor of $\frac{1}{2}$ from the y -axis, translation of 1 unit to the right, translation of 3 units up

$$y = \frac{1}{\sqrt{x}} \text{ to } y = 3 - \frac{2}{\sqrt{x+1}}$$

- dilation by a factor of 2 from the x -axis
- translation of 1 unit to the left
- reflection over the x -axis
- translation of 3 units up

QUESTION 12

If $f(x) = -\sqrt{-(x+1)} - 2$, then the domain of the inverse function f^{-1} is

- A $(-\infty, -1)$
- B $(-\infty, -2)$
- C $(-2, \infty)$
- D $[2, \infty)$
- E $(-\infty, -2]$

$$f(x) = -\sqrt{-(x+1)} - 2$$

Domain of f^{-1} = range $f(x) = (-\infty, -2]$.

QUESTION 13

The functionality equation $f(x) \times f(y) = f(x+y)$ is satisfied by

- A $f(x) = e^x$
- B $f(x) = \frac{1}{x}$
- C $f(x) = \log_e(x)$
- D $f(x) = x$
- E $f(x) = 2x^2$

$f(x) \times f(y) = f(x+y)$ looks like the rule $e^x \times e^y = e^{x+y}$ and is satisfied by $f(x) = e^x$.

QUESTION 14

Which of the following is **not** a polynomial?

- A $f(x) = 1 + 2x$
- B $f(x) = -3 + 4x + 2x^2$
- C $f(x) = 4x + 2x^3$
- D $f(x) = x^6 - 1$
- E $f(x) = x^2 + \frac{1}{x}$

$f(x) = x^2 + \frac{1}{x}$ is not a polynomial because of the $\frac{1}{x}$ term, which is the power of -1 .

QUESTION 15

The sum of the solutions to the equation $\tan(2x) = -1$ for $-\pi \leq x \leq 0$ is

- A $\frac{3\pi}{4}$
- B $-\frac{3\pi}{4}$
- C $-\frac{\pi}{2}$
- D $-\frac{\pi}{8}$ and $-\frac{5\pi}{8}$
- E $\frac{5\pi^2}{64}$

$$\tan(2x) = -1 \text{ for } -\pi \leq x \leq 0$$

$$\text{solve}(\tan(2 \cdot x) = -1 \mid -\pi \leq x \leq 0, x)$$

$$\left\{ x = -\frac{5\pi}{8}, x = -\frac{\pi}{8} \right\}$$

$$\text{Reference angle} = \frac{\pi}{4}$$

$$2x = -\frac{\pi}{4}, -\pi - \frac{\pi}{4}$$

$$x = -\frac{\pi}{8}, -\frac{5\pi}{8}$$

$$\text{Sum} = -\frac{\pi}{8} - \frac{5\pi}{8} = -\frac{6\pi}{8} = -\frac{3\pi}{4}$$

QUESTION 16

$\sin\left(\frac{11\pi}{6}\right)$ is equivalent to

- A $\cos\left(\frac{11\pi}{6}\right)$
- B $\sin\left(\frac{\pi}{6}\right)$
- C $\cos\left(\frac{16\pi}{3}\right)$

D $\cos\left(\frac{11\pi}{3}\right)$

E $\tan\left(\frac{11\pi}{3}\right)$

$$\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2} \text{ and } \cos\left(\frac{16\pi}{3}\right) = -\frac{1}{2}$$

ONE ANSWER PER LINE					USE PENCIL ONLY						
1	A	B	C	D		9	A	B	C	D	E
2		B	C	D	E	10	A		C	D	E
3	A	B	C	D		11	A	B		D	E
4	A	B	C		E	12	A	B	C	D	
5		B	C	D	E	13		B	C	D	E
6	A	B	C		E	14	A	B	C	D	
7	A	B		D	E	15	A		C	D	E
8		B	C	D	E	16	A	B		D	E

Section B: Extended-response questions

Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 17

- a If $3x^3 + ax^2 + 5x = 3(x + b)^3 + c$, find all possible values of a , b and c .

$$3x^3 + ax^2 + 5x = 3(x + b)^3 + c = 3(x^3 + 3x^2b + 3xb^2 + b^3) + c$$

$$\text{So } 3x^3 + ax^2 + 5x = 3x^3 + 9x^2b + 9xb^2 + 3b^3 + c$$

Equating coefficients

$$a = 9b \text{ (coefficients of } x^2)$$

$$5 = 9b^2 \text{ (coefficients of } x^1)$$

$$0 = 3b^3 + c \text{ (coefficient of } x^0)$$

$$\text{expand}(3 \cdot (x+b)^3 + c)$$

$$3 \cdot x^2 + 3 \cdot b^2 + c^2 + 6 \cdot b \cdot x$$

$$\begin{cases} a=9b \\ 5=9b^2 \\ 0=c+3b^3 \end{cases} \quad a, b, c$$

$$\left\{ \left\{ a=-3\sqrt{5}, b=-\frac{\sqrt{5}}{3}, c=\frac{5\sqrt{5}}{9} \right\}, \left\{ a=3\sqrt{5}, b=\frac{\sqrt{5}}{3}, c=-\frac{5\sqrt{5}}{9} \right\} \right\}$$

Hence $a = \pm 3\sqrt{5}$, $b = \mp \frac{\sqrt{5}}{3}$, $c = \pm \frac{5\sqrt{5}}{9}$

$$\therefore 3x^3 + ax^2 + 5x = 3\left(x - \frac{\sqrt{5}}{3}\right)^3 + \frac{5\sqrt{5}}{9}$$

Or

$$3x^3 + ax^2 + 5x = 3\left(x + \frac{\sqrt{5}}{3}\right)^3 - \frac{5\sqrt{5}}{9}$$

4 marks

- b Letting $f_1(x) = 3(x + b)^3 + c$, and using values from part a, where $b < 0$ and $c > 0$, find the x -coordinates of the point(s) where $f(x) = 0$.

$$b < 0 \text{ and } c > 0$$

$$f(x) = 3\left(x - \frac{\sqrt{5}}{3}\right)^3 + \frac{5\sqrt{5}}{9}$$

$$f(x) = 0 \text{ gives}$$

$$\left(x - \frac{\sqrt{5}}{3}\right)^3 = -\frac{5\sqrt{5}}{27}$$

$$x = -\frac{\sqrt{5}}{3} + \frac{\sqrt{5}}{3}$$

$$x = 0$$

1 mark

- c Letting $f_2(x) = 3(x + b)^3 + c$, and using values from part a, where $b > 0$ and $c < 0$, find the x -coordinates of the point(s) where $f(x) = 0$.

$$b > 0 \text{ and } c < 0$$

$$f(x) = 3\left(x + \frac{\sqrt{5}}{3}\right)^3 - \frac{5\sqrt{5}}{9}$$

$$f(x) = 0 \text{ gives}$$

$$\left(x + \frac{\sqrt{5}}{3}\right)^3 = \frac{5\sqrt{5}}{27}$$

$$x = \frac{\sqrt{5}}{3} - \frac{\sqrt{5}}{3}$$

$$x = 0$$

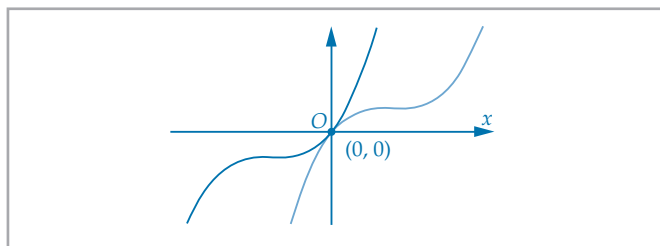
1 mark

- d Hence, find where $f_1(x) = f_2(x)$.

Point of intersection at $(0, 0)$.

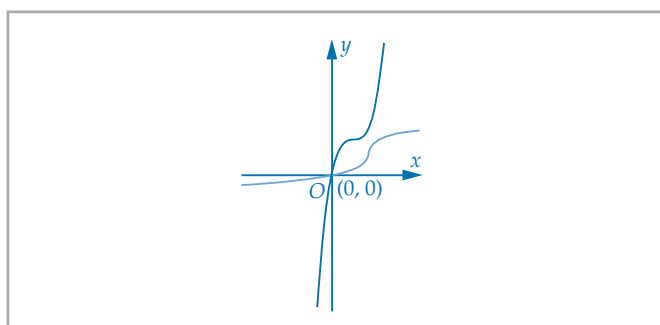
1 mark

- e Sketch the graphs of $f_1(x)$ and $f_2(x)$ on the same axes, labelling point(s) of intersection.



3 marks

- f Sketch the inverse of graph $f_1(x)$ on the same axes, labelling point(s) of intersection.



2 marks

(Total: 12 marks)

QUESTION 18

- a Find the general solution to the equation $2 \cos(2x) = 1$.

$$\begin{aligned}\cos(2x) &= \frac{1}{2} \\ 2x &= 2n\pi \pm \cos^{-1}\left(\frac{1}{2}\right) \\ 2x &= 2n\pi \pm \frac{\pi}{3} \\ x &= n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}\end{aligned}$$

2 marks

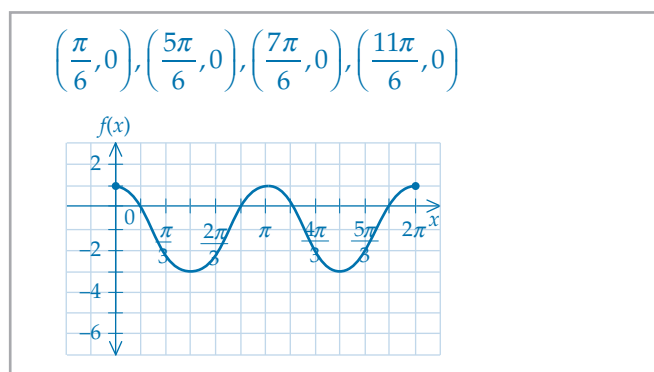
- b Hence, give the first four positive solutions to $\cos(2x) = \frac{1}{2}$.

N	0	1	2
$\frac{\pi}{6} + n\pi$	$\frac{\pi}{6}$	$\frac{7\pi}{6}$	$\frac{13\pi}{6}$
$-\frac{\pi}{6} + n\pi$	$-\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{11\pi}{6}$

Answer: $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

4 marks

- c Sketch the graph of $f(x) = 2 \cos(2x) - 1$ for the domain $x \in [0, 2\pi]$, showing the coordinates of the axial intercepts.



3 marks

(Total: 9 marks)

QUESTION 19

The temperature of a cup of tea cools according to the rule $T = T_0 \times 2^{-kt}$, where T is the temperature in degrees Celsius and t is time in hours. The original temperature of the cup of tea is 90°C .

- a What is T_0 ?

$$T_0 = 90^\circ\text{C}$$

1 mark

It takes 30 minutes for the temperature to halve.

- b What fraction of the original temperature is the temperature of the cup of tea after 60 minutes?

$$T = T_0 \times 2^{-kt}$$

It takes 30 minutes for the temperature to halve,

$$\text{so } t = \frac{1}{2} \text{ and } T = \frac{1}{2} T_0.$$

$$\Rightarrow \frac{1}{2} T_0 = T_0 \times 2^{-0.5k}$$

$$\Rightarrow \frac{1}{2} = 2^{-0.5k}$$

$$2^{-1} = 2^{-0.5k}$$

$$\text{This gives } \frac{1}{2} k = 1$$

$$k = 2$$

After 60 minutes, i.e. at $t = 1$

$$T = T_0 \times 2^{-2 \times 1}$$

$$T = T_0 \times \frac{1}{4}$$

The fraction of the original temperature after 60 minutes = $\frac{1}{4}$

3 marks

- c What is the temperature of the cup of tea after 60 minutes?

Using the result from part b,

$$T = 90 \times \frac{1}{4} = 22.5^\circ\text{C}$$

1 mark

- d A different drink's temperature follows the formula $T = T_0 \times 2^{-kt} + 20$, where T is the temperature in degrees Celsius and t is time in hours. It takes 20 minutes for the temperature to halve and the original temperature of the cup of tea is 90°C . Find the value of k in this case, giving your answer correct to 2 decimal places.

$$T = T_0 \times 2^{-kt} + 20$$

$$t = 0, \text{ gives } T = T_0 + 20$$

Original temperature of the cup of tea is 90°C .

$$90 = T_0 + 20, \text{ so } T_0 = 70$$

It takes 20 minutes for the temperature to halve,

$$\text{so } t = \frac{1}{3}.$$

$$\Rightarrow \frac{1}{2} \times 90 = 70 \times 2^{-k \cdot \frac{1}{3}} + 20$$

$$45 = 70 \times 2^{-k \cdot \frac{1}{3}} + 20$$

This gives $k \approx 4.46$

$$\left\| \begin{array}{l} \text{solve} \left(45 = 70 \cdot 2^{-\frac{k}{3}} + 20, k \right) \\ \{k = 4.456280482\} \end{array} \right\|$$

2 marks

The formula is changed to suit another drink.

The graph of $T = T_0 \times 2^{-kt}$ has a sequence of transformations applied to it, in the order given.

- The graph is translated 15 units in the negative direction of the t -axis.
 - It is then translated 7 units in the negative direction of the T -axis
 - It is then dilated by a factor of 5 from the t -axis
 - It is then dilated by a factor of $\frac{1}{2}$ from the T -axis
 - The graph is then reflected in the T -axis
- e After all these transformations are applied, what is the new rule for $T(t)$?

$T = T_0 \times 2^{-kt}$	
Step 1: translated 15 units in the negative direction of the t -axis	$T_1 = T_0 \times 2^{-k(t+15)}$
Step 2: translated 7 units in the negative direction of the T -axis	$T_2 = T_0 \times 2^{-k(t+15)} - 7$
Step 3: dilated by a factor of 5 from the t -axis	$T_3 = 5T_0 \times 2^{-k(t+15)} - 35$
Step 4: dilated by a factor of $\frac{1}{2}$ from the T -axis	$T_4 = 5T_0 \times 2^{-k(2t+15)} - 35$
Step 5: reflected in the T -axis	$T_5 = 5T_0 \times 2^{-k(-2t+15)} - 35$
New rule for $T(t)$	$T_5 = 5T_0 \times 2^{k(2t-15)} - 35$

2 marks

(Total: 9 marks)