

Section A: Multiple-choice questions

Specific instructions to students

- A correct answer scores 1 mark, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one letter is shaded on the answer sheet.
- Choose the alternative that most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 ☐ A ☐ B ☐ C ☐ D



- Use pencil only.
- Working space is provided under those questions that require working out.

QUESTION 1

$$\log_{10}(1000) =$$

- A 1
B 2
C 3
D 4
E 10^3

$$\log_{10}(1000) = \log_{10}(10^3) = 3 \log_{10}(10) = 3$$

QUESTION 2

An equivalent statement for $a = \log_2(3 - 6x)$ is

- A $6x = \frac{6-2^a}{3}$
B $x = 2^a - 3$
C $x = 1 - \frac{2^a}{6}$
D $2 = a^{3-6x}$
E $x = \frac{3-2^a}{6}$

$$a = \log_2(3 - 6x) \Leftrightarrow 2^a = 3 - 6x$$

$$\text{so } x = \frac{1}{6}(3 - 2^a) = \frac{3 - 2^a}{6}$$

QUESTION 3

The rule for the inverse of the function

$$f(x) = 2 + \log_e(3 - x) \text{ is } f^{-1}(x) =$$

- A $2 + \log_e(3 - y)$
B $3 + \log_e(2 - x)$
C $3 - e^{x-2}$
D $2 + e^{3-x}$
E $\frac{1}{2 + \log_e(3 - x)}$

$$y = 2 + \log_e(3 - x)$$

Swap x and y and rearrange to find the inverse.

$$x = 2 + \log_e(3 - y)$$

$$\Rightarrow \log_e(3 - y) = x - 2$$

$$\Leftrightarrow e^{x-2} = 3 - y$$

$$y = 3 - e^{x-2}$$

$$\text{Thus, } f^{-1}(x) = 3 - e^{x-2}$$

QUESTION 4

$$\text{For } f(x) = \begin{cases} e^x, & x \geq 0 \\ \sin(x), & x < 0 \end{cases}$$

$$f\left(-\frac{\pi}{2}\right) =$$

- A $e^{-\frac{\pi}{2}}$ and -1
B $e^{-\frac{\pi}{2}}$
C -1
D e^{-1} and $-\frac{\pi}{2}$
E 1

$$f(x) = \begin{cases} e^x, & x \geq 0 \\ \sin(x), & x < 0 \end{cases}$$

$$x = -\frac{\pi}{2} \text{ is in domain } x < 0$$

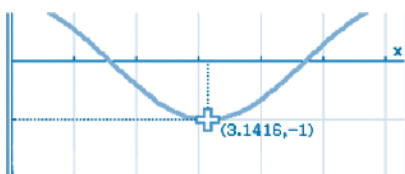
$$f\left(-\frac{\pi}{2}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$$

QUESTION 5

Consider $f(x) = \cos(x)$ and $g(x) = x^2 + 2x$ for their maximal domains. The domain of $f(g(x))$ is

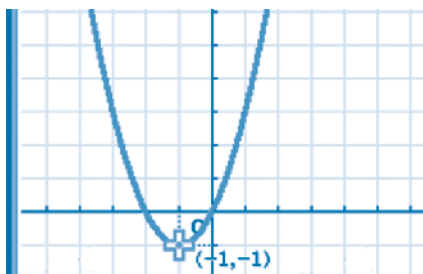
- A $\mathbb{R}$
- B $[-1, \infty)$
- C $(2, \infty)$
- D $[-1, 1]$
- E $[-1, 0]$

$$f(x) = \cos(x)$$



and

$$g(x) = x^2 + 2x$$



Test $f(g(x))$.

$$\text{ran}(\text{inner}) \subseteq \text{dom}(\text{outer})$$

$$\text{ran}(g) \subseteq \text{dom}(f)$$

$$[-1, \infty) \subseteq \mathbb{R}$$

$$\text{Dom } f(g(x)) = \text{dom } g(x) = \mathbb{R}$$

QUESTION 6

Consider $f(x) = \cos(x)$ and $g(x) = x^2 + 2x$ for their maximal domains. The rule and domain respectively for $g(f(x))$ is

- A $y = \cos(x^2 + 2x), [-1, 1]$
- B $y = \cos(x), [-1, 1]$
- C $y = \cos^2(x) + 2 \cos(x), [-1, \infty)$
- D $y = \cos(x^2 + 2x), \mathbb{R}$
- E $y = \cos^2(x) + 2 \cos(x), \mathbb{R}$

$$f(x) = \cos(x) \text{ and } g(x) = x^2 + 2x.$$

Test $g(f(x))$.

$$\text{ran}(\text{inner}) \subseteq \text{dom}(\text{outer})$$

$$\text{ran}(f) \subseteq \text{dom}(g)$$

$$[-1, 1] \subseteq \mathbb{R}$$

$$\text{dom } g(f(x)) = \text{dom } f(x) = \mathbb{R}$$

$$\text{Rule } g(f(x)) = \cos^2(x) + 2 \cos(x)$$

QUESTION 7

$P(x) = -0.5x^3 + x^2 + 0.5x - 1$ is a cubic polynomial. Factorised into the product of linear factors, it can be expressed as

- A $P(x) = (x+1)(x-1)(x-2)$
- B $P(x) = x(x+1)(x-2)$
- C $P(x) = 0.5x(x+1)(2-x)$
- D $P(x) = 0.5x(x+1)(x-2)$
- E $P(x) = 0.5(1-x)(x+1)(x-2)$

$$\begin{aligned} P(x) &= -0.5x^3 + x^2 + 0.5x - 1 \\ &= -0.5(x+1)(x-1)(x-2) \\ &= 0.5(1-x)(x+1)(x-2) \end{aligned}$$

QUESTION 8

The solution of the equation $e^{x+1} - 2 = 1$ for x is

- A $e^{x+1} = 3$
- B $x = \log_e(2) - 1$
- C $x = \log_e(3) - 1$
- D $x + 1 = \log_e(3)$
- E $x = \log_e(y+2) - 1$

$$\begin{aligned} e^{x+1} - 2 &= 1 \Rightarrow e^{x+1} = 3 \\ \Leftrightarrow x + 1 &= \log_e(3) \\ x &= \log_e(3) - 1 \end{aligned}$$

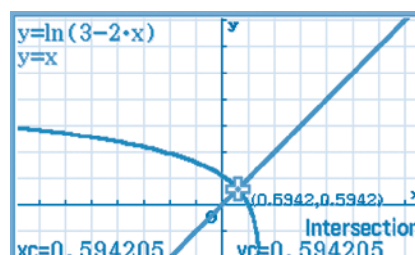
QUESTION 9

The point(s) of intersection between the graph of $y = \log_e(3-2x)$ and its inverse is/are closest to

- A $x = -2, y = -2$
- B $x = 1, y = 1$
- C $x = 0.6, y = 0.6$
- D $x = 0.594, y = 0.594$
- E $x = 0.058, y = 0.594$

Equate $y = \log_e(3-2x)$ with $y = x$, over which the inverse is reflected.

$$\text{Solve } \log_e(3-2x) = x$$



This gives $x = 0.594, y = 0.594$.

QUESTION 10

The graph of $y = -2 \cos(2x - 1) + 7$ has an amplitude and period respectively of

- A -2 and π
- B 10 and 2π
- C 2 and $\frac{\pi}{2}$
- D 10 and π
- E 2 and π

$$y = -2 \cos(2x - 1) + 7$$

$$\text{Amplitude} = 2$$

$$\text{Period} = \frac{2\pi}{2} = \pi$$

QUESTION 11

The coordinates of A and B are $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and $(5, 1)$ respectively. The equation of the line perpendicular to AB , which goes through the point B is

- A $11y = x + 6$
- B $9y = 14 - x$
- C $y = 9x - 44$
- D $y + 11x = 56$
- E $y = -9x + 46$

$$A\left(-\frac{1}{2}, \frac{1}{2}\right), B(5, 1).$$

$$\text{Gradient } AB = \frac{1 - \frac{1}{2}}{5 + \frac{1}{2}} = \frac{1}{11}$$

$$m_T = \frac{1}{11}, m_N = -11$$

For the equation of the line, use $y - y_1 = m(x - x_1)$ with gradient $= -11$ and point $B(5, 1)$.

$$y - 1 = -11(x - 5)$$

$$y = -11x + 56$$

$$y + 11x = 56$$

$$\begin{array}{|l} \frac{1 - \frac{1}{2}}{5 + \frac{1}{2}} \\ \hline \text{solve}(y - 1 = -11 \cdot (x - 5), y) \\ \hline \{y = -11 \cdot x + 56\} \end{array}$$

QUESTION 12

For the polynomial $P(x) = x^3 - x^2 + 2x$, the remainder when $P(x)$ is divided by $(2x + 1)$ is

- A $-\frac{1}{2}$
- B $-\frac{11}{8}$
- C $\frac{11}{8}$
- D $\frac{15}{8}$
- E -3

$$P(x) = x^3 - x^2 + 2x$$

$$P\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) = -\frac{11}{8} = \text{remainder}$$

QUESTION 13

The functionality equation $f(x) + f(y) = f(x + y)$ is satisfied by

- A $f(x) = e^{2x}$
- B $f(x) = \frac{2}{x}$
- C $f(x) = \log_e(2x)$
- D $f(x) = 2x$
- E $f(x) = 2x^2$

$$f(x) + f(y) = f(x + y)$$

By inspection, it is not A (exp) or C (log).

It looks linear.

Test D.

If $f(x) = 2x$, show that $f(x) + f(y) = f(x + y)$.

$$\text{LHS} = f(x) + f(y)$$

$$= 2x + 2y$$

$$= 2(x + y)$$

$$= f(x + y) = \text{RHS}$$

QUESTION 14

Simplified, the expression

$$\frac{1}{2} \log_a(y) + 3 \log_a(x) - \frac{1}{3} \log_a(a)$$

can be expressed as

- A $\log_a(x^3 \sqrt{y}) - \frac{1}{3}$
 B $\frac{1}{6} \log_a(ax^3 \sqrt{y})$
 C $\frac{19}{6} \log_a(a)$
 D $\log_e\left(\frac{x^3 \sqrt{y}}{\sqrt[3]{a}}\right)$
 E $\log_a(x^3 \sqrt{y})$

$$\begin{aligned} \frac{1}{2} \log_a(y) + 3 \log_a(x) - \frac{1}{3} \log_a(a) &= \log_a(y^{\frac{1}{2}}) + \log_a(x^3) - \frac{1}{3} \\ &= \log_a(x^3 \sqrt{y}) - \frac{1}{3} \end{aligned}$$

QUESTION 15

The system of linear graphs of $3x - my = m$ and $6mx - 8y = 2$, where m is a constant, has no solutions for

- A $m \in \mathbb{R} \setminus \{-2, 2\}$
 B $m \in \mathbb{R} \setminus \{-2\}$
 C $m = -2, 2$
 D $m = -2$
 E $m = 2$

$$3x - my = m \text{ and } 6mx - 8y = 2$$

$$\det \begin{bmatrix} 3 & -m \\ 6m & -8 \end{bmatrix} = -24 + 6m^2 = 0 \text{ for a singular matrix.}$$

This gives $m = -2, 2$.

$$\left\| \text{solve} \left(\det \begin{bmatrix} 3 & -m \\ 6 \cdot m & -8 \end{bmatrix} = 0, m \right) \right\|$$

$\{m = -2, m = 2\}$

Test both $m = -2$ and $m = 2$ in the equations of the lines.

$$3x - my = m, 6mx - 8y = 2$$

$$m = 2 \Rightarrow 3x - 2y = 2 \quad 12x - 8y = 2 \quad \text{parallel lines}$$

$$m = -2 \Rightarrow 3x + 2y = -2 \quad -12x - 8y = 2 \quad \text{parallel lines}$$

For parallel lines with no solutions, $m = -2, 2$.

Alternative solution

Equate gradients for lines

$$3x - my = m \text{ and } 6mx - 8y = 2$$

$$\frac{3}{m} = \frac{6m}{8}$$

Gives $m = 2, -2$

ONE ANSWER PER LINE

1	A	B	☒	D	E
2	A	B	C	D	☒
3	A	B	☒	D	E
4	A	B	☒	D	E
5	☒	B	C	D	E
6	A	B	C	D	☒
7	A	B	C	D	☒
8	A	B	☒	D	E

USE PENCIL ONLY

9	A	B	C	☒	E
10	A	B	C	D	☒
11	A	B	C	☒	E
12	A	☒	C	D	E
13	A	B	C	☒	E
14	☒	B	C	D	E
15	A	B	☒	D	E

Section B: Extended-response questions

Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 16

Two lines are defined by $y = 3x - k$ and $y = 3kx + k$ for their maximal domains, where k is a real constant.

- a What are the value(s) of k for the two lines to have no point of intersection?

$$y = 3x - k \text{ and } y = 3kx + k$$

Rearranging, we have $3x - y = k$ and $3kx - y = -k$

$$\det \begin{bmatrix} 3 & -1 \\ 3k & -1 \end{bmatrix} = -3 + 3k = 0 \text{ for a singular matrix.}$$

Thus, $k = 1$.

Test $k = 1$ in the equations of the lines.

$$y = 3x - k \quad y = 3kx + k$$

$$y = 3x - 1 \quad y = 3x + 1 \quad \text{parallel lines}$$

So there is no point of intersection for $k = 1$.

Alternative solution

Two lines are defined by $y = 3x - k$ (line 1) and $y = 3kx + k$ (line 2).

The gradient of line 1 is 3.

The gradient of line 2 is $3k$.

Solving $3 = 3k$ gives $k = 1$.

3 marks

- b If the lines $y = 3x - k$ and $y = 3kx + k$ are now perpendicular, show that the value of k is equal to $-\frac{1}{9}$.

Perpendicular lines for $m_1 \times m_2 = -1$

$$3 \times 3k = -1$$

$$k = -\frac{1}{9}$$

1 mark

- c Hence, using $k = -\frac{1}{9}$, find the coordinates of the point of intersection of the lines.

$$y = 3x - k \text{ and } y = 3kx + k \text{ with } k = -\frac{1}{9}$$

$$\text{become } y = 3x + \frac{1}{9} \text{ and } y = -\frac{1}{3}x - \frac{1}{9}$$

Solve for the intersection point.

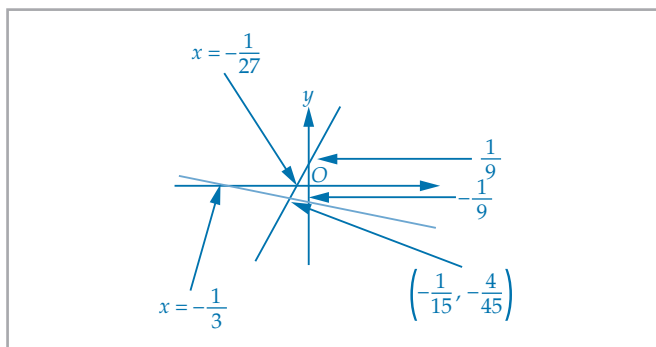
$$\begin{cases} y = 3x + \frac{1}{9} \\ y = -\frac{x}{3} - \frac{1}{9} \end{cases} \quad x, y$$

$$\left\{ x = -\frac{1}{15}, y = -\frac{4}{45} \right\}$$

The intersection point is $\left(-\frac{1}{15}, -\frac{4}{45}\right)$.

2 marks

- d Sketch the graphs of the lines $y = 3x + \frac{1}{9}$ and $y = -\frac{1}{3}x - \frac{1}{9}$, labelling the point of intersection found in part c.



2 marks

- e Find the area of the shape bounded by the x -intercept of the line with the $-ve$ gradient, the x -intercept of the line with the $+ve$ gradient and the x -axis.

$$x\text{-intercept of line with } -ve \text{ gradient, } x = -\frac{1}{3}$$

$$x\text{-intercept of line with } +ve \text{ gradient, } x = -\frac{1}{27}$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \text{ base} \times \text{height} \\ &= \frac{1}{2} \times \frac{8}{27} \times \frac{4}{45} \\ &= \frac{16}{1215} \text{ square units} \end{aligned}$$

2 marks

(Total: 10 marks)

QUESTION 17

Consider $f: [-2, 2] \rightarrow \mathbb{R}, f(x) = ax^4 - 2bx^2 + c$

- a It is known that both $x - 2$ and $2x + 1$ are factors of $f(x)$, and when $f(x)$ is divided by $x + 1$, the remainder is 6. Find a , b and c .

$$f(x) = ax^4 - 2bx^2 + c$$

$$f(2) = 0$$

$$f\left(-\frac{1}{2}\right) = 0$$

$$f(-1) = 6$$

Solve simultaneously.

$$\begin{aligned} &\text{define } f(x) = ax^4 - 2bx^2 + c \\ &\left\{ \begin{array}{l} f(2) = 0 \\ f(-\frac{1}{2}) = 0 \\ f(-1) = 6 \end{array} \right. \quad a, b, c \\ &\left\{ a = -\frac{8}{3}, b = -\frac{17}{3}, c = -\frac{8}{3} \right\} \end{aligned}$$

$$\text{Thus } a = -\frac{8}{3}, b = -\frac{17}{3}, c = -\frac{8}{3}$$

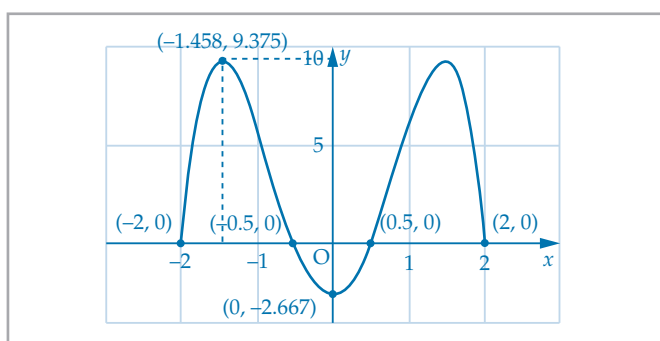
4 marks

- b Hence, fully factorise $f(x)$.

$$\begin{aligned} f(x) &= ax^4 - 2bx^2 + c \\ f(x) &= -\frac{8}{3}x^4 + \frac{34}{3}x^2 - \frac{8}{3} \\ &= -\frac{2}{3}(4x^4 - 17x^2 + 4) \\ &= -\frac{2}{3}(x^2 - 4)(4x^2 - 1) \\ &= -\frac{2}{3}(x + 2)(x - 2)(2x + 1)(2x - 1) \end{aligned}$$

1 mark

- c Sketch the graph of $f(x)$ for $x \in [-2, 2]$, labelling exact x -intercepts as well as the turning points, correct to 2 decimal places.



3 marks

- d Find a new function, $g(x)$, if it is the image of $f(x)$ after it is translated 2 units in the +ve direction of the x -axis, $\frac{8}{3}$ units in the +ve direction of the y -axis, and then dilated by 2 units from the y -axis.

$$\begin{aligned} f(x-2) + \frac{8}{3} &= -\frac{2}{3}[4(x-2)^4 - 17(x-2)^2 + 4] + \frac{8}{3} \\ g(x) &= -\frac{2}{3}\left[4\left(\frac{1}{2}x-2\right)^4 - 17\left(\frac{1}{2}x-2\right)^2 + 4\right] + \frac{8}{3} \end{aligned}$$

3 marks

- e Express this new function, $g(x)$, in fully factorised form, hence showing that one quadratic factor has rational roots and the other quadratic factor has irrational roots.

$$\begin{aligned} &\text{factor}\left(g\left(\frac{1}{2}x-2\right)+\frac{8}{3}\right) \\ &= \frac{-(x^2-8x-1) \cdot (x-4)^2}{6} \\ g(x) &= -\frac{1}{6}(x-4)^2(x^2-8x-1) \end{aligned}$$

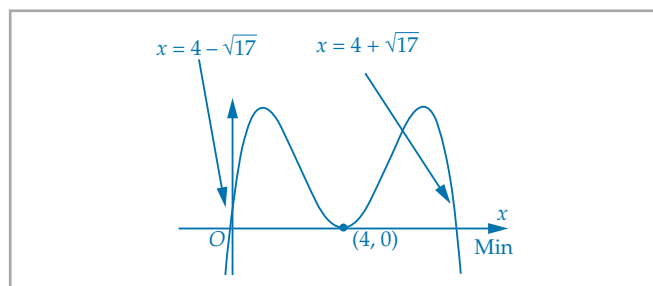
$$\begin{aligned} &\text{rFactor}(x^2-8x-1) \\ &= (x+\sqrt{17}-4) \cdot (x-\sqrt{17}-4) \\ g(x) &= -\frac{1}{6}(x-4)^2(x-4+\sqrt{17})(x-4-\sqrt{17}) \end{aligned}$$

Rational and irrational roots are shown.

Alternatively, find the discriminant of $(x^2 - 8x - 1)$.

2 marks

- f Sketch a graph of $g(x)$, labelling all axial intercepts.



2 marks

(Total: 15 marks)