

TEST 1

Technology active end-of-year examination

Functions and graphs

Section A: 15 marks

Section B: 25 marks

Suggested writing time: 60 minutes

Section A: Multiple-choice questions

Specific instructions to students

- A correct answer scores 1 mark, and an incorrect answer scores 0.
- Marks are not deducted for incorrect answers.
- No marks are given if more than one letter is shaded on the answer sheet.
- Choose the alternative that most correctly answers the question and mark your choice on the multiple-choice answer section at the bottom of each page, as shown in the example below.

1 ☐ A ☐ B ☐ C ☐ D



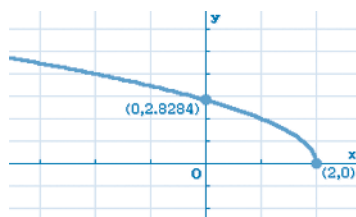
- Use pencil only.
- Working space is provided under those questions that require working out.

QUESTION 1

The maximal domain for the function $y = 2\sqrt{2-x}$ is

- A R
- B $[2, \infty)$
- C $(2, \infty)$
- D $(-\infty, 2)$
- E $(-\infty, 2]$

$y = 2\sqrt{2-x}$ has the graph below with domain $(-\infty, 2]$.



QUESTION 2

The derivative of the function $f(x) = 3 - 7x$ is $f'(x) =$

- A 3
- B -3
- C -7
- D 7
- E $3x - \frac{7x^2}{2}$

$f(x) = 3 - 7x$ so $f'(x) = \text{gradient} = -7$

QUESTION 3

The rule of the inverse of the function $f(x) = 3 - 7x$ is $f^{-1}(x) =$

- A $\frac{3-x}{7}$
- B $\frac{x-3}{7}$
- C $3 - 7y$
- D $3 - 7x$
- E $7 - 3x$

Let $y = 3 - 7x$

Swap x and y and rearrange to determine the inverse.

$$x = 3 - 7y$$

$$\text{So } f^{-1}(x) = \frac{3-x}{7}$$

QUESTION 4

For $f(x) = \begin{cases} x, & x \geq 0 \\ 1, & x < 0 \end{cases}$

$f(2) =$

- A 1
- B 2
- C 3
- D 4
- E 0

$$f(x) = \begin{cases} x, & x \geq 0 \\ 1, & x < 0 \end{cases}$$

$x = 2$ is in domain $x \geq 0$, so $f(2) = 2$

QUESTION 5

For $f(x) = \begin{cases} x^2, & x \geq 0 \\ 1, & x < 0 \end{cases}$

$f'(2) =$

- A 1
- B 2
- C 3
- D 4
- E 0

$$f(x) = \begin{cases} x^2, & x \geq 0 \\ 1, & x < 0 \end{cases}$$

$x = 2$ is in domain $x \geq 0$, so $f'(2) = \text{gradient at } x = 2$
Answer = 4.

QUESTION 6

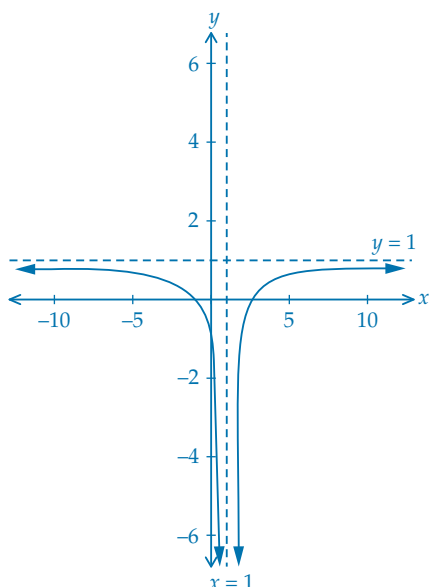
The domain and range, respectively, of the function

$$y = -\frac{2}{(x-1)^2} + 1$$
 are

- A R and R
- B $R \setminus \{0\}$ and $R \setminus \{0\}$
- C $R \setminus \{0\}$ and $R \setminus \{1\}$
- D $R \setminus \{1\}$ and $(-\infty, 2)$
- E $R \setminus \{1\}$ and $(-\infty, 1)$

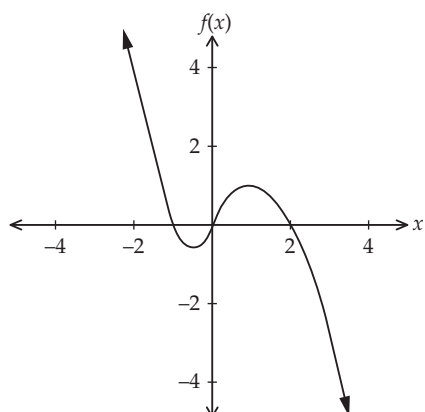
$$y = -\frac{2}{(x-1)^2} + 1.$$

Domain = $R \setminus \{1\}$ and range = $(-\infty, 1)$.



QUESTION 7

A cubic graph is shown.



The equation of the graph could be

- A $y = (x+1)(x-1)(x-2)$
- B $y = x(x+1)(x-2)$
- C $y = 0.5x(x+1)(2-x)$
- D $y = 0.5x(x+1)(x-2)$
- E $y = -0.5(x+1)(x-1)(x-2)$

-ve cubic graph with x -intercepts of $-1, 0, 2$.

The equation of the graph could be

$$y = 0.5x(x+1)(2-x)$$

QUESTION 8

The graph of $y = e^{x+1} - 2$ has asymptotes at

- A $y = -2$ only
- B $y = -2$ and $x = -1$
- C $x = -1$ only
- D $x = -2$ and $y = -1$
- E $y = 1$ only

$y = e^{x+1} - 2$ has an asymptote at $y = -2$ only.

QUESTION 9

The graph of $y = 2 \log_e (2x+2)$ has asymptotes at

- A $y = -2$ only
- B $y = 2$ and $x = -1$
- C $x = -1$ only
- D $x = -2$ and $y = -1$
- E $x = -2$ only

$$2x+2=0 \Rightarrow x=-1$$

$y = 2 \log_e (2x+2)$ has an asymptote at $x = -1$ only.

QUESTION 10

The graph of $y = -\cos(2x)$ has a period and amplitude respectively of

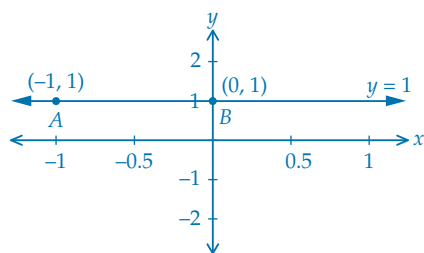
- A 1 and 2π
- B 2π and 1
- C π and 1
- D π and -1
- E 1 and π

$y = -\cos(2x)$ has a period = $\frac{2\pi}{2} = \pi$ and amplitude = 1.

QUESTION 11

The coordinates of A and B are $(-1, 1)$ and $(0, 1)$ respectively. The equation of the line that passes through the points A and B is

- A $y = 1$
- B $x = 0$
- C $y = x$
- D $x = -1$
- E $y = x + 1$

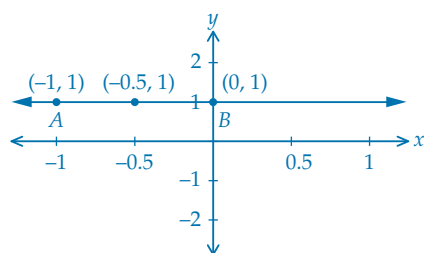


The y -coordinate in the two points $(-1, 1)$ and $(0, 1)$ is the same. The equation of the line is $y = 1$.

QUESTION 12

The coordinates of A and B are $(-1, 1)$ and $(0, 1)$ respectively. The coordinates of the midpoint between A and B are

- A $(0, 0)$
- B $(0, 1)$
- C $(-0.5, 0)$
- D $(-0.5, 1)$
- E $(0.5, 2)$



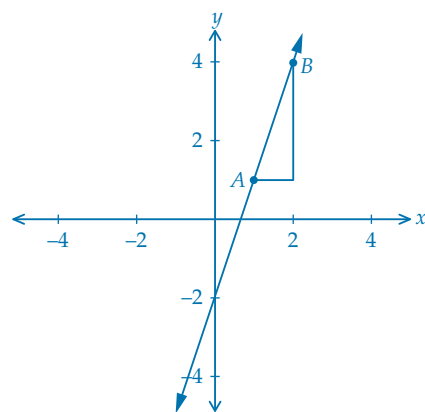
The y -coordinate in the two points $(-1, 1)$ and $(0, 1)$ is the same, so find the middle x value.

Midpoint = $(-0.5, 1)$

QUESTION 13

The coordinates of A and B are $(1, 1)$ and $(2, 4)$, respectively. The distance between A and B is

- A 10
- B $\sqrt{10}$
- C $\pm\sqrt{10}$
- D 2
- E 4



$(1, 1)$ and $(2, 4)$.

$$\text{Distance} = \sqrt{(2-1)^2 + (4-1)^2} = \sqrt{1+9} = \sqrt{10}$$

QUESTION 14

The maximal domain of the inverse of the graph of $y = -(x-2)^2 - 1$ is

- A $\mathbb{R}$
- B $[2, \infty)$
- C $(-1, \infty)$
- D $(-\infty, 1]$
- E $(-\infty, -1]$

Upside-down parabola with turning point $(2, -1)$.

Domain of inverse = range of $y = -(x-2)^2 - 1$, and is $(-\infty, -1]$.

QUESTION 15

An equivalent expression to $y = a^{2x}$ is

- A $x = \log_a(y)$
- B $y = \log_a(x)$
- C $x = \frac{1}{2} \log_a(y)$
- D $y = \frac{1}{2} \log_a(2x)$
- E $x = a^{2y}$

$$\begin{aligned} y = a^{2x} &\Leftrightarrow 2x = \log_a(y) \\ &\Rightarrow x = \frac{1}{2} \log_a(y) \end{aligned}$$

ONE ANSWER PER LINE

1	A	B	C	D	☐
2	A	B	☐	D	E
3	☐	B	C	D	E
4	A	☐	C	D	E
5	A	B	C	☐	E
6	A	B	C	D	☐
7	A	B	☐	D	E
8	☐	B	C	D	E

USE PENCIL ONLY

9	A	B	☐	D	E
10	A	B	☐	D	E
11	☐	B	C	D	E
12	A	B	C	☐	E
13	A	☐	C	D	E
14	A	B	C	D	☐
15	A	B	☐	D	E

Section B: Extended-response questions

Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 16

A polynomial is defined as $P(x) = ax^3 + bx^2 + 2x + 10$

- a It is known that $x - 2$ is a factor of $P(x)$ and that if $P(x)$ is divided by $x + 1$, the remainder is -3 . Show that $a = \frac{5}{2}$ and $b = -\frac{17}{2}$

Let $f(x) = ax^3 + bx^2 + 2x + 10$

And $f(2) = 0$

And $f(-1) = -3$

```

define f(x)=ax^3+bx^2+2x+10
done
{f(2)=0
f(-1)=-3|a,b
{a=5/2, b=-17/2}

```

Solve simultaneously to get $a = \frac{5}{2}$ and $b = -\frac{17}{2}$

(For a 'show that' question it would be necessary to solve these equations simultaneously by hand.)

3 marks

- b Hence find the solutions for the equation $y = P(x) = 0$.

$$y = P(x) = \frac{5}{2}x^3 - \frac{17}{2}x^2 + 2x + 10$$

We know that $f(2) = 0$, so $x - 2$ is a factor.

By division, the quadratic factor is $\frac{5}{2}x^2 - \frac{7}{2}x - 5$

$$\text{propFrac}\left(\left(\frac{5}{2}x^3 - \frac{17}{2}x^2 + 2x + 10\right) / (x - 2)\right)$$

$$\frac{5 \cdot x^2}{2} - \frac{7 \cdot x}{2} - 5$$

$$y = (x - 2)\left(\frac{5}{2}x^2 - \frac{7}{2}x - 5\right)$$

$$= \frac{1}{2}(x - 2)(5x^2 - 7x - 10)$$

$$\text{rFactor}\left(\frac{5 \cdot x^2}{2} - \frac{7 \cdot x}{2} - 5\right)$$

$$\frac{5 \cdot \left(x + \frac{\sqrt{249}}{10} - \frac{7}{10}\right) \cdot \left(x - \frac{\sqrt{249}}{10} - \frac{7}{10}\right)}{2}$$

$$y = \frac{5}{2}(x - 2)\left(x - \frac{7 \pm \sqrt{249}}{10}\right)$$

$$y = 0 \text{ at } x = 2, x = \frac{7 \pm \sqrt{249}}{10}$$

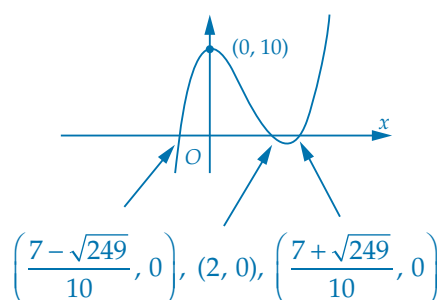
Or solve can be used.

$$\text{solve}\left(\frac{5}{2}x^3 - \frac{17}{2}x^2 + 2x + 10 = 0, x\right)$$

$$\left\{x = 2, x = \frac{-\sqrt{249}}{10} + \frac{7}{10}, x = \frac{\sqrt{249}}{10} + \frac{7}{10}\right\}$$

3 marks

- c Sketch the graph of $y = P(x)$, labelling the coordinates of any axial intercepts.

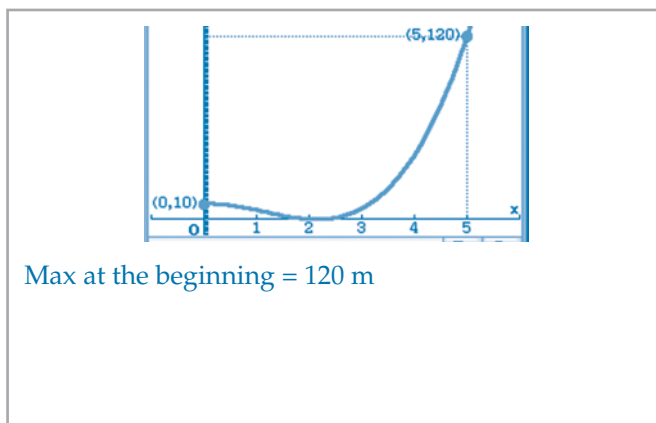


4 marks

A section of the graph drawn in part c represents a proposed ski slope, where y metres is the height above ground level and x metres is the horizontal distance from the end of the ski slope. The ski slope ends at the point $(0, 10)$.

Consider $f: [0, 5] \rightarrow \mathbb{R}, f(x) = \frac{5}{2}x^3 - \frac{17}{2}x^2 + 2x + 10$

- d Find the maximum height of the ski slope.



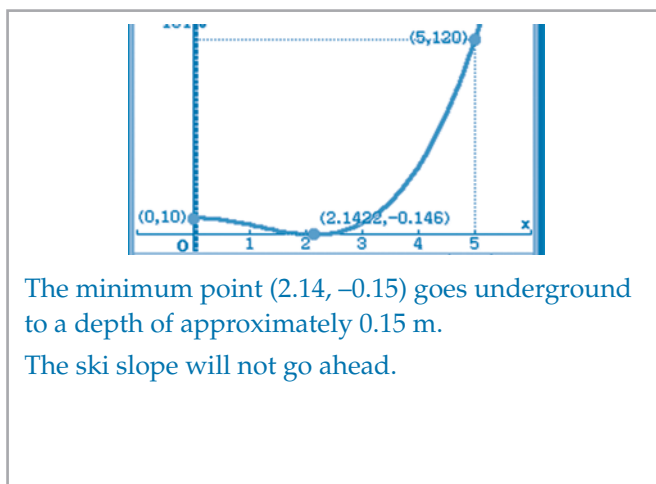
2 marks

- e How high will the engineers have to build the platform for the end of the slope?

The ski slope ends at the point (0, 10).
Height at the end = 10 m

1 mark

- f The engineers do not permit the ski slope to be opened if the path goes underground. Will the ski slope go ahead?



2 marks

(Total: 15 marks)

QUESTION 17

A jet of water from a hose is pointed over the fence by children into their neighbour's garden. The water stream from the hose starts at a distance from the ground of 2.88 metres. Amy, who is holding the hose, is standing at a horizontal distance of 14 metres from the fence. The water goes over the fence and lands on the other side in the neighbour's garden at a horizontal distance of 4 metres from the fence.

- a Use the function $h(x) = a + b(x + 6)^2$ to model the path of the water in the air, where a and b are constants and the variables, h and x , are in metres. Find the values of a and b .

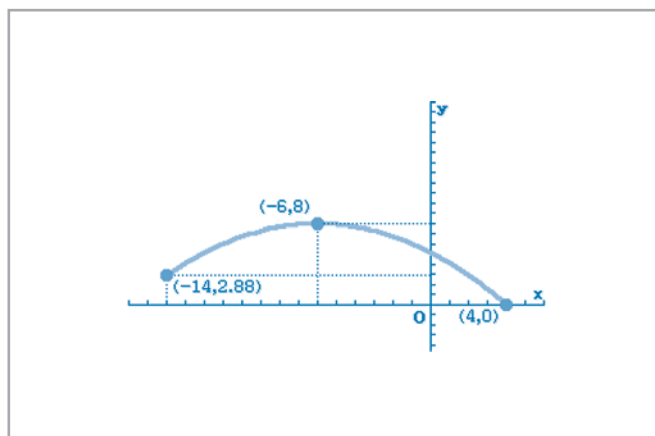
```
define h(x)=a+b(x+6)^2
done
{h(-14)=2.88 | a,b
 {h(4)=0      |
 {a=8, b=-2/25}
```

Define the function $h(x) = a + b(x + 6)^2$
and use $h(4) = 0$ and $h(-14) = 2.88$

Solve simultaneously to get $a = 8$ and $b = -\frac{2}{25}$.

3 marks

- b Sketch the function $h(x) = a + b(x + 6)^2$ that models the path of the water in the air, with your values of a and b . Use the domain $x \in [-14, 4]$.



2 marks

- c The neighbours are sick of the children next door and have built a fence that is 5 metres high. Will the water go over the fence? Why/why not?

y-intercept (0, 5.12)

$5.12 > 5$

Yes, the water will go over the fence.

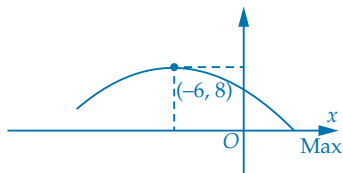
2 marks

- d There are power lines above the children's garden that are 10 metres off the ground. Will the water hit the power lines?

Maximum point is $(-6, 8)$.

$$8 < 10$$

Water will not hit the power lines.



1 mark

- e The children's dog likes to jump up and lap at the water in the air. The dog, Scruffy, can jump 2 metres in the air to reach the water. Where does Scruffy manage to drink the water?

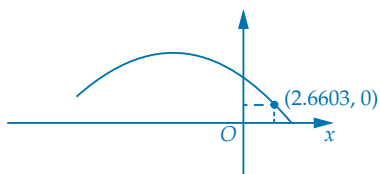
Solve $h(x) = 2$

This gives $x = -6 \pm 5\sqrt{3}$

The only solution in the domain $x \in [-14, 4]$ is

$$x = -6 + 5\sqrt{3} \approx 2.66$$

Scruffy will reach the water after the maximum point and in the neighbour's garden, approximately 2.66 metres beyond the fence and until it reaches the ground.



2 marks

(Total: 10 marks)