

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 2



2014 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: B

Explanation:

Period is: $\frac{\pi}{\pi/3} = 3$

Question 2

Answer: A

Explanation:

The domain of $f(x) + g(x)$ is given by *dom of $f \cap$ dom of g*

Question 3

Answer: C

Explanation:

$$d^2 = \left(\sqrt{(2a - a)^2 + (2b + b)^2} \right)^2 = a^2 + 9b^2$$

Question 4

Answer: E

Explanation:

$$-1 + a - 41 + 56 = 0$$

$$a = -14$$

Question 5

Answer: D

Explanation:

The domain of the inverse is the range of the function $= (-\infty, 3)$

$$x = 3 - e^y$$

$$e^y = 3 - x \text{ which implies } y = \log_e(3 - x)$$

Question 6

Answer: C

Explanation:

$$\frac{1}{3-1/2} \int_{1/2}^3 2\sin\left(x - \frac{3\pi}{4}\right) dx$$

Question 7

Answer: C

Explanation:

Draw the probability table and use $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$ to find the value of p .

Question 8

Answer: A

Explanation:

$$y - 2e^{a+2} = 2e^{a+2}(x - a)$$

$$\text{Solve : } -2e^{a+2} = 2e^{a+2}(-a) \text{ for } a$$

Question 9

Answer: C

Explanation:

$$kx^2 - 5x = x - 3$$

$$\Delta = 0 \text{ implies } k = 3$$

Question 10

Answer: D

Explanation:

Chain rule. First differentiate f , then $\cos$, then g

Question 11

Answer: B

Explanation:

Shaded Area = Area of rectangle – Area under the curve

$$\text{Shaded Area} = 3 \times 9 - \int_2^{11} \sqrt{x-2} \, dx$$

Question 12

Answer: A

Explanation:

For $f(g(x))$ to be defined (Range of g) should be a subset of (domain of f)

Question 13

Answer: A

Explanation:

Use matrix transformations to get $y + 1 = (-x + 4 - 1)^2$

Question 14

Answer: E

Explanation:

$$\Pr(X < 90) = \Pr\left(Z < \frac{-30}{\sigma}\right)$$
$$\frac{-30}{\sigma} = \text{invnorm}\left(\frac{3}{40}, 0, 1\right)$$

Question 15

Answer: C

Explanation:

$$f(2x) = 2(2x) = 2f(x)$$

Question 16

Answer: B

Explanation:

solve $\left(\frac{d}{dx}(5e^x \sin(x)) = 0\right)$ for x and choose the second value of x .

Question 17

Answer: E

Explanation:

$$0 = (1 + a)^2 \text{ implies } a = -1$$
$$f(x) = \frac{(x-1)^3}{3} + c$$
$$0 = c$$

Question 18

Answer: B

Explanation:

$$f(109) \approx f(100) + 9 \times f'(100)$$

Question 19

Answer: A

Explanation:

As the graph of the function passes through 3, its gradient changes from negative to positive hence a point of minimum.

Question 20

Answer: C

Explanation:

$$X \sim \text{Bi}(5, 0.2) \\ \Pr(X = 3) = C(5, 3)(0.2)^3(0.8)^2$$

Question 21

Answer: E

Explanation:

$$1(f(3) + f(4) + f(5))$$

Question 22

Answer: A

Explanation:

$$\frac{f(3) - f(1)}{3 - 1} = 7$$

Solve for **a**.

SECTION 2: Analysis Questions

Question 1

- a. $f(x) = x^3 e^{-2x}$
 $f'(x) = -2x^3 e^{-2x} + 3x^2 e^{-2x}$
 $f'(x) = x^2 e^{-2x} (3 - 2x)$
 $a = 2$ and $b = -2$

M1+A2
3 marks

- b. $f(0) = 0$
 $(0,0)$

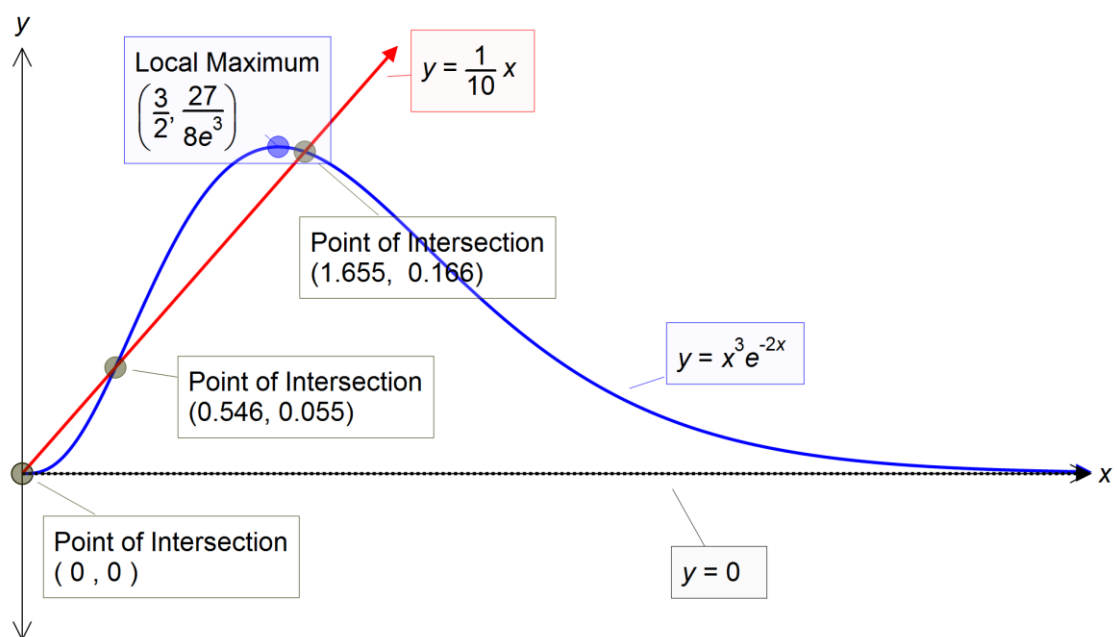
A1
1 mark

- c.
 $f'(x) = 0 \rightarrow x^2 e^{-2x} (3 - 2x) = 0$
 $x = 0, x = \frac{3}{2}$

Stationary points are $(0,0)$ and $\left(\frac{3}{2}, \frac{27}{8e^3}\right)$

M1+A2
3 marks

- d.



1 mark for turning point, 1 mark for asymptote, 1 mark for shape, 1 mark for axes intercepts
4 marks

e. See the graph above.

1 mark for sketching the line, 1 mark for the intersection points correct to 3 dp.

2 marks

f.

$$Area = \int_0^{0.546} \left(\frac{1}{10}x - x^3 e^{-2x} \right) dx + \int_{0.546}^{1.655} \left(x^3 e^{-2x} - \frac{1}{10}x \right) dx$$

$$Area = 0.0322 \text{ sq units}$$

M2+A1

3 marks

Question 2

a. $Max d(t) = 6m$

$$6 = 4 + 2\sin\left(\frac{\pi(t+2)}{6}\right)$$

$$t = 1, 13$$

At 10.00am and 10.00pm

M1+A2

3

marks

b. $Period = \frac{2\pi}{\pi/6} = 12 \text{ hours}$

A1

1 mark

c. $3.6 = 4 + 2\sin\left(\frac{\pi(t+2)}{6}\right)$

$$t = 4.38457, 9.61543, 16.3846, 21.6154$$

At 6:37pm

M2+A1

3 marks

d. $2 = 4 + 2\sin\left(\frac{\pi(t+2)}{6}\right)$

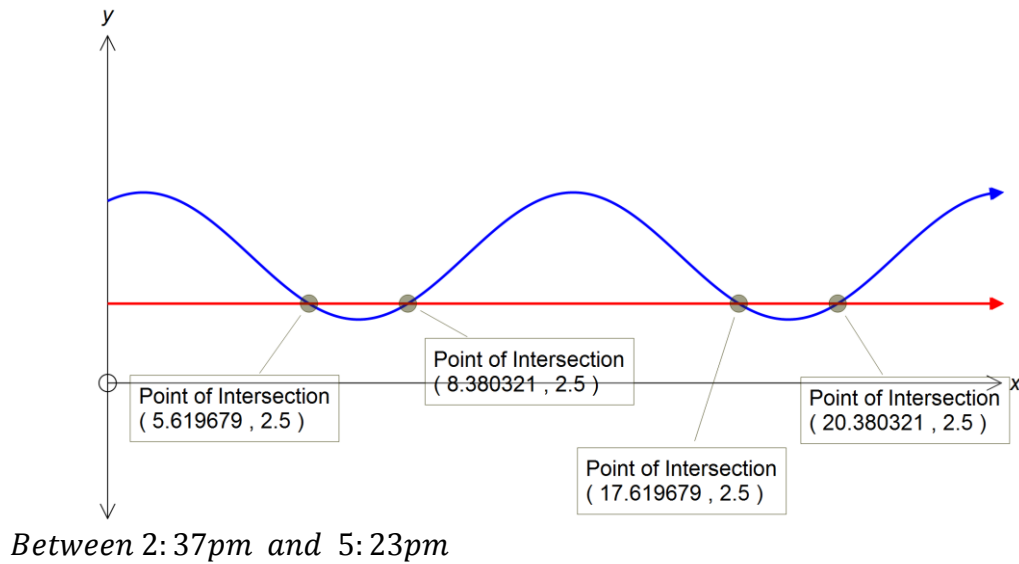
$$t = 7, 19$$

At 4pm and 4am (next day)

A2

2 marks

e. $4 + 2\sin\left(\frac{\pi(t+2)}{6}\right) \leq 2.5$



M2+A2
4 marks

Question 3

a. $(-1, \infty)$

1 mark

- b. *Translate by -1 unit along the x – axis*
Reflect the graph across the x – axis
Translate by $+2$ units along the y – axis

A3
3 marks

c. *let $y = 2 - \log_e(x + 1)$*
 $x = 2 - \log_e(y + 1)$
 $\log_e(y + 1) = 2 - x$
 $f^{-1}(x) = e^{2-x} - 1$
Domain of f^{-1} is R

M2+A2
4 marks

d. $2 - \log_e(x + 1) = x$
 $(1.2079, 1.2079)$

M1+A1

2 marks

e. $\frac{d}{dx}(2 - \log_e(x + 1)) = \frac{-1}{x+1}$
 $m = \frac{-1}{5}$

M1+A1
2 marks

f. $(4, 2 - \log_e 5)$
 $y - (2 - \log_e 5) = 5(x - 4)$
 $y = 5x - 18 - \log_e 5$

M2+A1
3 marks

Question 4

a. $0.85^5 = 0.4437$

M1+A1
2 marks

b. $C(5,1) \times 0.15 \times 0.85^4 = 0.3915$

M1+A1
2 marks

c. $1 - C(5,0) \times 0.15^0 \times 0.85^5 = 0.5563$

M1+A1
2 marks

d. $\text{binomcdf}(110, 0.15, 15, 20) = 0.5520$

M2+A1
3 marks

e. $\Pr(X > 21000) = \text{normcdf}(21000, \infty, 20700, 2915) = 0.4590$

M2+A1
3 marks

f. $\Pr(18000 < X < 25000) = \text{normcdf}(18000, 25000, 20700, 2915) = 0.7528$

M1+A1
2 marks