

2014 Trial Examination

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 17 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 –Multiple-choice questions**Instructions for Section 1**

Answer all questions on the answer sheet provided for multiple choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The function with the rule $f(x) = -\frac{1}{2}\tan\left(\frac{\pi}{3}x\right)$ has period:

- A. $\frac{1}{3}$
- B. 3
- C. 3π
- D. $\frac{\pi}{3}$
- E. 6

Question 2

If the function $f: (-\infty, 3) \rightarrow R$, $f(x) = \log_e(3 - x)$ and $g: [-3, \infty) \rightarrow R$, $g(x) = -\sqrt{x + 3}$ then the maximal domain of the function $f(x) + g(x)$ is:

- A. $[-3, 3)$
- B. $(-3, 3)$
- C. $(-\infty, 3)$
- D. $[-3, \infty)$
- E. $(-\infty, \infty)$

Question 3

The square of the distance between the two points $(a, -b)$ and $(2a, 2b)$ is given by:

- A. $\sqrt{9a^2 + b^2}$
- B. $\sqrt{a^2 + 9b^2}$
- C. $a^2 + 9b^2$
- D. $9a^2 + b^2$
- E. $9a^2 - b^2$

Question 4

If the equation $x^3 + ax^2 + 41x + 56 = 0$ has a solution $x = -1$, then the value of a is:

- A. 14
- B. ± 14
- C. 16
- D. -16
- E. -14

Question 5

The inverse of the function $f: R \rightarrow R$, $f(x) = 3 - e^x$ is given by:

- A. $f^{-1}: R \rightarrow R, f^{-1}(x) = \log_e(x - 3)$
- B. $f^{-1}: R \rightarrow R, f^{-1}(x) = \log_e(3 - x)$
- C. $f^{-1}: (3, \infty) \rightarrow R, f^{-1}(x) = \log_e(x - 3)$
- D. $f^{-1}: (-\infty, 3) \rightarrow R, f^{-1}(x) = \log_e(3 - x)$
- E. $f^{-1}: R \rightarrow R, f^{-1}(x) = -\log_e(x - 3)$

Question 6

The average value of the function $f(x) = 2\sin\left(x - \frac{3\pi}{4}\right)$ over the interval $\left[\frac{1}{2}, 3\right]$ is closest to:

- A. -2.16
- B. 2.16
- C. -0.87
- D. 0.87
- E. -0.22

SECTION 1 – continued
TURN OVER

Question 7

For independent events A and B , $\Pr(A \cap B) = p$, $\Pr(A' \cap B') = \Pr(A \cap B') = p + \frac{1}{4}$
then the value of p is:

A. 0

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{2}$

E. $\frac{3}{8}$

Question 8

If the tangent to the graph of $f(x) = 2e^{x+2}$ at $x = a$ passes through the origin, then a equals:

A. 1

B. 0

C. -2

D. 2

E. 3

Question 9

The graph of $y = kx^2 - 5x$ intersects the graph of $x - y - 3 = 0$ at one point. The value of k is:

A. $\frac{1}{3}$

B. -3

C. 3

D. $\frac{4}{3}$

E. $\frac{3}{4}$

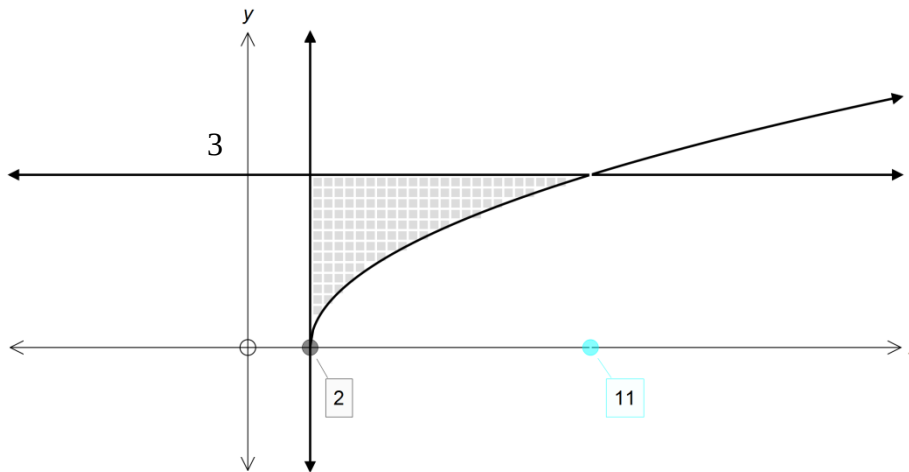
Question 10

Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function. The derivative of $f(\cos(g(x)))$ is:

- A. $f'(\cos(g(x))) \times -\sin(g(x))$
- B. $f'(\cos(g(x))) \times -\sin(g'(x))$
- C. $f'(-\sin(g'(x)))$
- D. $f'(\cos(g(x))) \times -\sin(g(x)) \times g'(x)$
- E. $f'(\cos(g(x)))$

Question 11

The graph of the function $f: [2, 11] \rightarrow \mathbb{R}$, $f(x) = \sqrt{x-2}$ is shown below:



Which one of the following definite integrals could be used to find the area of the shaded region?

- A. $\int_2^{11} \sqrt{x-2} \, dx$
- B. $27 - \int_2^{11} \sqrt{x-2} \, dx$
- C. $33 - \int_2^{11} \sqrt{x-2} \, dx$
- D. $27 - \int_0^3 (x^2 + 2) \, dx$
- E. $33 - \int_0^3 (x^2 + 2) \, dx$

Question 12

If $g(x) = x^3 + 1$ and $f(x) = \sqrt{2 - x}$ then the largest possible range of g for which $f(g(x))$ is defined is:

- A. $(-\infty, 2]$
- B. $[-2, \infty)$
- C. $(-1, \infty)$
- D. R^+
- E. $[0, 2]$

Question 13

A transformation $T: R^2 \rightarrow R^2$, $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ maps the graph of a function f to the graph of $y = (x - 1)^2$, $x \in R$. The rule of f is:

- A. $y = (x - 3)^2 - 1$
- B. $y = -(x - 3)^2 - 1$
- C. $y = (x - 3)^2 + 1$
- D. $y = -(x - 3)^2 + 1$
- E. $y = x^2 - 4$

Question 14

A random variable X is normally distributed with mean 120 and standard deviation of σ . If $\Pr(X < 90) = \frac{3}{40}$, the value of σ is:

- A. 7
- B. 13
- C. 17
- D. 18
- E. 21

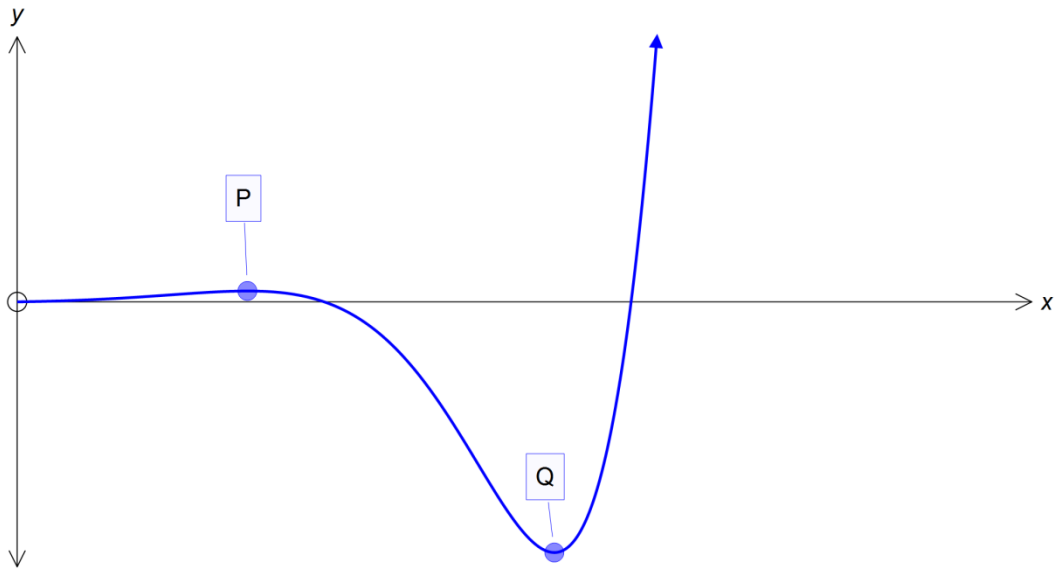
Question 15

Which of the following functions satisfies $f(2x) = 2f(x)$ for all values of x ?

- A. $f(x) = 2x^2$
- B. $f(x) = \sqrt{2x}$
- C. $f(x) = 2x$
- D. $f(x) = x + 2$
- E. $f(x) = \frac{1}{2} \log_e x$

Question 16

Part of the graph of the function $f: (0, \infty) \rightarrow \mathbb{R}, f(x) = 5e^x \sin(x)$ is shown in the diagram below.



The first two turning points are labelled as P and Q . The x -coordinate of the point Q is:

- A. $\frac{\pi}{4}$
- B. $\frac{7\pi}{4}$
- C. $\frac{9\pi}{4}$
- D. $\frac{5\pi}{6}$
- E. π

SECTION 1 – continued
TURN OVER

Question 17

If $f'(x) = (x + a)^2$, where a is a real constant, and $f'(1) = 0$ then $f(x)$ is equal to:

- A. $(x - 1)^3$
- B. $\frac{(x)^3}{3} + 1$
- C. $\frac{(x+1)^3}{3}$
- D. $\frac{(x+1)^3}{3} - 1$
- E. $\frac{(x-1)^3}{3}$

Question 18

Using Euler's rule for approximation, the best approximation for $\sqrt{109}$ is given by:

- A. $f(10) + 9f'(10)$
- B. $f(100) + 9f'(100)$
- C. $f(121) - 12f'(121)$
- D. $f(11) - 12f'(11)$
- E. $f(100 - 9f'(100))$

Question 19

Let $f: R \rightarrow R$ be a function such that $f'(3) = 0$ and $f'(x) > 0$ when $x > 3$, and $f'(x) < 0$, when $x < 3$. The graph of f at $x = 3$ has a:

- A. *local minimum*
- B. *local maximum*
- C. *stationary point of inflection*
- D. *point of discontinuity*
- E. *gradient 3*

Question 20

The probability of hitting a target with any particular arrow is 0.8. If 5 arrows are shot, the probability that 3 targets are missed can be best represented by:

- A. $(0.2)^3$
- B. $C(5,3)(0.8)^3(0.2)^2$
- C. $C(5,3)(0.2)^3(0.8)^2$
- D. $(0.2)^3(0.8)^2$
- E. $(0.8)^3(0.2)$

Question 21

Using the right rectangle approximation with rectangles of width 1, the area bounded by the curve $y = \frac{1}{x}$, the x -axis and the lines $x = 2$ and $x = 5$ is approximated by:

- A. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$
- B. $2\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)$
- C. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
- D. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$
- E. $\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

Question 22

If the average rate of change of the function $f(x) = 2x^2 + ax - 3$ over the interval $[1, 3]$ is 7, the value of a is:

- A. -1
- B. 1
- C. 15
- D. 3
- E. 0

END OF SECTION 1
TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (16 marks)

Consider the function $f(x) = x^3 e^{-2x}$.

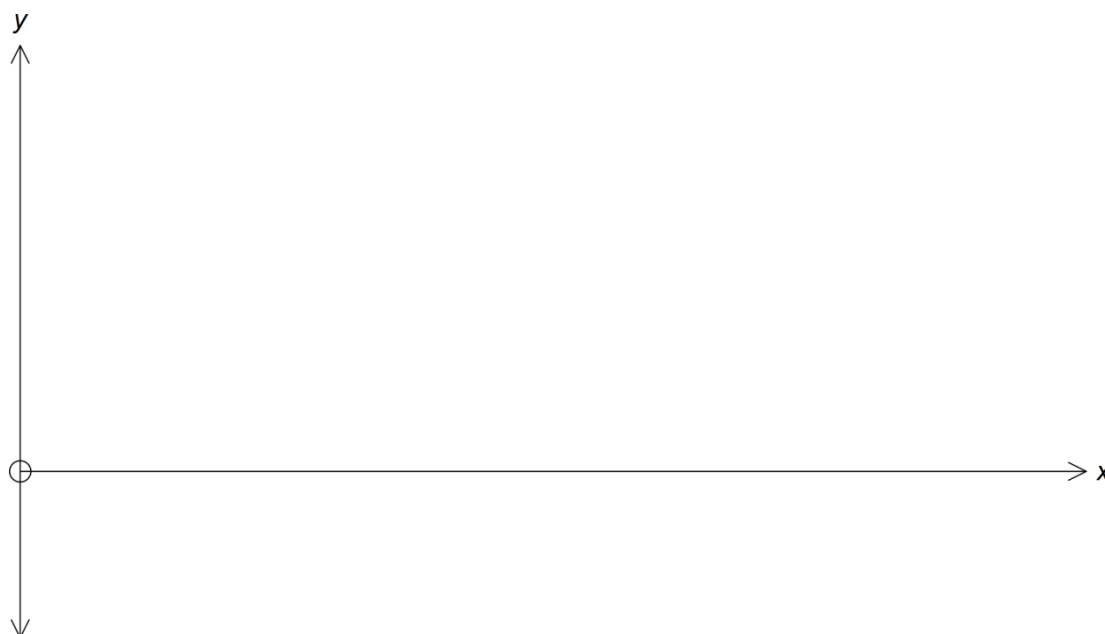
- a.** Find the values of a and b so that $f'(x) = x^a e^{-2x} (3 + bx)$. 3 marks

- b.** Find the coordinates of the axes intercepts, if any. 1 mark

- c. Find the stationary points of $f(x) = x^3 e^{-2x}$.

3 marks

- d. Sketch the graph of $f(x)$ clearly labelling all axes intercepts and turning point(s). 4 marks



- e. On the axes above, sketch the line $y = \frac{1}{10}x$, labelling the point(s) of intersection of the two graphs correct to three decimal places. 2 marks

SECTION 2 – Question 1 - continued
TURN OVER

- f. Find the area, correct to 4 decimal places, between the curve $y = f(x)$ and the line $y = \frac{1}{10}x$. 3 marks

Question 2 (13 marks)

The distance, d metres, of water from a fixed point O on the sand at any time t hours after 9.00am is modelled by the equation

$$d(t) = 4 + 2\sin\left(\frac{\pi(t+2)}{6}\right)$$

- a. State the maximum distance of water from the fixed point and the time when it occurs in the first 24 hours. 3 marks

- b. State the period of the function $d(t)$. 1 mark

- c. When will the distance of the water be 3.6m from O the second time in the first 24 hours? 3 marks

- d. At what time(s) is the water closest to the fixed point on the sand, in the first 24 hours? 2 marks

Children are allowed to play on the sand when the water is 2.5 m or less from the fixed point O .

- e. At what time(s) between 9am and 6pm are the children likely to play? 4 marks

SECTION 2 – continued
TURN OVER

Question 3 (15 marks)

Let $f: D \rightarrow \mathbb{R}$, $f(x) = 2 - \log_e(x + 1)$ and D is the largest possible domain for which f is defined.

- a.** Find D . 1 mark

- b.** Describe the sequence of transformations which when applied to the graph of $y = \log_e x$, produces the graph of $f(x)$. 3 marks

- c.** Find the rule for f^{-1} , the inverse of f . State the domain of f^{-1} . 4 marks

- d.** Find the point of intersection of f and f^{-1} . 2 marks

- e.** Use calculus to find the exact value of the gradient of the graph of f when $x = 4$. 2 marks

- f. Find the equation of the normal to the graph of $y = f(x)$ when $x = 4$. 3 marks

Question 4 (14 marks)

The probability of a student getting into a TAFE institution is 0.15. Five students of the same school submit an application to gain entry into TAFE, correct to 4 decimal places.

- a. Find the probability that none of the five gain entry into TAFE. 2 marks

- b. Find the probability that only one of the five gain entry into TAFE. 2 marks

- c. Find the probability that at least one of the five gain entry into TAFE. 2 marks

TURN OVER

- d.** If 110 candidates were applying for TAFE, find the probability of between 15 to 20 (inclusive) students gaining an entry into TAFE. 3 marks

Over the past five years, the number of students gaining entry into TAFE is normally distributed with mean 20 700 and standard deviation 2915.

- e.** Find the probability of the number of TAFE students exceeding 21 000. 3 marks

- f.** Find the probability that a particular year will have between 18 000 and 25 000 TAFE students. 2 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name: _____

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E