

**‘2016 Examination Package’ -
Trial Examination 4 of 5**

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1

(TSSM’s 2014 trial exam updated for the current study design)

Reading time: 15 minutes

Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 9 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (6 marks)

- a.** If $f(x) = 2x^2 \log_e(3x)$, find $f'(x)$. 2 marks

- b.** Find $f'\left(\frac{1}{3}\right)$. 1 mark

- c.** Find the value(s) of x for which $f'(x) = 0$. 3 marks

Question 2 (6 marks)

- a.** Find the derivative of $(2x - 3)\sin(x)$. 2 marks

- b.** Hence, find an anti-derivative of $xcos(x)$. 4 marks

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

TURN OVER

Question 3 (2 marks)

Solve the equation $2 \cos\left(\frac{x}{3}\right) + 1 = 0$, $x \in [3\pi, 6\pi]$

Question 4 (9 marks)

a. Find the positive value of k for which $4e^{2x} - ke^x + 9 = 0$ has a unique solution.

2 marks

b. Find the unique solution for the equation $4e^{2x} - ke^x + 9 = 0$ for the positive value of k found in part **a**.

3 marks

Question 4 – continued

Let $g(x) = 2e^x - 3$

- c. Find the rule for the inverse $g^{-1}(x)$. 2 marks

- d. State the domain and range of $g^{-1}(x)$. 2 marks

TURN OVER

Question 5 (4 marks)

Let $f: R \rightarrow R$, $f(x) = 4x + 9$.

The average value of f on the interval $[0, a]$ is $\frac{1}{a}$, where a is a real constant.

Find all possible values of a .

[illegible]

Question 6 (6 marks)

The probability distribution of a discrete random variable, X , is given by the table below.

x	0	1	2	3	4
$\Pr(X = x)$	$\frac{1}{2k}$	$\frac{k}{10}$	$\frac{k}{10}$	$\frac{3}{10k}$	$\frac{2}{5k}$

- a. Show that $k = 3$ or $k = 2$.

3 marks

Let $k = 3$.

- b. Calculate $E(X)$.

2 marks

- c. Find $\Pr(x \leq E(X))$.

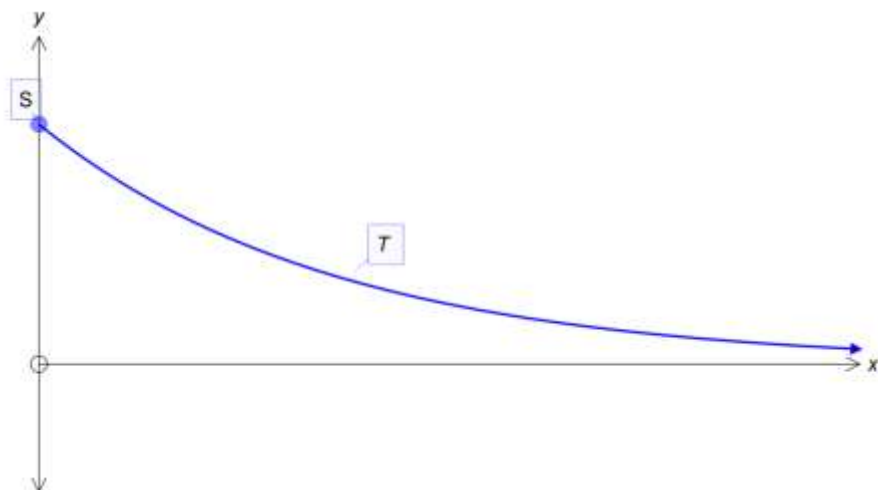
1 mark

TURN OVER

Question 7 (4 marks)

Let $f: [0, \infty) \rightarrow \mathbb{R}$, $f(x) = 2e^{-\frac{x}{3}}$

Let S be the point on the graph of f on the y -axis and let T be the point on the graph of f with the y -coordinate $\frac{1}{2}$.



Find the area of the region bounded by the graph of f and the line segment ST .

Question 8 (3 marks)

A continuous random variable, X , has a probability density function

$$f(x) = \begin{cases} \frac{1}{2} \sin(x), & 0 \leq x \leq \pi \\ 0, & \text{elsewhere} \end{cases}$$

a. Find the median of X .

2 marks

b. Find the mode of X .

1 mark

END OF QUESTION AND ANSWER BOOK