

SACRED HEART GIRLS' COLLEGE

OAKLEIGH



Mathematical Methods CAS 2014

Unit 3 SAC 1: TEST

Part B

Name: SOLUTIONS (GARKEL)

Teacher (please circle): Ms Gates

Mr Smith

Ms Garkel

Part B: 10 multiple choice questions and 2 extended response questions.

CAS and a bound reference of summary notes permitted

Writing Time: 40 minutes

Marks: 25

MULTIPLE CHOICE**Instructions:**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for that question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Only the answers on the Answer Sheet will be marked.

Question 1

The maximum domain for the function $f(x) = 1 - \sqrt{25 - x^2}$ is

- A. $(-5, 5)$
- B. $(-\infty, -5] \cup [5, \infty)$
- C. $[-4, 6]$
- ☒ D. $[-5, 5]$
- E. R

Question 2

Which of the following does **not** have an inverse function?

- A. $f: [2, 4) \rightarrow R, f(x) = \sqrt{x - 2}$
- ☒ B. $g: R \setminus \{0\} \rightarrow R, g(x) = \frac{1}{x^2}$
- C. $h: R^+ \rightarrow R, h(x) = x^3$
- D. $k: (-\infty, 0] \rightarrow R, k(x) = x^2 + 1$
- E. $m: R^+ \rightarrow R, m(x) = \frac{1}{x+3}$

Question 3

The simultaneous linear equations

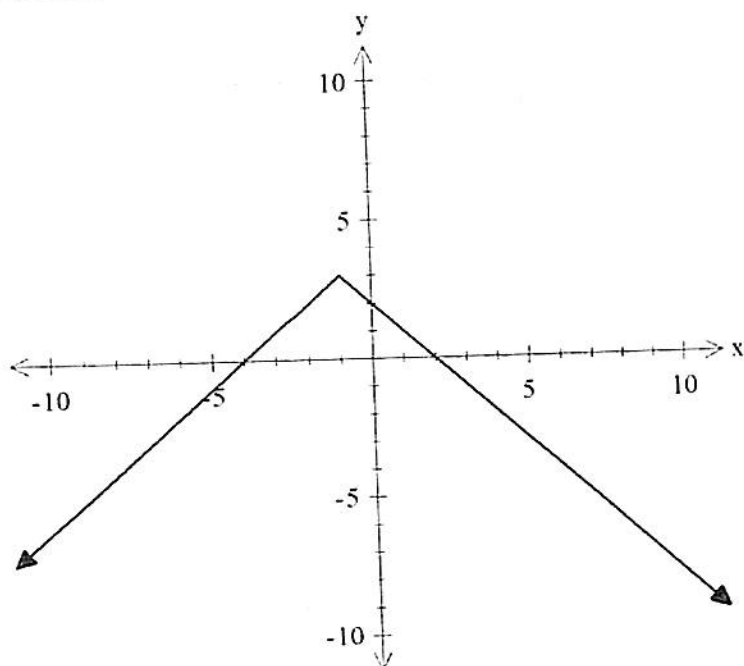
$$4x - ky = 16$$

$$(k - 1)x - 3y = 3k$$

where k is a real constant, have infinitely many solutions for

- A. $k \in \{-3, 4\}$
- B. $k \in R \setminus \{-3, 4\}$
- ☒ C. $k = 4$
- D. $k = -3$
- E. $k \in R \setminus \{0\}$

Question 4



The graph above is best represented by the equation

- A. $y = |x + 1| + 3$
- B. $y = |1 - x| + 3$
- ☒ C. $y = 3 - |x + 1|$
- D. $y = 3 - |1 - x|$
- E. $y = |x + 1| - 3$

Question 5

Let $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3$. Which of the following is **not** true?

- A. $f(x) + f(-x) = 0$
- B. $f(xy) = f(x)f(y)$
- C. $f(2x) = 8f(x)$
- ☒ D. $f(x - y) = f(x) - f(y)$
- E. $f^{-1}(x) = x^{\frac{1}{3}}$

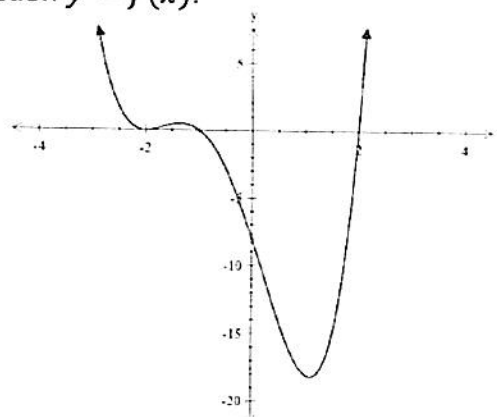
Question 6

The linear function $f: D \rightarrow \mathbb{R}, f(x) = 6 - 2x$ has range $[-4, 12]$. The domain D is

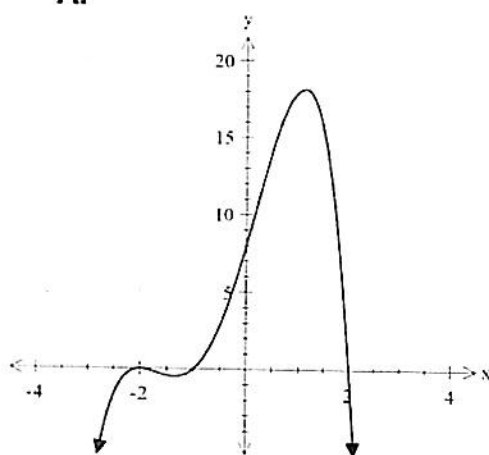
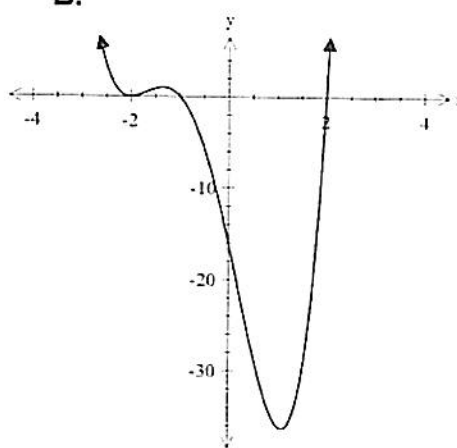
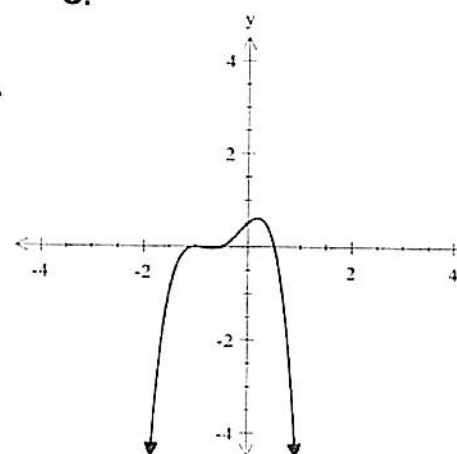
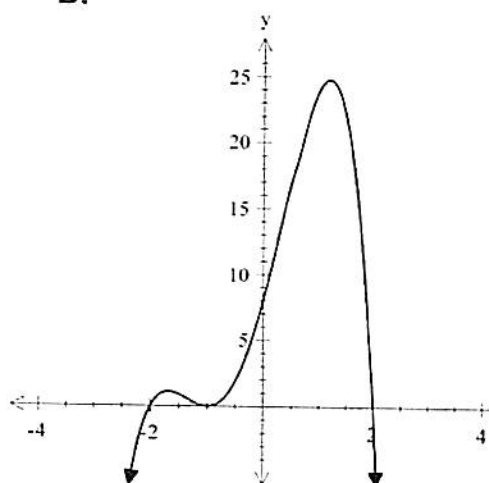
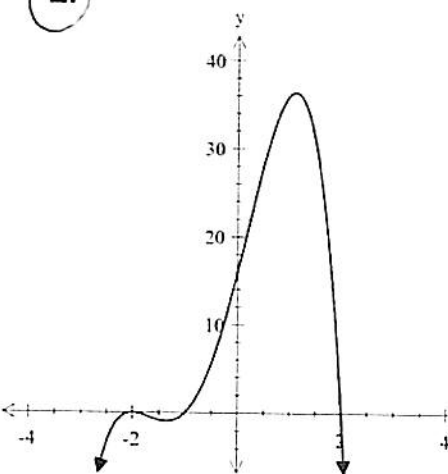
- ☒ A. $[-3, 5]$
- B. $[-5, 3]$
- C. $\mathbb{R}$
- D. $[-14, 18]$

Question 7

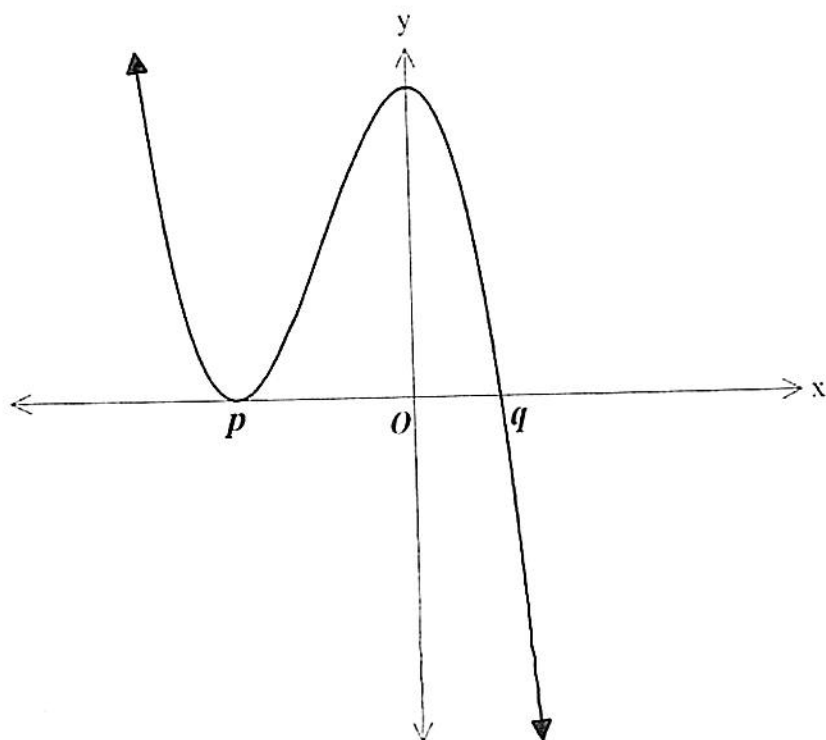
The graph below shows the function $y = f(x)$.



Which of the following is the graph of $y = -2f(x)$?

A.**B.****C.****D.****E.**

Question 8



The graph above could be that of the function f with rule

- A. $y = (x + p)^2(q - x)$
- B. $y = (x + p)^2(x - q)$
- ☒ C. $y = (x - p)^2(q - x)$
- D. $y = -(x + p)^2(x + q)$
- E. $y = (x - p)^2(x - q)$

Question 9

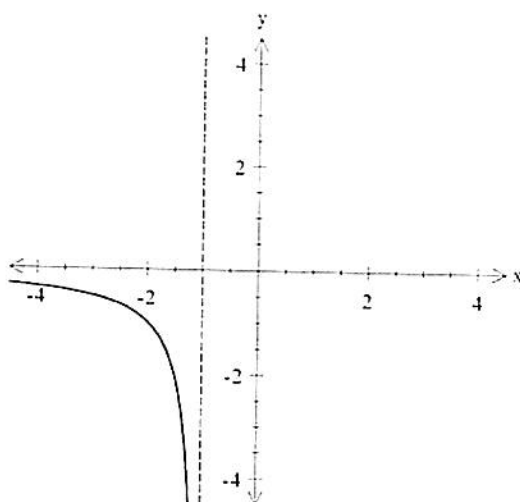
The graph of $y = f(x)$ takes the following transformations (1) to (3) in order:

- (1) Dilation by a factor of 2 from the x - axis
- (2) Reflection in the y - axis
- (3) Translation 3 units to the right.

If the transformed function is in the form $y = af(n(x + b)) + c$, then the values of a , n , b and c , respectively are:

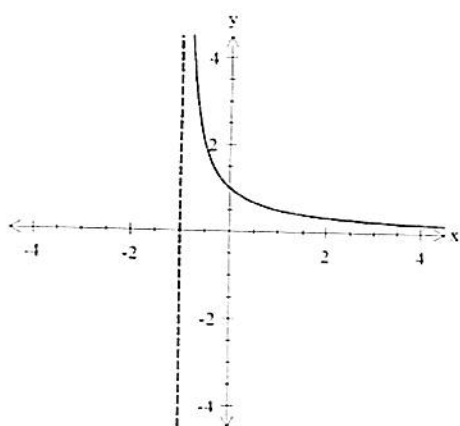
- A. 2, -1, 3, 0
- B. 2, -1, 0, 3
- ☒ C. 2, -1, -3, 0
- D. $-1, \frac{1}{2}, 3, 0$
- E. $1, -\frac{1}{2}, 0, 3$

Question 10

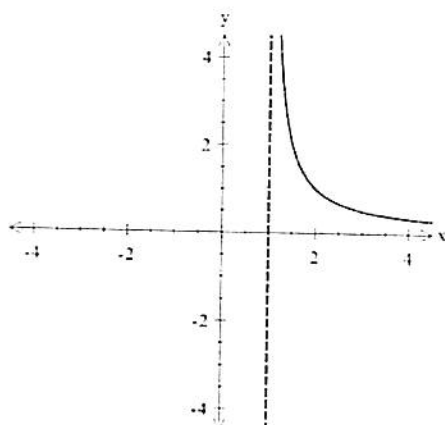


For the graph of $y = f(x)$ shown above, which of the following is most likely to be the graph of $y = f^{-1}(x)$?

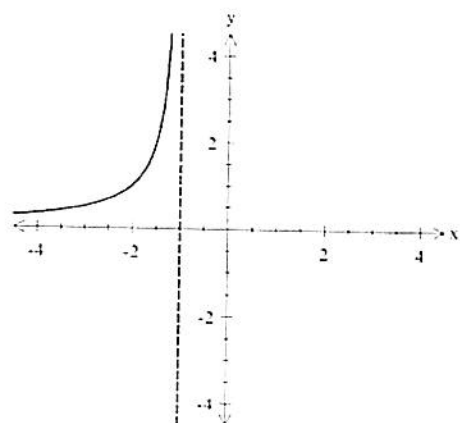
A.



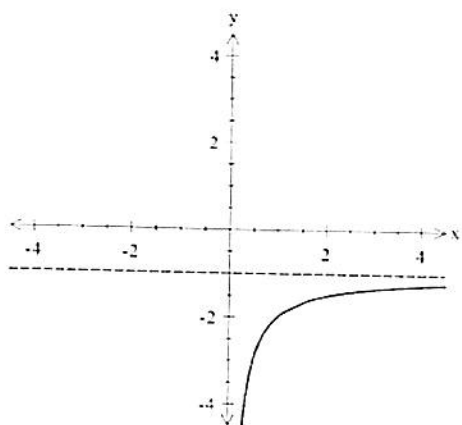
B.



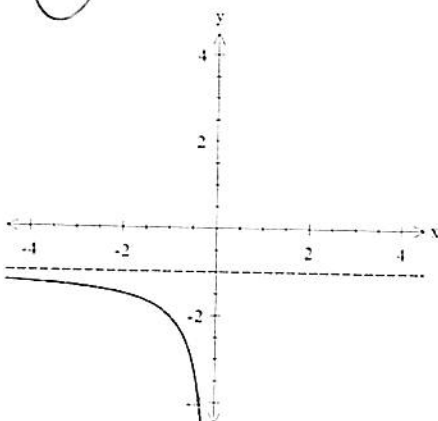
C.



D.



E.



EXTENDED RESPONSE

Instructions:

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

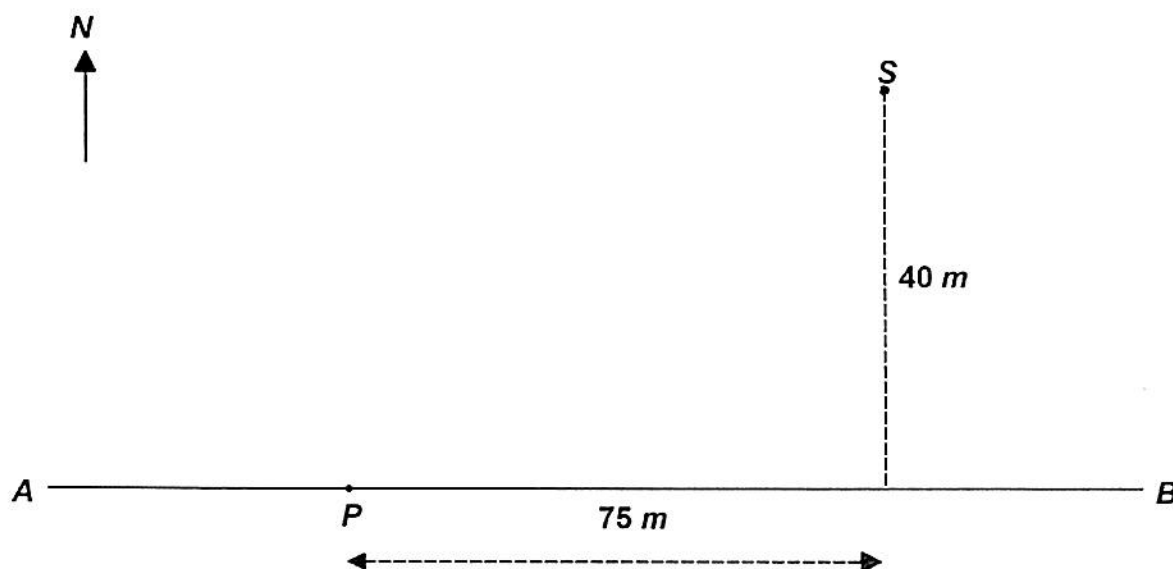
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this test are **not** drawn to scale.

Question 1 (7 marks)

Gunamatta Beach lifeguard Dorothy Smart, situated at point P on the beach, spots a swimmer in difficulty in the water at point S, 75 metres along the beach and 40 metres to the north in the water. The line AB represents the shoreline; to the north is water.

Dorothy can travel at 5m/second along the beach and 3m/second in the water.



- a) How long will it take Dorothy to reach the swimmer if she travels due east 75m along the beach and then straight to the swimmer 40m in the water? 1 mark

$$t = \frac{d}{s} = \frac{75}{5} + \frac{40}{3} = \frac{85}{3} \text{ seconds}$$

- b) How long will it take Dorothy to reach the swimmer if she swims straight to the swimmer from point P to point S? 1 mark

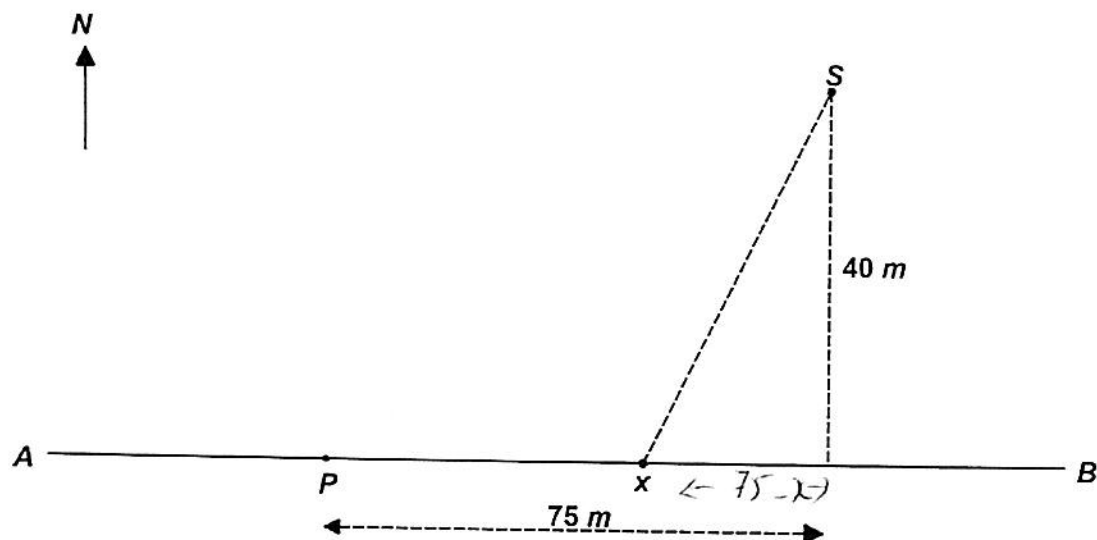
$$t = \frac{\sqrt{40^2 + 75^2}}{3} = \frac{85}{3} \text{ seconds}$$

- c) Correct to 1 decimal place, how long will it take Dorothy to reach the swimmer if she travels due east 25m along the beach and then straight to the swimmer? 1 mark

$$t = \frac{25}{5} + \frac{\sqrt{50^2 + 40^2}}{3}$$

$$t = 26.3 \text{ seconds}$$

Dorothy decides to run along the beach to a point X which is x metres from point P, then swim directly to the swimmer.



- d) Show that the time taken for Dorothy to go from point P to point X and then to point S is given by the rule: 2 marks

$$t(x) = \frac{x}{5} + \frac{\sqrt{x^2 - 150x + 7225}}{3}$$

$$t(x) = \frac{x}{5} + \frac{\sqrt{(75-x)^2 + 40^2}}{3}$$

$$= \frac{x}{5} + \frac{\sqrt{x^2 - 150x + 5625 + 1600}}{3}$$

$$= \frac{x}{5} + \frac{\sqrt{x^2 - 150x + 7225}}{3}$$

shown as required

- e) Find the minimum time for Dorothy to reach the swimmer. 1 mark

$$\frac{77}{3} = 25 \frac{2}{3} \text{ seconds}$$

- f) How far should Dorothy run along the beach to ensure she reaches the swimmer in the shortest possible time? 1 mark

$$41.5 \text{ m}$$

Question 2 (8 marks)

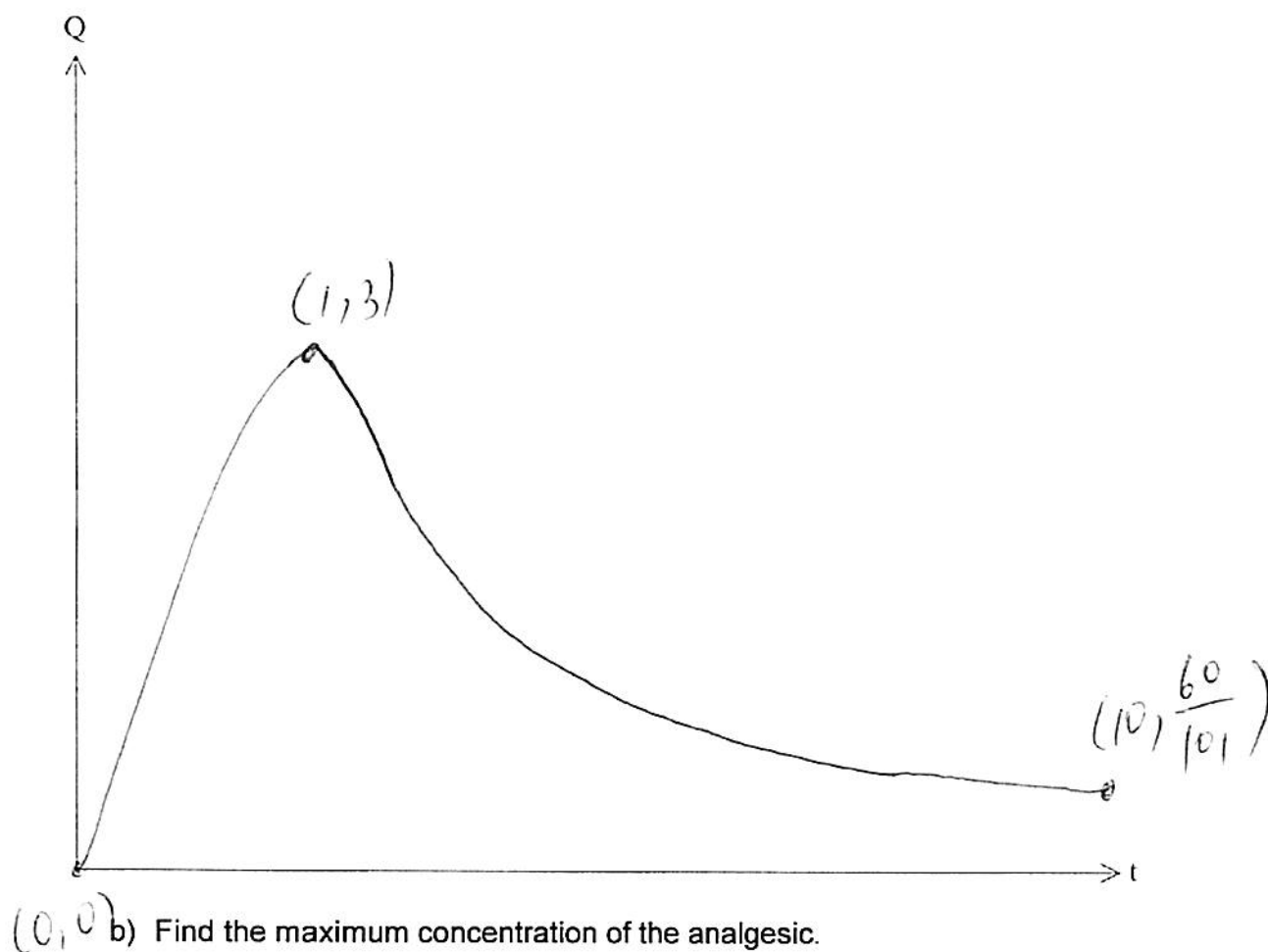
Dorothy is treating a bottlenose dolphin injured on the rocky reef at the end of the beach. She administers a dose of an analgesic (pain reliever) at 12 pm. The concentration, Q units/cm³, of analgesic in the dolphin's bloodstream t hours after it is administered is modelled by the function

$$Q: [0, 10] \rightarrow \mathbb{R}, Q(t) = \frac{6t}{t^2 + 1}$$

where t is time in hours after 12 pm.

- a) Sketch the graph of Q on the axes below. Label any stationary points and endpoints with coordinates.

2 marks



- b) Find the maximum concentration of the analgesic.

1 mark

3 units/cm³

- c) How long after the dose is administered does the maximum occur?

1 mark

1 hour

The analgesic will provide pain relief when the concentration is above 1.25 units/cm^3 .

- d) For what length of time, in hours correct to 2 decimal places, will the dolphin experience relief from the pain?

2 marks

$$4.58... - 0.21... = 4.36 \text{ hours}$$

$$\text{solve } 1.25 = \frac{6t}{t^2 + 1}$$

and find difference b/w values

Dorothy changes the dose to give the dolphin pain relief for twice as long.

- e) Find the rule for such a function. Give your answer in the form $f(t) = \frac{at}{t^2 + b}$.

2 marks

dilation from y-axis by factor 2

$$g\left(\frac{t}{2}\right) = \frac{6\left(\frac{t}{2}\right)}{\left(\frac{t}{2}\right)^2 + 1}$$

$$= \frac{3t}{\frac{t^2 + 4}{4}}$$

$$= \frac{12t}{t^2 + 4}$$