

The Mathematical Association of Victoria

Trial Exam 2014

MATHEMATICAL METHODS (CAS)

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 9 pages, a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1 (3 marks)

For what value(s) of m is/are there no real solutions to the following system of equations?

$$mx + 2y = 6$$

$$x + (m - 1)y = -3$$

[illegible]

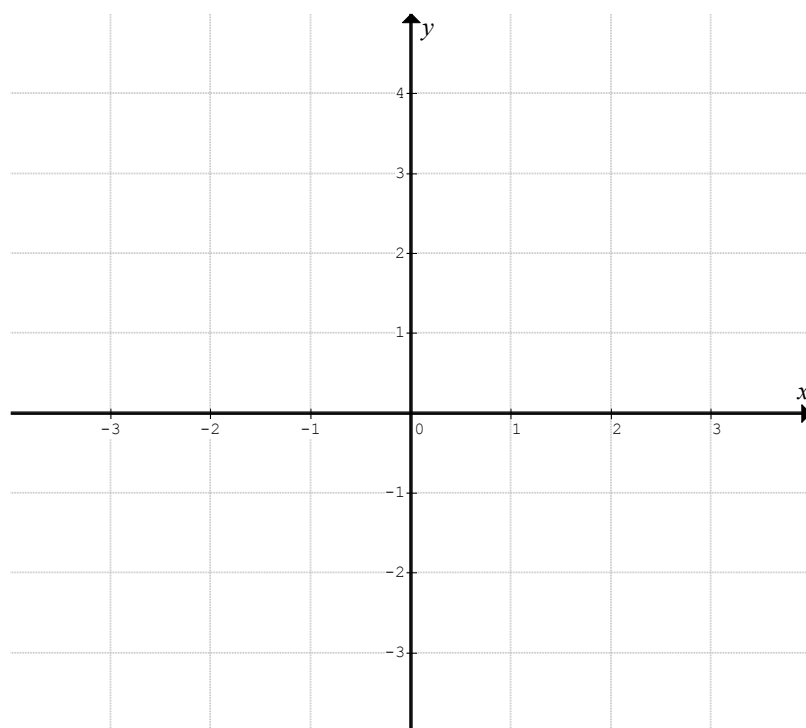
Question 2 (3 marks)

Solve the equation $2\sin^2(2x) - 1 = 0$ for x where $x \in [0, \pi]$.

Question 3 (3 marks)Solve $2\log_2(2x+1) + \log_2(3) = 3$ for x .

Question 4 (4 marks)

- a. Sketch the graphs of $f : [-2, 2) \rightarrow \mathbb{R}, f(x) = |x - 1|$ and $g : (-2, 2] \rightarrow \mathbb{R}, g(x) = |x + 1|$ on the set of axes below. 2 marks
- b. Hence sketch $h(x) = f(x) + g(x)$ over its maximal domain on the same set of axes. 2 marks



Question 5 (5 marks)

Consider the function $f(x) = kx^2 \tan(2x)$, where k is a positive real constant.

- a. Find $f'(x)$.

2 marks

- b. Show that $f'\left(\frac{\pi}{8}\right) = \frac{k\pi(4+\pi)}{16}$.

1 mark

- c. If $f'\left(\frac{\pi}{8}\right) = 0.75$, find the value of k .

2 marks

Question 6 (4 marks)

Let $f : (-\infty, A] \rightarrow \mathbb{R}, f(x) = x^2 + \frac{2}{3}x + 3$.

- a. Find the maximum value of A if f^{-1} exists.

1 mark

- b. Define f^{-1} .

3 marks

Question 7 (4 marks)

- a. Show that the equation of the **normal** to the curve with equation $f(x) = e^{3x+2}$ at $x = 0$ is

$$y = -\frac{1}{3e^2}x + e^2.$$

1 mark

- b. Hence, find the area bounded by f , the normal line and the line $x = -2$.

3 marks

Question 8 (4 marks)

For events A and B , $\Pr(A) = \frac{1}{4}$ and $\Pr(A \cap B) = \frac{1}{5}$.

- a. If $\Pr(A' \cap B) = \Pr(A)$ evaluate $\Pr(B)$.

2 marks

Working Space

- b. If events A and B are independent evaluate $\Pr(A \cup B)$.

2 marks

Working Space

Question 9 (6 marks)

The probability density function for the random variable, X , is given by

$$f(x) = \begin{cases} \frac{k}{2x} & 1 \leq x \leq e^3 \\ 0 & \text{elsewhere} \end{cases}.$$

- a.** Show that $k = \frac{2}{3}$. 2 marks

- b.** Find $\Pr(X < 10 | X \geq 1)$. 2 marks

- c.** Find $E(X)$ the mean of X . 2 marks

Question 10 (4 marks)

a. Find $\frac{d}{dx}\left(\frac{5}{2x^2-1}\right)$.

1 mark

b. Hence find the value of a if $\int_1^a \left(\frac{20x}{(2x^2-1)^2} + 1 \right) dx = \frac{37}{7}$.

3 marks

END OF QUESTION AND ANSWER BOOKLET