

Year 2014

VCE

Mathematical Methods

Trial Examination 1



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9018 5376
FAX: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.com.au>

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Victorian Certificate of Education 2014

STUDENT NUMBER

Figures
Words

Letter

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MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown. Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. If $y = \frac{\cos(2x)}{3x}$, find $\frac{dy}{dx}$. 2 marks

b. Let $f(x) = \tan(\sqrt{x})$. Find $f'\left(\frac{\pi^2}{16}\right)$. 2 marks

Question 2 (4 marks)

- a. Describe in words, giving scale factors, the transformations in a suitable order, required to sketch the graph of $y = 3 - \sqrt{2 - x}$ from the graph of $y = \sqrt{x}$. 2 marks

- b. The function $f(x) = 3 - \sqrt{2 - x}$ is defined on its maximal domain. Find the inverse function f^{-1} . 2 marks

Question 3 (3 marks)

Consider the linear simultaneous equations

$$\begin{aligned} 3x - (k+2)y &= k+1 \\ kx - 5y &= 4 \end{aligned} \quad \text{where } k \text{ is a constant.}$$

- i. Find the value(s) of k , for which there is a unique solution.
- ii. Find the value(s) of k , for which there is an infinite number of solutions.

Question 4 (2 marks)

Find the general solution to the equation $\sqrt{3}\cos(2x) + \sin(2x) = 0$.

Question 5 (5 marks)

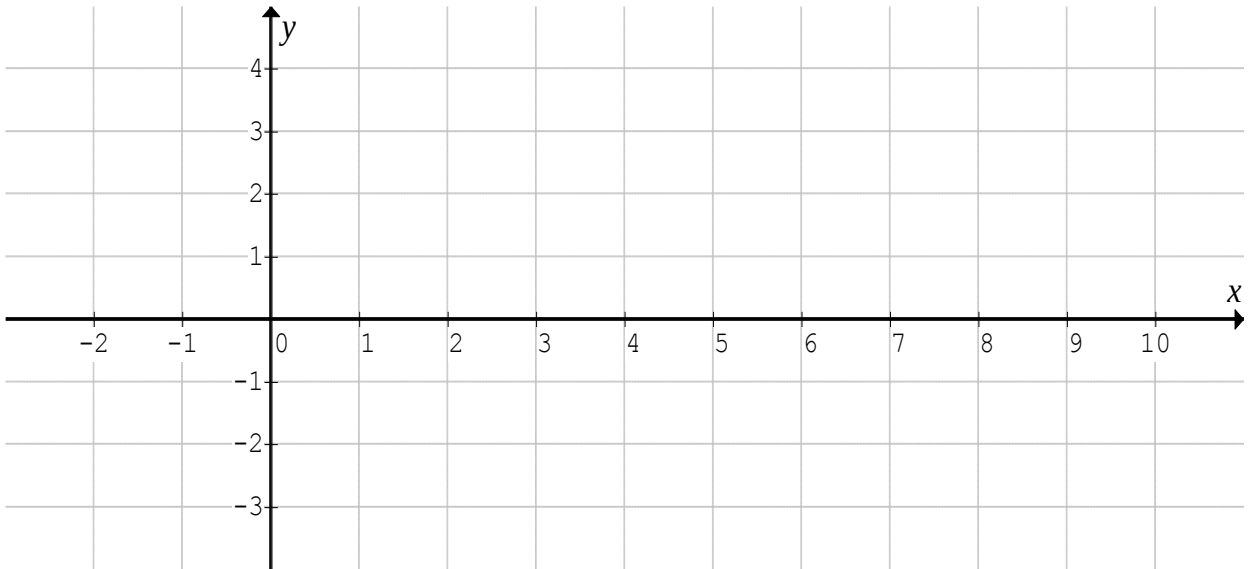
- a. Solve the equation $2\log_4(x-1) + \log_4(2) - \log_4(x) = \frac{3}{2}$ for x . 3 marks

- b. Solve the equation $8 \times 4^x + 16 \times 4^{-x} = 129$ for x . 2 marks

Question 6 (6 marks)

Consider the function $f : [0, 8] \rightarrow \mathbb{R}$, $f(x) = \left| 2 \sin\left(\frac{\pi x}{4}\right) \right| + 1$

- i. Sketch the graph of the function on the axes below, stating the coordinates of any maximum and minimum points and the endpoints. 2 marks



- ii. On the axes above, sketch the graph of $y = f'(x)$. 2 marks

- iii. Determine the average value of the function.

2 marks

Question 7 (4 marks)

- a. The probability distribution function for the continuous random variable X is given by

$$f(x) = \begin{cases} \frac{k}{\sqrt{9-2x}} & \text{for } 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of k .

2 marks

- b. If X is a normally distributed random variable with mean 20 and variance 16 and Z is the random variable with the standard normal distribution. If $\Pr(1 < Z < 2) = p$ and $\Pr(Z > 2) = q$, then express $\Pr(X < 24 | X > 12)$ in terms of p and q .

2 marks

Question 8 (4 marks)

- a. A binomial distribution of the random variable X , with three independent trials, is such that $\Pr(X=1) = \frac{p}{3}$, where p is the probability of a success on any trial and $0 < p < 1$. Determine the value of p .

2 marks

- b. A discrete random variable X has a probability distribution given by

X	0	1	2
$\Pr(X=x)$	k^2	$\frac{3k}{2}$	$\frac{k}{2}$

Find $E(X)$.

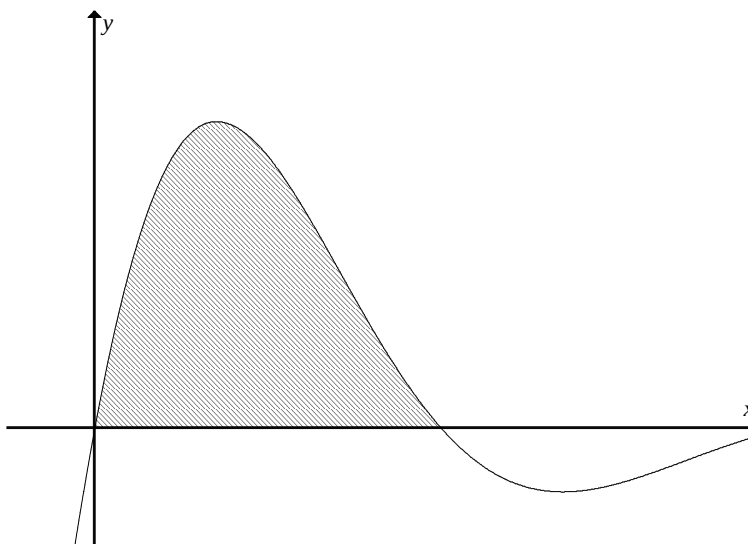
2 marks

Question 9 (4 marks)

- a.** Differentiate $e^{-x}(2\cos(2x) + \sin(2x))$ with respect to x . 1 mark

- b.** The diagram shows part of the graph of $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = e^{-x} \sin(2x)$.
Use your answer to **a.** to determine the area of the shaded region.

3 marks



MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of triangle:	$\frac{1}{2}bc \sin(A)$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h f'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$