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Mathematical Methods (CAS)

2014

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use CAS/calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Consider the line $y = mx + c$ and the parabolas $y_1 = -4x - x^2$ and $y_2 = (x - 2)^2 - 2$.

- a. Given that $y = mx + c$ is a common tangent to the two parabolas, show that $m^2 + 8m + 4(4 - c) = 0$ and $m^2 + 8m + 4(2 + c) = 0$. 2 marks

- b. Determine the equations of the common tangents to the parabolas $y_1 = -4x - x^2$ and $y_2 = (x - 2)^2 - 2$. 2 marks

Question 2

Given $f(x) = a \cos\left(\frac{x}{3}\right) + 1$, find the value(s) of k such that $f(x + k) = 2 - f(x)$ and $-3\pi \leq k \leq 3\pi$.

2 marks

Question 3

Consider $f(x) = ax^3 + bx^2 + cx + d$ and its transformation $g(x) = f(1 - 2x) = 8x^3 - 4x^2 - 2x + 1$.

- a. Sketch the graph of $y = g(x)$. Show the coordinates of the axis-intercepts. Coordinates of turning point(s) and inflection point(s) are not required. 2 marks

- b. Find the values of coefficients a , b , c and d . 3 marks

Question 4

A worker always has 4 cups of drink (tea T or coffee C) each day. If he has tea in one drink, the probability that the next cup is also tea is 0.8. If he has coffee in one drink, the probability that the next cup is also coffee is 0.6. The worker always starts a day with a cup of coffee.

- a. Find the probability that the worker drinks 2 cups of coffee in a day. 1 mark

- b. Find the probability that the worker drinks at least 2 cups of coffee in a day. 1 mark

- c. Find the expected number of cups of coffee consumed by the worker in a day. 2 marks

Question 5

Consider $f(x) = \log_{10}(9 - x^2)$.

- a. State the domain of the function. 1 mark

- b. Find the exact value of $f'(-1)$. 2 marks

- c. Estimate the area of the region bounded by $y = 0$, $y = \log_{10}(9 - x^2)$, $x = -2$ and $x = 2$ using four vertical rectangles of unit width with the top left corner of each rectangle on the curve $y = \log_{10}(9 - x^2)$. Write your answer in simplest form. 2 marks

Question 6

Find the average value of $f : [-6, 6] \rightarrow \mathbb{R}$, $f(x) = 1 - 12 \sin \frac{\pi x}{3} + \frac{1}{12} \cos \frac{\pi x}{6}$.

2 marks

Question 7

The graph of $y = f(x)$ is shown below.

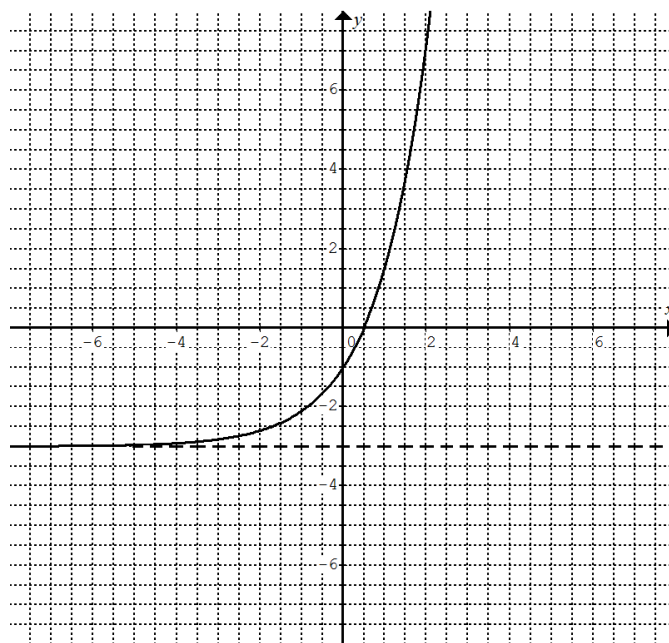
The following sequence of transformations is applied to $y = f(x)$:

Reflection in the line $y = x$

Translation in the positive x -direction by 2 units

Dilation from the y -axis by a factor of 3

Let $y = g(x)$ be the result after the sequence of transformations.



a. Sketch the graph of $y = g(x)$ on the grid shown above. Show the asymptote and its equation. Label the x -intercept with its coordinates.

3 marks

b. Express $g(x)$ in terms of f^{-1} .

2 marks

Question 8

Consider $y = |x|e^{-|x|}$ where $x \in \mathbb{R}$.

- a. Find $\frac{dy}{dx}$. Write your answer as a hybrid function.

2 marks

- b. Hence find the exact value of $\int_{-\log_e 2}^{\log_e 2} |x|e^{-|x|} dx$.

3 marks

Question 9

Solve $\frac{2^{2x} + 2^{-2x} - 2}{2^x - 2^{-x}} = 2$ for $x \in \mathbb{R}$.

3 marks

Question 10

The probability density function of random variable X is given by $f(x) = \begin{cases} 0.75 - k|x-1|, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$ where $k > 0$.

a. Show that $k = 0.5$.

1 mark

b. What is the expected value of X ?

2 marks

c. Find b such that $\Pr(X < b) = \frac{13}{16}$.

2 marks

End of exam 1