

insight™
YEAR 12 Trial Exam Paper

2014

MATHEMATICAL METHODS (CAS)

Written examination 1

STUDENT NAME:

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 15 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, show appropriate working.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (7 marks)

a. Find $\frac{d}{dx}(x \sin(2x))$.

2 marks

b. For $f(x) = e^{\sqrt{x}}$, find $f'(4)$.

2 marks

- c. The average value of the function $f : (-\infty, 5) \rightarrow \mathbb{R}$, $f(x) = \frac{1}{5-x}$ over the interval $[1, k]$ is $\frac{1}{2} \log_e 2$.
Find the value of k .

3 marks

Question 2 (2 marks)

A transformation is defined by the matrix $\begin{bmatrix} 0 & 4 \\ -3 & 0 \end{bmatrix}$. Find the equation of the image of the graph of the line with the equation $2y - 3x = 5$ under this transformation.

Question 3 (5 marks)

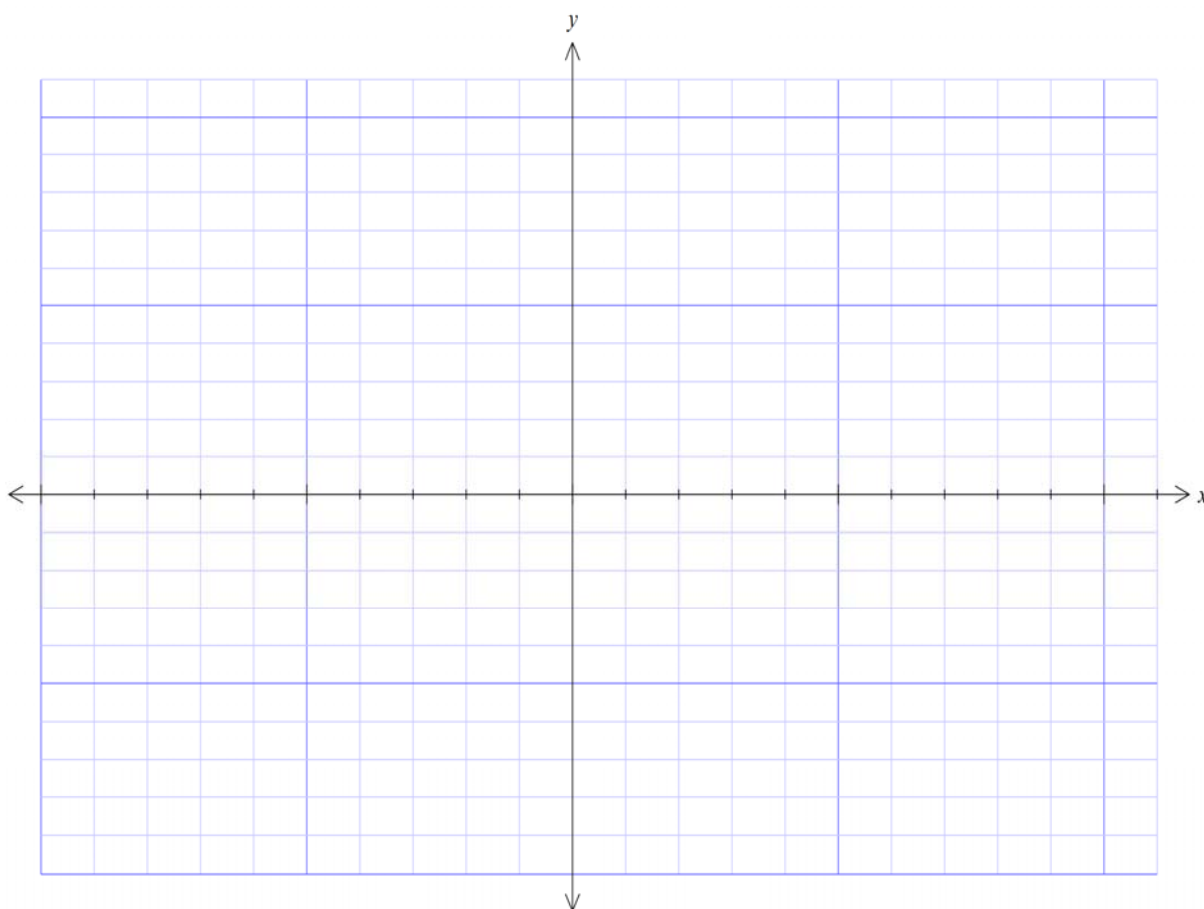
Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2 - 4x + 3$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = |x|$.

- a. Find the rule for the function $f(g(x))$.

1 mark

- b.** Sketch the graph of $y = f(g(x))$ on the axes below. Label axes intercepts and turning point(s) with their coordinates.

3 marks



- c.** State the domain for which the derivative of $f(g(x))$ is defined.

1 mark

TURN OVER

Question 4 (6 marks)

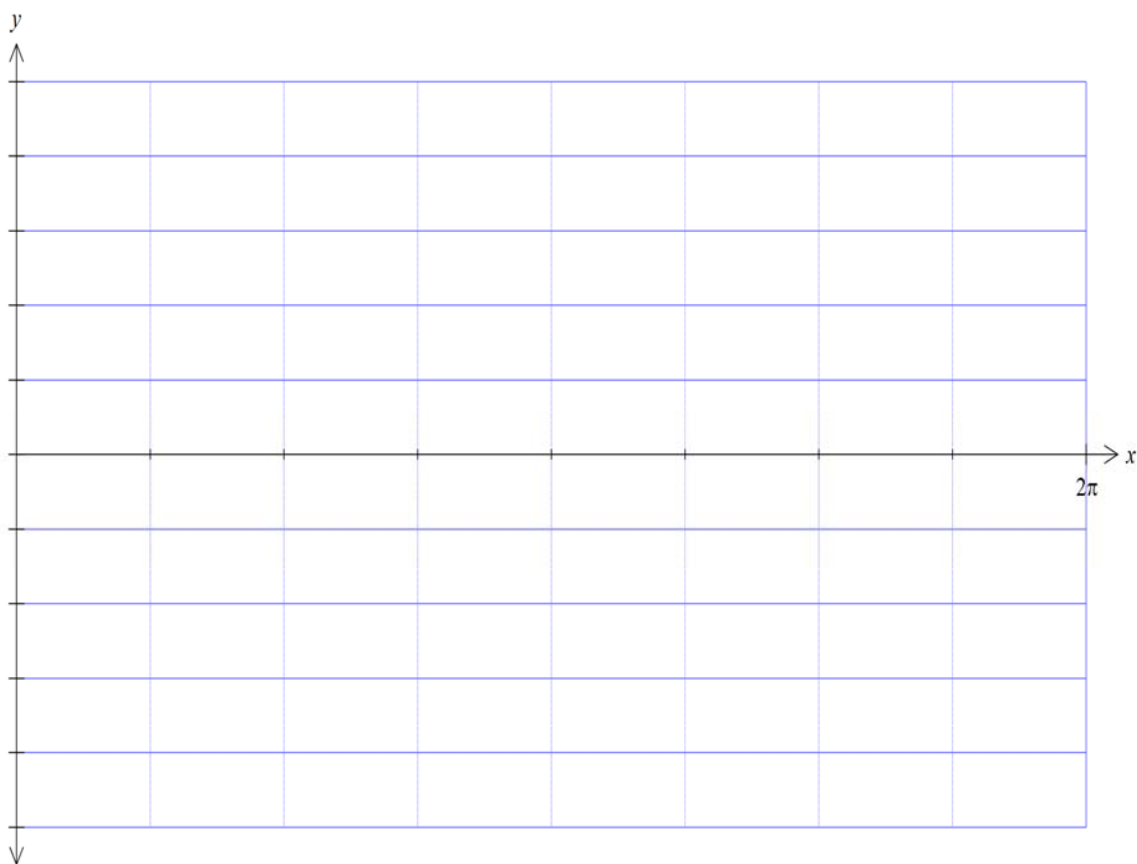
For $f : [0, 2\pi] \rightarrow \mathbb{R}$, $f(x) = -2 \sin(2x) - 1$.

- a. State the range and period of the function.

2 marks

- b. Sketch the graph of f on the axes below. Label axes intercepts and end points with coordinates.

3 marks



- c. $2\sin(2x) - 1 = p$ has four solutions over the domain $[0, 2\pi]$.

State the interval of values of p for which this will occur.

1 mark

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Question 5 (3 marks)

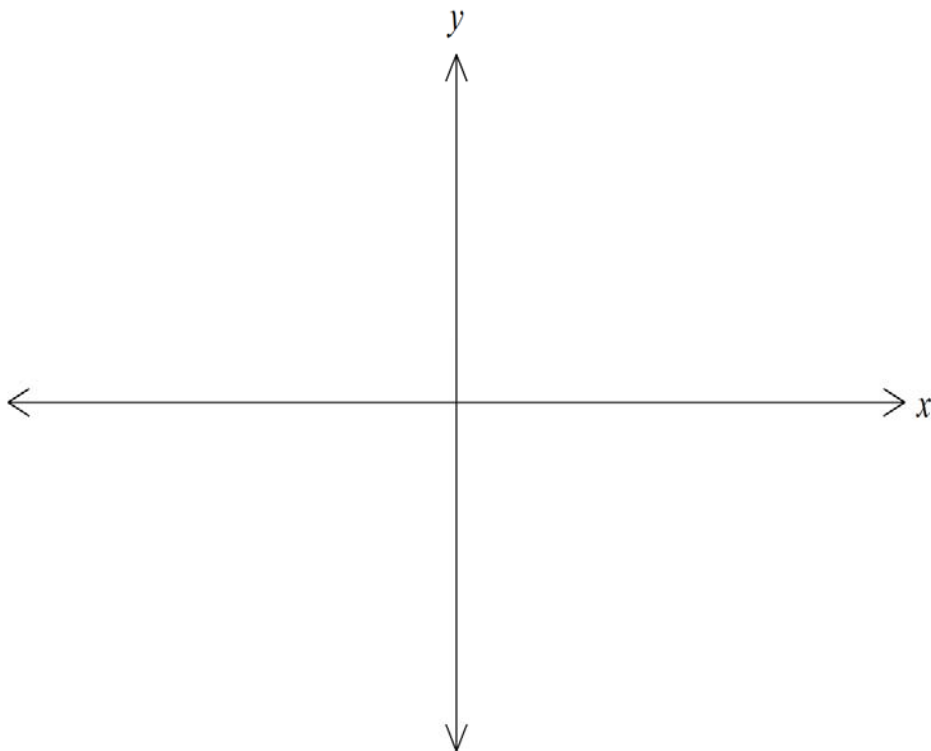
Consider the function $f : (-\infty, 4) \rightarrow \mathbb{R}$, $f(x) = 3\log_e(4 - x)$.

- a. Find the rule for the inverse function, f^{-1} .

2 marks

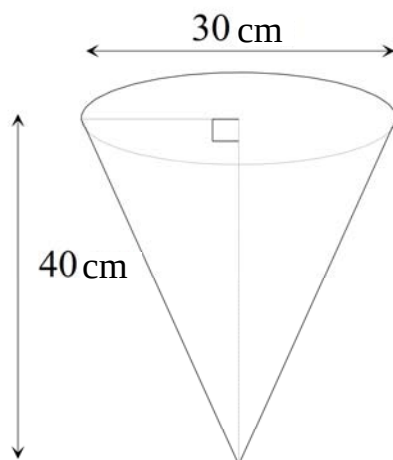
- b. On the axes below, sketch the graph of $y = f^{-1}(f(x))$.

1 mark

**TURN OVER**

Question 6 (3 marks)

An inverted right circular cone has a diameter of 30 cm and a height of 40 cm, as shown in the diagram below



Water is poured into the cone at a rate of $6 \text{ cm}^3/\text{min}$.

Find the rate at which the height is increasing when the volume is $24\pi \text{ cm}^3$.

Question 7 (2 marks)

Joel has either an apple or a banana at morning tea break. If he has an apple one morning, the probability he has an apple the next morning is 0.3. If he has a banana one morning, the probability he has a banana the next morning is 0.4. Suppose he has a banana on a Monday morning. What is the probability that the next time he has a banana is not until the Friday morning?

Question 8 (2 marks)

Let X be a normally distributed random variable with a mean of 36 and a variance of 64. Let Z be the standard normal random variable.

- a. Find the value of c if $\Pr(Z > -1) = \Pr(X < c)$.

1 mark

- b. Find the value of d when $\Pr(Z < d) = \Pr(X > 20)$.

1 mark

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Question 9 (4 marks)

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} k(2x+3) & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- a.** Show that $k = \frac{1}{28}$.

2 marks

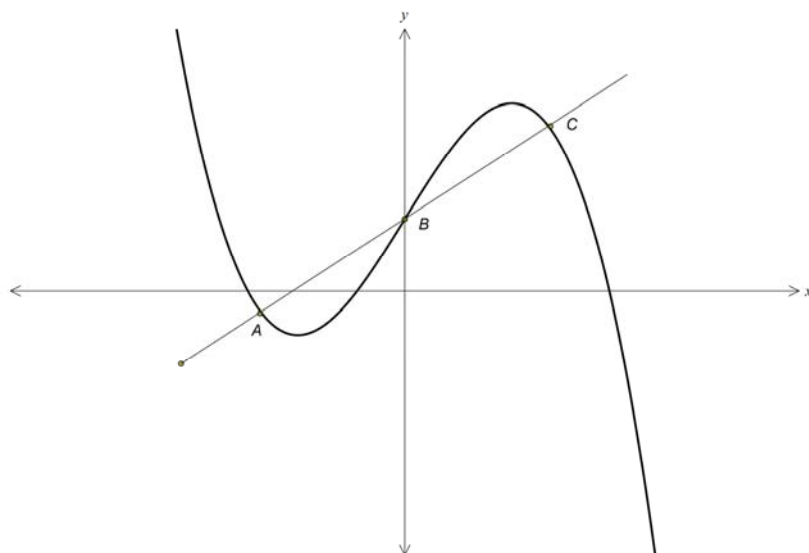
- b.** Find $\Pr(X \leq 2 \mid X < 3)$.

2 marks

TURN OVER

Question 10 (6 marks)

Part of the graph of $y = f(x)$, where $f : \mathbb{R} \rightarrow \mathbb{R}$, is given below. The three points A , B and C lie on the curve with coordinates $(-p, f(-p))$, $(0, f(0))$ and $(p, f(p))$, respectively. The three points A , B and C lie on a straight line.



- a. Find the gradient of the line segments AB and BC and, **hence**, show that $\frac{f(p) + f(-p)}{2} = f(0)$.

1 mark

- b. i. Let $p = 3$ and $f(x) = -x^3 + bx^2 + cx + d$ for $b, c, d \in \mathbb{R}$. Show that $b = 0$.

2 marks

Question 10 – continued

- ii. Find the coordinates of the points on the curve, in terms of c and d , such that a tangent line drawn to the curve at these points is parallel to the line AC .

3 marks

END OF QUESTION AND ANSWER BOOK