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Student Name.....

MATHEMATICAL METHODS (CAS) UNITS 3 & 4
TRIAL EXAMINATION 1

2014

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where an exact answer is required a decimal approximation will not be accepted.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on page 12 of this exam.

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Question 1 (4 marks)

a. If $y = 3x^2 e^{4x}$, find $\frac{dy}{dx}$.

2 marks

b. For $g(x) = \frac{\sin(x)}{2x}$, find $g'\left(\frac{\pi}{2}\right)$.

2 marks

Question 2 (2 marks)

Find an antiderivative of $\frac{1}{2x-3}$ with respect to x .

Question 3 (2 marks)

Let $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R}$, $f(x) = \frac{5}{x}$.

Show that $f(f(x)) = f(f^{-1}(x))$ where f^{-1} is the inverse of f .

Question 4 (5 marks)

a. Solve $\log_6(3) - 2\log_6(x) + \log_6(2) = 1$ for x .

3 marks

b. Solve $8^{1-2x} = 2^{4+x}$ for x .

2 marks

Question 5 (2 marks)

Solve the equation $\sin\left(\frac{x}{3}\right) = \frac{1}{\sqrt{2}}$ for $x \in [0, 6\pi]$.

Question 6 (5 marks)

For the discrete random variable X , with probability distribution shown below, $E(X) = 2$.

x	0	1	2	3	4
$\Pr(X = x)$	0.2	0.2	0.1	0.4	0.1

- a.** Find the median of X . 1 mark

- b.** Find $\Pr(X \leq 2 \mid X > 0)$. 2 marks

- c.** Find the variance of X , $\text{Var}(X)$. 2 marks

Question 7 (4 marks)

The continuous random variable X , has a probability density function given by

$$f(x) = \begin{cases} a \cos\left(\frac{\pi x}{2}\right) & \text{for } x \in [0,1] \\ 0 & \text{elsewhere} \end{cases}$$

where a is a positive constant.

- a.** Find the value of a .

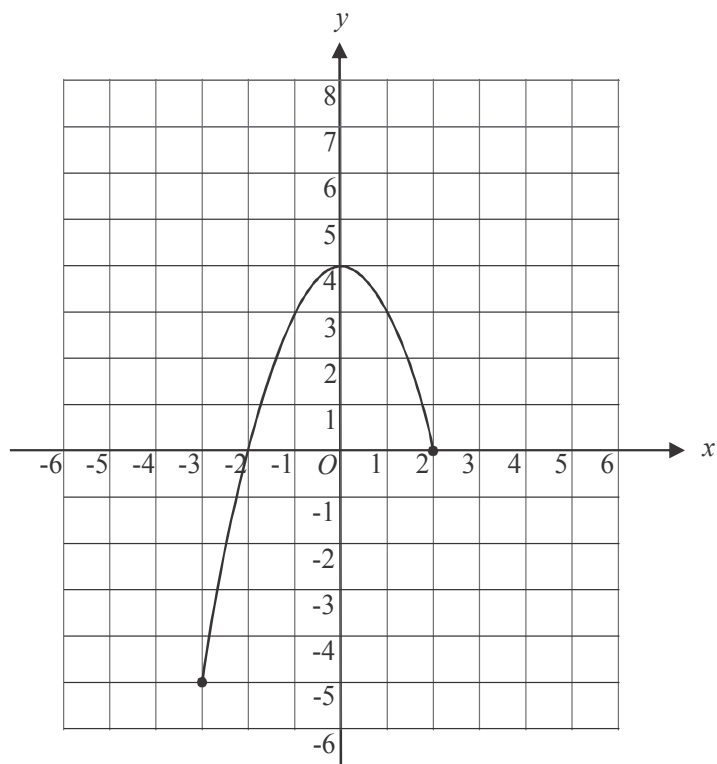
3 marks

- b.** Find the mode of X .

1 mark

Question 8 (6 marks)

The graph of $y = 4 - x^2$, where $x \in [-3, 2]$ is shown below.



Let $f: [-3, 2] \rightarrow \mathbb{R}, f(x) = |4 - x^2| + 2$.

- a.** Sketch the graph of $y = f(x)$ on the set of axes above. Indicate clearly the coordinates of the endpoints.

2 marks

b. The graph of f is

- translated 2 units in the negative direction of the y -axis
- translated 2 units in the positive direction of the x -axis
- dilated from the y -axis by a factor of $\frac{1}{2}$

to become the graph of the function h .

For this function h , find

i. the rule. 2 marks

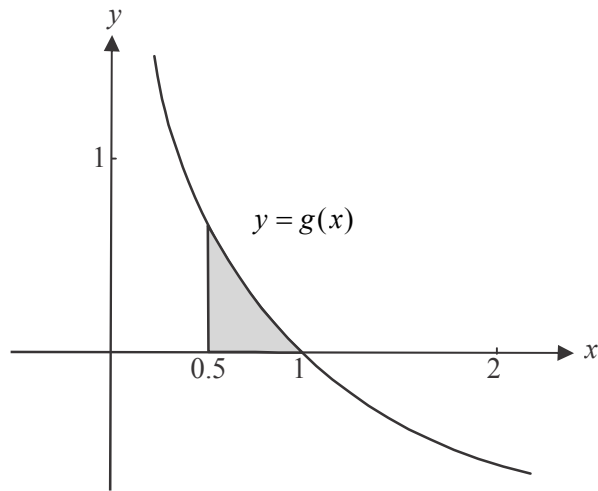
ii. the domain. 1 mark

iii. the range. 1 mark

Question 9 (4 marks)

Let $g : (0, \infty) \rightarrow \mathbb{R}, g(x) = -\log_e(x)$.

The graph of g is shown below.

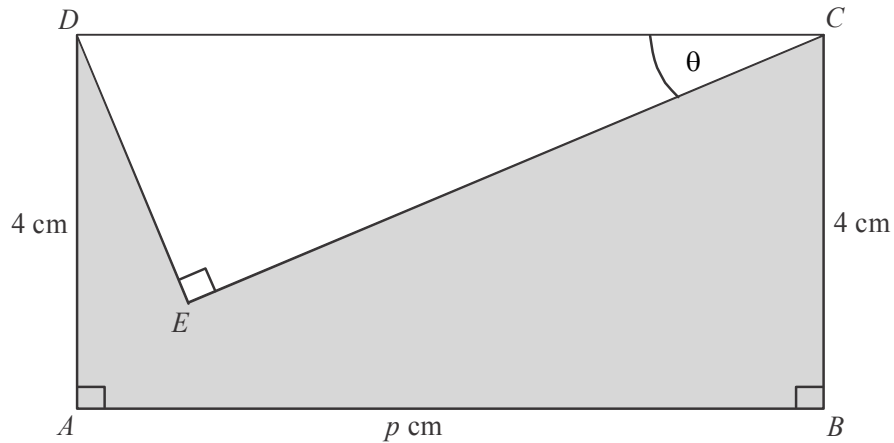


- a.** Differentiate $(1-x)\log_e(x)$ with respect to x .

1 mark

- b.** Hence find the area of the shaded region shown above.
Express your answer in the form $a(\log_e(a) + 1)$ where a is a positive, real constant.

3 marks

Question 10 (6 marks)

In the diagram above, $ABCD$ is a rectangle and CDE is a right-angled triangle where $\angle DCE = \theta$ and $0 < \theta < \frac{\pi}{2}$.

Also, $AD = BC = 4$ cm and $AB = p$ cm where p is a positive constant.

a. Find an expression in terms of p and θ for

i. DE 1 mark

ii. CE 1 mark

The area of the shaded region in the diagram is $A \text{ cm}^2$.

b. Find an expression for A in terms of p and θ . 1 mark

- c.** Use the derivative $\frac{dA}{d\theta}$ to find the value of θ when A is a minimum. 2 marks

- d.** If the minimum area of the shaded region is 16cm^2 , find the value of p . 1 mark

Mathematical Methods (CAS) Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc \sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	transition matrices: $S_n = T^n \times S_0$
mean: $\mu = E(X)$	variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

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