



Victorian Certificate of Education 2013

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS (CAS)

Written examination 1

Wednesday 6 November 2013

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (5 marks)

- a. If $y = x^2 \log_e(x)$, find $\frac{dy}{dx}$. 2 marks

- b. Let $f(x) = e^{x^2}$.
Find $f'(3)$. 3 marks

Question 2 (2 marks)

Find an anti-derivative of $(4 - 2x)^{-5}$ with respect to x .

Question 3 (2 marks)

The function with rule $g(x)$ has derivative $g'(x) = \sin(2\pi x)$.

Given that $g(1) = \frac{1}{\pi}$, find $g(x)$.

Question 4 (2 marks)

Solve the equation $\sin\left(\frac{x}{2}\right) = -\frac{1}{2}$ for $x \in [2\pi, 4\pi]$.

Question 5 (4 marks)

a. Solve the equation $2 \log_3(5) - \log_3(2) + \log_3(x) = 2$ for x .

2 marks

b. Solve the equation $3^{-4x} = 9^{6-x}$ for x .

2 marks

Question 6 (3 marks)

Let $g: \mathbb{R} \rightarrow \mathbb{R}$, $g(x) = (a - x)^2$, where a is a real constant.

The average value of g on the interval $[-1, 1]$ is $\frac{31}{12}$.

Find all possible values of a .

Question 7 (6 marks)

The probability distribution of a discrete random variable, X , is given by the table below.

x	0	1	2	3	4
$\Pr(X = x)$	0.2	$0.6p^2$	0.1	$1 - p$	0.1

- a. Show that $p = \frac{2}{3}$ or $p = 1$.

3 marks

b. Let $p = \frac{2}{3}$.

i. Calculate $E(X)$.

2 marks

ii. Find $\Pr(X \geq E(X))$.

1 mark

A continuous random variable, X , has a probability density function

$$f(x) = \begin{cases} \frac{\pi}{4} \cos\left(\frac{\pi x}{4}\right) & \text{if } x \in [0, 2] \\ 0 & \text{otherwise} \end{cases}$$

Given that $\frac{d}{dx}\left(x \sin\left(\frac{\pi x}{4}\right)\right) = \frac{\pi x}{4} \cos\left(\frac{\pi x}{4}\right) + \sin\left(\frac{\pi x}{4}\right)$, find $E(X)$.

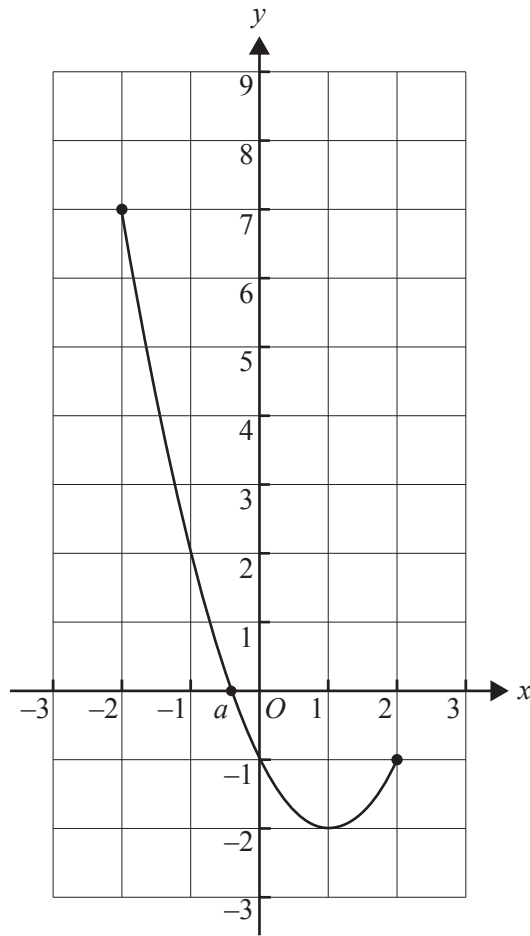
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Question 9 (6 marks)

The graph of $f(x) = (x - 1)^2 - 2$, $x \in [-2, 2]$, is shown below. The graph intersects the x -axis where $x = a$.



- a. Find the value of a .

1 mark

- b. On the axes above, sketch the graph of $g(x) = |f(x)| + 1$, for $x \in [-2, 2]$. Label the end points with their coordinates.

2 marks

- c. The following sequence of transformations is applied to the graph of the function

$$g: [-2, 2] \rightarrow R, g(x) = |f(x)| + 1.$$

- a translation of one unit in the negative direction of the x -axis
- a translation of one unit in the negative direction of the y -axis
- a dilation from the x -axis of factor $\frac{1}{3}$

Find

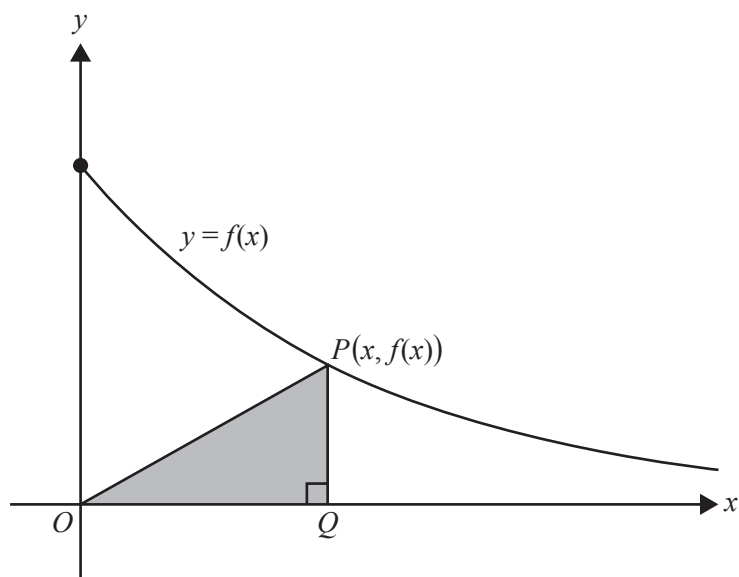
- i. the rule of the image of g after the sequence of transformations has been applied 2 marks

- ii. the domain of the image of g after the sequence of transformations has been applied. 1 mark

Question 10 (7 marks)

Let $f: [0, \infty) \rightarrow \mathbb{R}$, $f(x) = 2e^{-\frac{x}{5}}$.

A right-angled triangle OQP has vertex O at the origin, vertex Q on the x -axis and vertex P on the graph of f , as shown. The coordinates of P are $(x, f(x))$.



- a. Find the area, A , of the triangle OQP in terms of x .

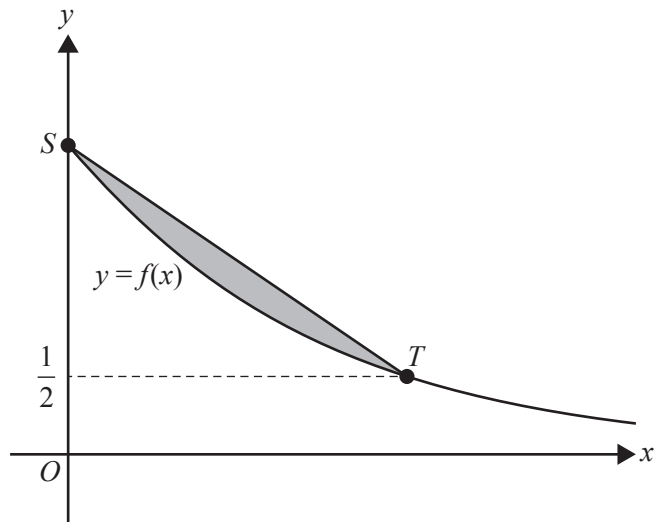
1 mark

- b. Find the maximum area of triangle OQP and the value of x for which the maximum occurs. 3 marks

- c. Let S be the point on the graph of f on the y -axis and let T be the point on the graph of f with the y -coordinate $\frac{1}{2}$.

Find the area of the region bounded by the graph of f and the line segment ST .

3 marks

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MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods (CAS)

Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc \sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	transition matrices: $S_n = T^n \times S_0$
mean: $\mu = E(X)$	variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$