



NAME: _____

VCE MATHEMATICAL METHODS (CAS)

Practice written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **not** permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 10 pages.
- Formula booklet PROVIDED BY YOUR TEACHER.
- Working space is provided throughout the book.

Instructions

- All written responses must be in English.
- Write your student name in the space provided above on this page.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 3

- a) Find an antiderivative of $\frac{3}{(3-3x)^2}$ with respect to x .

2 marks

- b) Determine the average value of the function $h(x) = 4x - \frac{x^3}{4}$ between $x = 0$ and $x = 4$.

3 marks

Question 4

Solve the equation $e^x + \frac{25}{e^x} = 10$ for x .

3 marks

Question 5

The equation for the function g is $g(x) = \frac{10}{(2x^2 + 5)}$.

- a)** Find an equation for the inverse of g .

2 marks

- b)** Determine a maximal domain for $g(x)$ that will make the inverse also a function.

3 marks

Question 6

Consider the function $h: R \rightarrow R, h(x) = 2 - 2 \left| \sin \left(\frac{\pi x}{2} \right) \right|$

a) State the range of $h(x)$.

1 mark

b) Determine the period of $h(x)$.

1 mark

c) Solve $h(x) = 0$ for all $x \in [-2, 2]$.

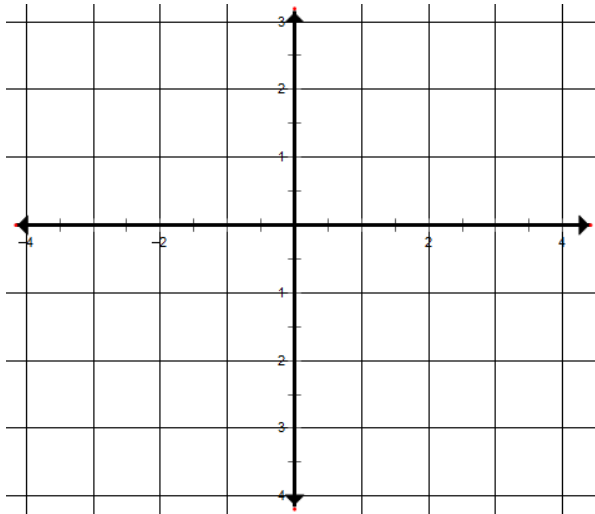
1 mark

- d)** For $0 \leq x \leq 2$ $h(x)$ is equivalent to $2 - 2\sin\left(\frac{\pi x}{2}\right)$. Use this information and calculus to determine the area bounded by $h(x)$, the x-axis, $x = -2$ and $x = 2$.

2 marks

Question 7

- a) Sketch the graph of $f(x) = \log_e((x - 1)^2)$ on the axes provided.



3 marks

- b) Differentiate $f(x) = \log_e((x - 1)^2)$.

1 mark

- c) Determine the equation of the normal through the point where $x = 2$.

2 marks

Question 8

A weighted six sided die has a distribution as given:

X	1	2	3	4	5	6
$\Pr(X = x)$	k	$k - 0.2$	0.1	0.2	0.1	2k

- a) Determine the value of k in the distribution.

2 marks

- b) Find the mean of the distribution.

1 mark

- c) What is the probability that the die will land face up on an even number?

1 mark

Question 9

- a) A binomial distribution has a mean of 20 and a variance of 12
Determine the values of n and p .

2 marks

- b) Hence determine the value of $\Pr(X = 0)$. Leave your answer in exact form.

1 mark

Question 10

A continuous random variable is defined by the hybrid function

$$p(x) = \begin{cases} \frac{x}{16} & 0 \leq x \leq 4 \\ \frac{(x-1)(7-x)}{36} & 4 < x \leq 7 \end{cases}$$

- a) What is the $Pr(X \leq 4)$?

2 marks

- b) Hence or otherwise determine the value of the 25th percentile.

2 marks

END OF QUESTION AND ANSWER BOOKLET

Solution Pathway**Question 1**

Reflection in the x-axis gives $-\sqrt{4-x^2}$ (1 mark)

Translation in the negative x-axis gives $-\sqrt{4-(x+2)^2}$

So $g(x) = -\sqrt{4-(x+2)^2}$ (1 mark)

Question 2

a)

Use chain rule. Let $u = x + 2x^4$ and then $y = u^2$

$$\frac{du}{dx} = 1 + 8x^3 \text{ and } \frac{dy}{du} = 2u$$

Then $\frac{dy}{dx} = 2(x + 2x^4) \times (1 + 8x^3)$ (1 mark)

b)

Use quotient rule: Let $u = \cos(2x)$ and $v = x$

$$\frac{du}{dx} = -2\sin(2x) \text{ and } \frac{dv}{dx} = 1$$

Then $\frac{dy}{dx} = \frac{x(-2\sin(2x)) - \cos(2x)}{x^2}$ (1 mark)

$\therefore g'\left(\frac{\pi}{2}\right) = \frac{\frac{1}{\pi^2}}{\frac{4}{4}} = \frac{4}{\pi^2}$ (1 mark)

Question 3

a)

$$\int \frac{3}{(3-3x)^2} dx = 3 \int (3-3x)^{-2} dx$$

$$= 3 \left(\frac{(3-3x)^{-2+1}}{-3x-1} \right) + c \quad (1 \text{ M})$$

$$= (3-3x)^{-1} + c$$

$$= \frac{1}{3-3x} + c \quad (1 \text{ A})$$

b)

$\frac{1}{4-0} \int_0^4 4x - \frac{x^3}{4} dx = \frac{1}{4} \left[\frac{4x^2}{2} - \frac{x^4}{16} \right]_0^4$	(1 M)
$= \frac{1}{4} \left(\frac{4 \times 16}{2} - \frac{16 \times 16}{16} - 0 \right)$	(1 M)
$= \frac{1}{4} (4 \times 8 - 16)$	
$= \frac{1}{4} \times 16 = 4$	(1 A)

Question 4

Multiply through by e^x :	$e^{2x} + 25 = 10e^x$	
	$e^{2x} - 10e^x + 25 = 0$	(1 M)
Let $A = e^x$	$A^2 - 10A + 25 = 0$	
Solve for A	$(A - 5)^2 = 0$	
	$(A - 5) = 0$	
	$A = 5$	(1 M)
Now solve for x	$e^x = 5$	
	$x = \log_e(5)$	(1 A)

Question 5

a)

	Let $g(x) = y$	
	$y = \frac{10}{(2x^2+5)}$	
Swap x and y	$x = \frac{10}{(2y^2+5)}$	(1 mark)
Rearrange	$2y^2 + 5 = \frac{10}{x}$	
	$y^2 = \frac{5}{x} - 2.5$	
	$y = \pm \sqrt{\frac{5}{x} - \frac{5}{2}}$	(1 mark)

b)

Domain restrictions must make the original function 1 to 1. Therefore it is necessary to find turning points of the original function.

	$g'(x) = \frac{-40x}{(2x^2+5)^2}$	(1 mark)
Let $g'(x) = 0$	$0 = \frac{-40x}{(2x^2+5)^2}$	
Hence	$0 = -40x$	
and	$x = 0$	(1 mark)
So maximal domain is	$0 \leq x < \infty$ or $-\infty < x \leq 0$	(1 mark for either)

Question 6

a)

All sine functions have a range of $-1 \leq x \leq 1$ The modulus converts this to a range $0 \leq x \leq 1$ $2 - 2 = 0$ and $2 - 0 = 2$.Hence $\text{Ran} = [0, 2]$

(1 mark)

b)

The $\sin\left(\frac{\pi x}{2}\right)$ component of the function has a period of $2\pi \times \frac{2}{\pi} = 4$ The modulus converts this to half the original period so $\text{Period} = 2$. (1 mark)

c)

$$0 = 2 - 2 \left| \sin\left(\frac{\pi x}{2}\right) \right|$$

$$2 = 2 \left| \sin\left(\frac{\pi x}{2}\right) \right|$$

$$1 = \left| \sin\left(\frac{\pi x}{2}\right) \right|$$

$$\pm 1 = \sin\left(\frac{\pi x}{2}\right)$$

$$\pm \frac{\pi}{2} = \frac{\pi x}{2}$$

$$x = \pm 1$$

(1 mark)

d)

$$\int_{-2}^2 h(x) dx = 2 \int_0^2 2 - 2 \sin\left(\frac{\pi x}{2}\right) dx$$

$$= 2 \left[2x + \frac{4}{\pi} \cos\left(\frac{\pi x}{2}\right) \right]_0^2 \quad (1 \text{ mark})$$

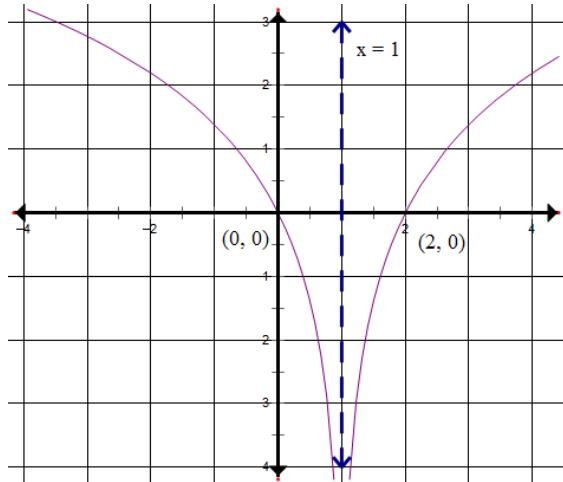
$$= 2 \left[\left(4 + \frac{4}{\pi} \cos(\pi) \right) - \left(0 + \frac{4}{\pi} \cos(0) \right) \right]$$

$$= 2 \left[\left(4 - \frac{4}{\pi} - \frac{4}{\pi} \right) \right]$$

$$= 8 - \frac{16}{\pi} \text{ units} \quad (1 \text{ mark})$$

Question 7

a)



Shape – 1, intercepts – 1, asymptote – 1

(3 marks)

b)

$$f'(x) = \frac{2(x-1)}{(x-1)^2}$$

$$f'(x) = \frac{2}{x-1}$$

(1 mark)

c)

$$m_T = f'(2) = \frac{2}{2-1} = 2$$

$$m_N = \frac{-1}{2}$$

(1 mark)

$$y - 0 = \frac{-1}{2}(x - 2)$$

$$y = \frac{-1}{2}x + 1$$

(1 mark)

Question 8

a)

$$\text{Sum} = 4k + 0.2$$

$$1 = 4k + 0.2$$

(1 mark)

$$4k = 0.8$$

$$k = 0.2$$

(1 mark)

b)

$$0.2 + 0 + 0.3 + 0.8 + 0.5 + 2.4 = 4.2 \quad (1 \text{ mark})$$

c)

$$0 + 0.2 + 2 \times 0.2 = 0.6 \quad (1 \text{ mark})$$

Question 9

a)

$$np = 20$$

$$npq = 12$$

$$q = \frac{12}{20}$$

$$\text{Therefore } p = 1 - \frac{12}{20} = \frac{8}{20} \quad (1 \text{ mark})$$

$$\text{And } n = \frac{20 \times 20}{8} = 50 \quad (1 \text{ mark})$$

b)

$$\Pr(X = 0) = {}^{50}C_0 \left(\frac{2}{5}\right)^0 \left(\frac{3}{5}\right)^{50} = 1 \times 1 \times \left(\frac{3}{5}\right)^{50} = \left(\frac{3}{5}\right)^{50} \quad (1 \text{ mark})$$

Question 10

a)

$$\int_0^4 \frac{x}{16} dx = \left[\frac{x^2}{32} \right]_0^4 \quad (1 \text{ mark})$$

$$= \frac{16}{32} - 0 = \frac{1}{2} \quad (1 \text{ mark})$$

b)

$$\frac{1}{4} = \int_0^q \frac{x}{16} dx = \left[\frac{x^2}{32} \right]_0^q \quad (1 \text{ mark})$$

$$\frac{1}{4} = \frac{q^2}{32}, \quad \text{Hence, } q^2 = 8, \text{ so } q = \sqrt{8} \quad (1 \text{ mark})$$

END OF SOLUTIONS