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Mathematical Methods (CAS)

2013

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1 The asymptote(s) of the graph of $f : [-1, 3) \rightarrow R, f(x) = \frac{3x-7}{x-3}$ is/are

- A. $x = 3$ only
- B. $x = -3$ only
- C. $x = 3$ and $y = 3$
- D. $x = -3$ and $y = -3$
- E. $x = -3$ and $y = 3$

Question 2 $\log_{\frac{1}{e}}(\sqrt[3]{e} \times 3)$ can be simplified to

- A. $3 - \log_e 3$
- B. $-3 - \log_e 3$
- C. $\frac{1}{3} - \log_e 3$
- D. $-\frac{1}{3} - \log_e 3$
- E. $\frac{1}{3} + \log_e 3$

Question 3 The solution to the equation $\sqrt{a-3x} + \log_b(3x) = \log_b a$, where $a, b \in R^+$, is

- A. 0
- B. 1
- C. $\frac{1}{3}$
- D. $\frac{a}{3}$
- E. $-\frac{3}{a}$

Question 4 Given $f : (a, 1] \rightarrow R$, $f(x) = x - a$ and $g : (a, 1] \rightarrow R$, $g(x) = \sqrt{x - a}$,

- A. $g \circ f$ is defined for $a \leq -10$
- B. $g \circ f$ is defined for $a < 1$
- C. $g \circ f$ is defined for $a = 0$ only
- D. $g \circ f$ is defined for $a \leq 0$
- E. $g \circ f$ is defined for $a \leq -1$

Question 5 $y = e^{ax}$ and $y = \frac{\log_e x}{a}$ intersect at $x = 2$ when

- A. $a = 1$
- B. $a = -1$
- C. $a = \log_e \sqrt{2}$
- D. $a = -\log_e 2$
- E. $a = \log_e 2$

Question 6 Given $f(x) = -(x+1)^2(x-5)^2(x-2)^3$ and $g(x) = -f(x-a)$, $g(-x) = -g(x)$ when

- A. $a \leq -1$
- B. $a > -1$
- C. $a = -2$
- D. $a \geq -2$
- E. $a = -5$

Question 7 The point of intersection of the family of lines $4ax + (a+b)y + 4b = 0$, where $a, b \in R \setminus \{0\}$, is

- A. $(-4, 1)$
- B. $(-1, 4)$
- C. $(4, -4)$
- D. $(1, -4)$
- E. $(-1, 1)$

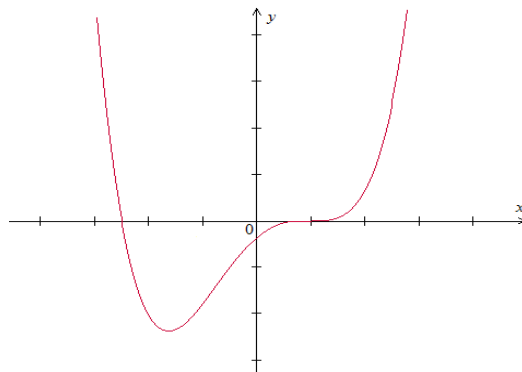
Question 8 $y = \sin(x)$ and $y = mx$ intersect at exactly 5 points for a particular **positive** value of m . Which one of the following statements is true?

- A. $0.1288 < m < 0.1290$
- B. $0.1286 < m < 0.1288$
- C. $0.1284 < m < 0.1286$
- D. $0.1282 < m < 0.1284$
- E. $0.1280 < m < 0.1282$

Question 9 The graph of $y = f(1 - x)$ is translated in the positive x direction by 1 unit and then reflected in the y -axis. The equation of the resulting graph is

- A. $y = f(x)$
- B. $y = -f(x)$
- C. $y = f(x - 2)$
- D. $y = f(x + 2)$
- E. $y = -f(2 - x)$

Question 10 The graph of $y = f(x)$ is shown below.



A possible rule of $f(x)$ is

- A. $2(x + a)(x + b)^5$
- B. $-2(x - a)^3(x + b)$
- C. $1 - 2(x - a)^3(x + b)$
- D. $2(x + b)(x^3 - a)$
- E. $2(x + a)^2(x + b)^3$

Question 11 Given hybrid function $f(x) = \begin{cases} 4a - 6x, & x < \frac{2a}{3} \\ 6x - 4a, & x \geq \frac{2a}{3} \end{cases}$ where a is a real constant, $f(x)$ can be expressed as

- A. $f(x) = 2\sqrt{(2a - 3x)^2}$
- B. $f(x) = \sqrt{2(3x - 2a)^2}$
- C. $f(x) = 2|3x| - 4a$
- D. $f(x) = -2|3x - 2a|$
- E. $f(x) = -2|2a - 3x|$

Question 12 The graph of a cubic polynomial function of x , where $x \in R$,

- A. has an inflection point and *only* one x -intercept
- B. has an inflection point and *at least* one x -intercept
- C. has 0, 1, 2 or 3 x -intercepts
- D. *always* has a stationary inflection point *or* a pair of local maximum and minimum points
- E. *always* has a maximum point and a minimum point

Question 13 If $f'(a) = b$ and $g(x) = b - f(-x)$, where a and b are real constants, then $g'(-a) =$

- A. $-a$
- B. b
- C. -1
- D. $\frac{1}{b}$
- E. $-\frac{1}{a}$

Question 14 If $(-2, -1)$ is the only inflection point on the graph of $f(x)$, then

- A. $(-2, 1)$ is an inflection point on the graph of $f(|x|)$
- B. $(2, 1)$ is an inflection point on the graph of $f(|x|)$
- C. $(2, -1)$ is an inflection point on the graph of $f(|x|)$
- D. $(-2, 1)$ and $(2, 1)$ are inflection points on the graph of $f(|x|)$
- E. the graph of $f(|x|)$ does not have an inflection point

Question 15 Given $\int_a^b f(x)dx = \log_e \left| \frac{b}{a} \right|$, which one of the following choices is *false*?

- A. $a = -2$ and $b = -1$
- B. $a = -1$ and $b = -2$
- C. $a = 1$ and $b = 2$
- D. $a = 2$ and $b = 1$
- E. $a = -1$ and $b = 2$

Question 16 Function g is defined as $g(x) = |x - \pi| + \sin x - \frac{\pi}{2}$. The average value of g in the interval

$\left[\frac{2\pi}{3}, \frac{4\pi}{3} \right]$ is closest to

- A. 0
- B. -1
- C. $-\frac{\pi}{3}$
- D. $-\frac{\pi}{2}$
- E. $-\pi$

Question 17 One of the factors of $f(\theta) = \cos^2 \theta + 2\sin^3 \theta - 1$ is

- A. $\sin \theta - 1$
- B. $\sin \theta + 1$
- C. $2 + \sin \theta$
- D. $\cos \theta$
- E. $\cos \theta - 1$

Question 18 X is a discrete random variable. A possible probability distribution of X is given by

- A.

x	-1	-3	1	-2	5
$\Pr(X = x)$	0.130	0.180	0.296	0.311	0.086
- B.

x	1	2	3	4	5
$\Pr(X = x)$	0.1	0.2	0.3	0.2	0.1
- C.

x	1	2	3	2	1
$\Pr(X = x)$	0	0.1	0.2	0.3	0.4
- D.

x	5	4	3	2	1
$\Pr(X = x)$	1.1	0.4	0.1	-0.8	0.2
- E.

x	1.1	0.4	0.1	-0.8	0.2
$\Pr(X = x)$	0	0.25	0.25	0.30	0.20

Question 19 A and B are the two states of a two-state Markov chain *starting with state A*. If $\Pr(A|B) = a$ and $\Pr(B|A) = b$, then $\Pr(ABABB) =$

- A. $b^2a(1-a)$
- B. b^2a^2
- C. $b(1-b)a^2$
- D. $b(1-b)a^2(1-a)$
- E. $b^2(1-b)(1-a)^2$

Question 20 A *biased* coin is **tossed 10 times** and the number of tails is recorded. This probability experiment is repeated many times.

In the long-run the probability of getting 5 tails is 0.10 (correct to the 2nd decimal place), and the probability of getting 6 tails is 0.20 (correct to the 2nd decimal place).

The probability of getting tail in a single toss of the biased coin is closest to

- A. 0.3
- B. 0.4
- C. 0.5
- D. 0.6
- E. 0.7

Question 21 $\Pr(A) - \Pr(A \mid B')\Pr(B') =$

- A. $\Pr(B)$
- B. $\Pr(B')$
- C. $\Pr(A')$
- D. $\Pr(A' \mid B)\Pr(B)$
- E. $\Pr(B) - \Pr(B \mid A')\Pr(A')$

Question 22 The probability distribution of continuous random variable X is symmetric and centred at $X = 1.3$.

Given $\Pr(X < 3) = 0.85$, the value of $\Pr(-0.4 < X < 3)$ is closest to

- A. 0.60
- B. 0.65
- C. 0.70
- D. 0.75
- E. 0.80

Instructions for Section 2

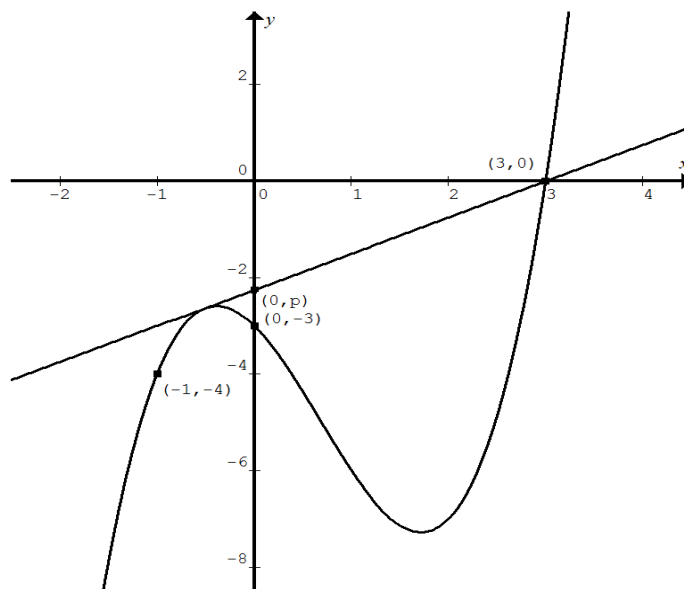
Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 The following diagram shows the graphs of a linear function and a cubic function.



- a.** The y-intercept of the graph of the linear function is $(0, p)$. Find the equation of the linear function in the form $y = A(x - B)$ in terms of p . 2 marks

- b.** The equation of the cubic function is $y = (x - 3)(x^2 + bx + c)$ where $b, c \in \mathbb{R}$. Find the values of b and c . 2 marks

c i. *By using the fact* that the two graphs intersect at *exactly* two points, find the value of p . 3 marks

c ii. *Hence* find the coordinates of the point where the graph of the linear function is a tangent to the graph of the cubic function. 2 marks

d. Find the exact coordinates of another point on the graph of the cubic function where a tangent line is parallel to the graph of the linear function. 2 marks

After the following sequence of transformations: reflection in the x -axis, dilation from the y -axis and dilation from the x -axis, the linear function and the cubic function become $f(x) = -3x + 18$ and $g(x) = -x^3 + 4x^2 + 8x + 24$ respectively.

- ei.** Find the x -coordinates of the intersections of the graphs of $y = f(x)$ and $y = g(x)$. 1 mark

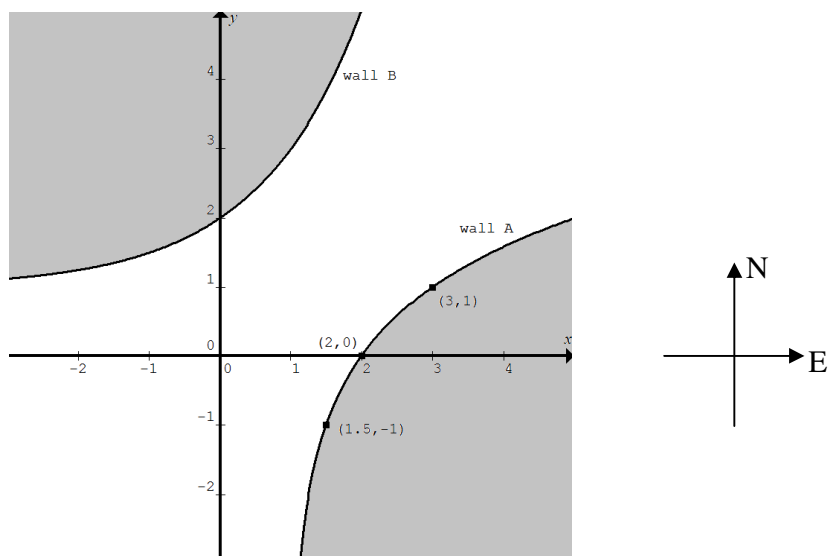
- eii.** State the factor of dilation from the y -axis. 1 mark

- f i.** Write down the definite integral for finding the area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$. 1 mark

- f ii.** Find the exact area of the region bounded by the graphs of $y = f(x)$ and $y = g(x)$. 1 mark

- g.** Determine the factor of dilation from the x -axis in the sequence of transformations. 1 mark

Question 2 Part of the floor plan of two structures (shaded) is shown below. All length measures are in metres.



Wall A has the equation $y = a \log_e(x+b) + c$. It passes through points (3,1), (2,0) and (1.5,-1). Wall B is the reflection of Wall A in the line $y = x$.

- a. Write down three simultaneous equations that can be used to find the parameters a , b and c in $y = a \log_e(x+b) + c$. 1 mark

- b. Solve the simultaneous equations without using CAS. Show that $a = \frac{1}{\log_e 2}$, $b = -1$ and $c = 0$. 3 marks

c. Show that the equation of Wall B is $y = 2^x + 1$.

2 marks

d. Find the exact coordinates of the point at Wall A where the tangent is pointing exactly in the NE direction.

3 marks

e. Find the shortest distance (correct to the nearest 0.01 m) between Wall A and Wall B.

2 marks

f. The horizontal ground between Wall A and Wall B, and bounded by $x = 0$, $x = 3$, $y = 0$ and $y = 3$ is to be covered by a synthetic lawn. Find the area (correct to nearest 0.01 m^2) of synthetic lawn required.

2 marks

Question 3 At $t = 0$ water is pumped into an empty cylindrical tank at a variable rate. The rate is given by $\frac{dV}{dt} = \sin \frac{2t}{\pi} + \cos \frac{2t}{\pi} + \pi$. Length is measured in metres, volume V is in m^3 and time t is in minutes.

- a. Given the radius of the tank is π metres and its height is 4 metres, find $\frac{dh}{dt}$ in terms of t minutes, where h metres is the depth of water in the tank. 2 marks

- b. Show that the period of $\frac{dV}{dt}$ is π^2 minutes. 1 mark

- c i. Show by calculus the average of $\frac{dV}{dt}$ over n periods is $\pi \text{ m}^3 \text{ min}^{-1}$, where n is a whole number. 2 marks

- c ii. Hence find the *number of periods* required to fill the tank. 1 mark

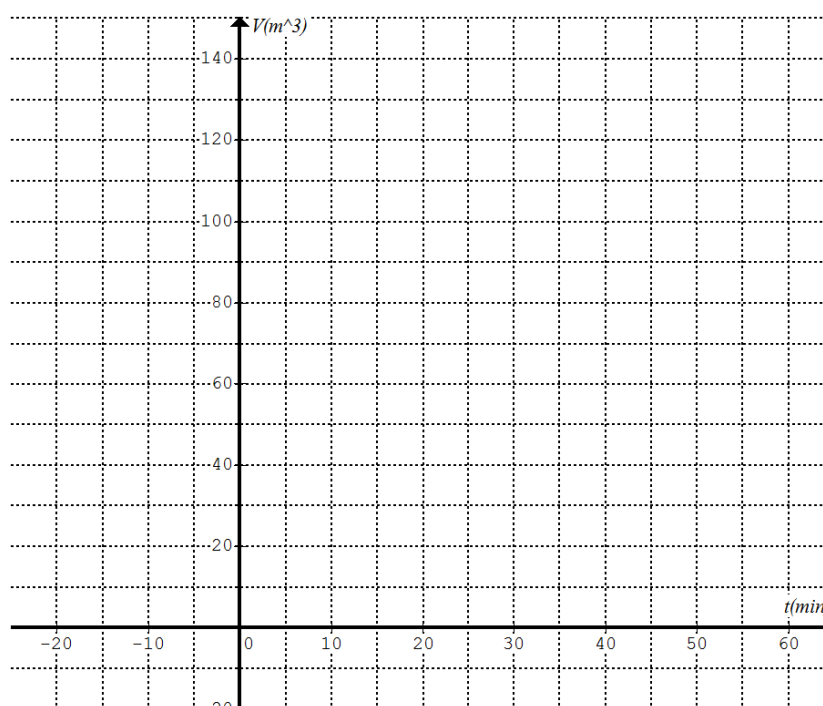
- d. Find the time (in min, correct to one decimal place) required to fill a second cylindrical tank of the same height but the radius is 3 metres. 2 marks

- e. Determine the exact time(s) when the depth of water in the second tank *increases* at the greatest rate. 3 marks

The pump is turned off when the **first tank** is filled completely and water is allowed to run out of the tank. The volume of water remains in the tank is given by $V(t) = 9.164(t + 10)^2 e^{-(0.2t+2)}$, and $t = 0$ when water starts to run out.

- f i. Find the time (in min, correct to one decimal place) when the depth of water is halved. 1 mark

- f ii. Sketch the graph of $V(t) = 9.164(t + 10)^2 e^{-(0.2t+2)}$. Show and label the important features. 2 marks



Question 4 The waiting time in minutes to check out of a supermarket is a random variable and has a normal distribution.

Let A be the waiting time to check out of Allthere Supermarket. It has a mean of 8.53 and a standard deviation of 5.23.

Let B be the waiting time in minutes to check out of Bestbuy Supermarket.

- a. Find the probability (correct to 2 decimal places) that an Allthere Supermarket patron has to wait for more than 5 minutes at the checkout. 1 mark

- b. Determine the proportion (correct to 2 decimal places) of Allthere Supermarket patrons expected to wait for 7.00 ± 5.23 minutes at the checkout. 1 mark

- c i. Find the probability (correct to 2 decimal places) that there are 7 among 10 randomly chosen Allthere Supermarket patrons who have waited for more than 5 minutes at the checkout. 1 mark

- c ii. Out of 10 randomly chosen patrons how many (correct to the nearest whole number) were expected to wait more than 5 minutes at the checkout? 1 mark

- d. 66.15% of Bestbuy Supermarket patrons have waited for more than 5 minutes at the checkout, and 40.13% have waited for less than 6 minutes. Find the mean and standard deviation (correct to 2 decimal places) of the waiting time at the checkout of Bestbuy Supermarket. 2 marks

- e. For how long (in minutes, correct to 2 decimal places) does a Bestbuy Supermarket patron expect to wait at the checkout? 1 mark

- f. Given that a Bestbuy Supermarket patron has waited for more than 5 minutes at the checkout, find the probability that the patron has waited for less than 6 minutes. 1 mark

Sofia moves into the neighbourhood and does her grocery shopping at Allthere Supermarket and Bestbuy Supermarket *only*. If she shops at Allthere the probability that she shops at Allthere again the next time is $\frac{2}{5}$. If she shops at Bestbuy the probability that she shops at Bestbuy again the next time is $\frac{1}{4}$.

- g. The probability that Sofia shops at Allthere Supermarket next time is $\frac{3}{4}$. Where did she shop last time? 1 mark

- h. Given that she shopped at Bestbuy Supermarket last 5 times, what is the probability that she shops once at each supermarket in her next two shopping trips assuming one supermarket in each trip? 2 marks

- i. In the long run what is the *exact* probability that Sofia does her grocery shopping at Bestbuy Supermarket? 1 mark

- j. If Sofia goes to Bestbuy Supermarket on her 210th shopping trip, what is the expected number of times (correct to 2 decimal places) that she goes to Bestbuy Supermarket in her next *three* (i.e. 211th, 212th and 213th) shopping trips? 3 marks

End of exam 2