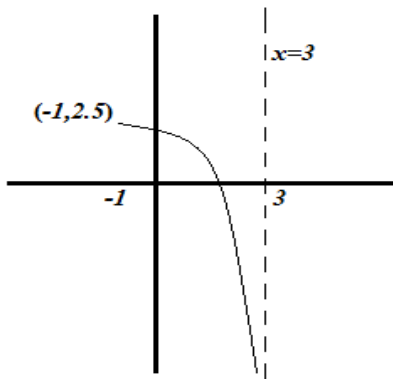


SECTION 1

1	2	3	4	5	6	7	8	9	10	11
A	D	D	C	C	C	D	D	D	A	A

12	13	14	15	16	17	18	19	20	21	22
B	B	E	E	B	E	E	A	E	E	C

Q1 $-1 \leq x < 3$, $f(x) \leq 2.5$, the only asymptote is $x = 3$. A



$$\begin{aligned} \text{Q2 } \log_{\frac{1}{e}}(\sqrt[3]{e} \times 3) &= \log_{\frac{1}{e}}\left(3e^{\frac{1}{3}}\right) = \frac{\log_e\left(3e^{\frac{1}{3}}\right)}{\log_e\left(\frac{1}{e}\right)} = \frac{\log_e 3 + \frac{1}{3}}{-1} \\ &= -\log_e 3 - \frac{1}{3} \end{aligned}$$

$$\begin{aligned} \text{Q3 } \sqrt{a-3x} + \log_b(3x) &= \log_b a, \sqrt{a-3x} = \log_b a - \log_b(3x), \\ \sqrt{a-3x} &= \log_b\left(\frac{a}{3x}\right), \text{ both sides equal zero when } 3x = a, \text{ i.e.} \\ x &= \frac{a}{3}. \end{aligned}$$

$$\begin{aligned} \text{Q4 } \text{Range of } f \subseteq \text{domain of } g, \therefore (0, 1-a] &\subseteq (a, 1] \\ \therefore a \leq 0 \text{ and } 1-a \leq 1, \text{ i.e. } a \leq 0 \text{ and } a &\geq 0 \\ \therefore a &= 0 \end{aligned}$$

$$\text{Q5 } \text{At } x = 2, e^{ax} = \frac{\log_e x}{a}, \therefore e^{2a} = \frac{\log_e 2}{a}, ae^{2a} = \log_e 2.$$

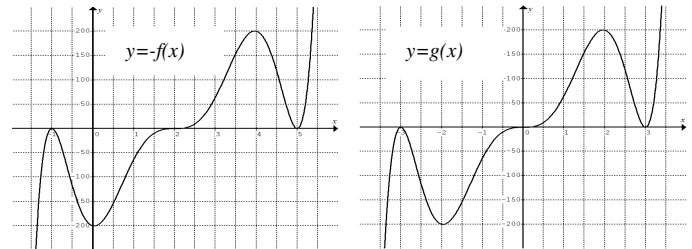
Since e^{2a} and $\log_e 2 \in \mathbb{R}^+$, $\therefore a > 0$.

$$\text{By inspection } a \neq 1, a \neq \log_e 2, \therefore a = \log_e \sqrt{2}$$

Check:

$$ae^{2a} = (\log_e \sqrt{2})e^{2\log_e \sqrt{2}} = (\log_e \sqrt{2})(e^{\log_e \sqrt{2}})^2 = (\log_e \sqrt{2})(2) = \log_e 2$$

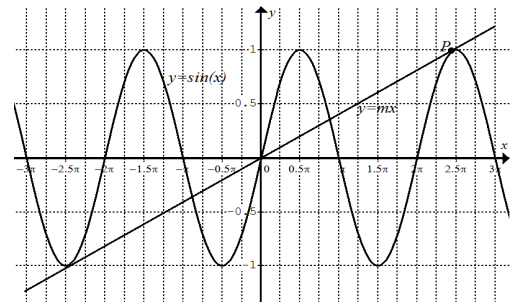
Q6 The graphs of $y = -f(x)$ and $y = g(x)$ are shown below.



$y = g(x)$ is the translation of $y = -f(x)$ to the left by 2 units, and $g(-x) = -g(x)$. $\therefore a = -2$ C

Q7 Let (p, q) be the point of intersection.
 $\therefore 4ap + (a+b)q + 4b = 0$ independent of the values of a and b .
Rearrange the equation: $(4p+q)a + (q+4)b = 0$
 $\therefore 4p+q = 0$ and $q+4 = 0$
 $\therefore q = -4$ and $p = 1$ D

Q8 The following graphs of $y = \sin(x)$ and $y = mx$ intersect at exactly 5 points.



The coordinates of point P is $(x, \sin(x))$.

$$\frac{dy}{dx} = m = \cos x \text{ at } P.$$

$$\therefore m = \frac{\sin(x)}{x} = \cos(x). \text{ By CAS, } m = 0.12837455 \quad \text{D}$$

$$\begin{aligned} \text{Q9 } y &= f(1-x) \\ \rightarrow y &= f(1-(x-1)), \text{ i.e. } y = f(2-x) \\ \rightarrow y &= f(2-(-x)), \text{ i.e. } y = f(2+x) \end{aligned} \quad \text{D}$$

Q10 Even degree, positive coefficient of leading term, two x -intercepts, one of them a stationary point of inflection. A

$$\begin{aligned} \text{Q11 } f(x) &= |4a-6x| = \sqrt{(4a-6x)^2} = \sqrt{4(2a-3x)^2} \\ &= 2\sqrt{(2a-3x)^2} \end{aligned} \quad \text{A}$$

Q12 B

Q13 $g(x) = b - f(-x)$, $g'(x) = -f'(-x) \times -1 = f'(-x)$
 $\therefore g'(-a) = f'(a) = b$

Q14 $y = f(|x|) = \begin{cases} f(x), & x \geq 0 \\ f(-x), & x < 0 \end{cases}$
 $y = f(-x)$, $x < 0$ is the reflection in the y-axis of $y = f(x)$,
 $x > 0$, $\therefore y = f(|x|)$ does not have an inflection point.

Q15 $\int_a^b f(x) dx = \log_e \left| \frac{b}{a} \right|$, $\therefore f(x) = \frac{1}{x}$ for $x \in \mathbb{R} \setminus \{0\}$

$\therefore f(x)$ is discontinuous at $x = 0$

$\therefore \int_{-1}^2 f(x) dx$ is undefined.

Q16 $\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} g(x) dx = \int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} \left(|x - \pi| + \sin x - \frac{\pi}{2} \right) dx \approx -2.19$ by CAS

Average value $= \frac{\int_{\frac{2\pi}{3}}^{\frac{4\pi}{3}} g(x) dx}{\frac{4\pi}{3} - \frac{2\pi}{3}} \approx \frac{-2.19}{\frac{2\pi}{3}} \approx -1.04$

Q17 $f(\theta) = \cos^2 \theta + 2 \sin^3 \theta - 1 = (\cos^2 \theta - 1) + 2 \sin \theta \sin^2 \theta$
 $= (\cos^2 \theta - 1) - 2 \sin \theta (\cos^2 \theta - 1)$
 $= (\cos^2 \theta - 1)(1 - 2 \sin \theta)$
 $= (\cos \theta - 1)(\cos \theta + 1)(1 - 2 \sin \theta)$

Q18

Q19 $\Pr(ABABB) = 1 \times b \times a \times b \times (1-a) = b^2 a (1-a)$

Q20 Binomial:

${}^{10}C_5 p^5 (1-p)^5 = 0.1$, ${}^{10}C_6 p^6 (1-p)^4 = 0.2$

$\frac{{}^{10}C_6 p^6 (1-p)^4}{{}^{10}C_5 p^5 (1-p)^5} = \frac{0.2}{0.1}$, $\frac{p}{1-p} = \frac{12}{5}$, $p = \frac{12}{17} \approx 0.7$

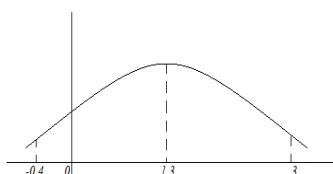
Q21 $\Pr(A) - \Pr(A|B')\Pr(B') = \Pr(A) - \Pr(A \cap B')$
 $= \Pr(A \cap B) = \Pr(B) - \Pr(B \cap A')$
 $= \Pr(B) - \Pr(B|A')\Pr(A')$

Q22 -0.4 and 3 are the same distance from 1.3 .

Given $\Pr(X < 3) = 0.85$, $\therefore \Pr(X > 3) = 0.15$ and

$\Pr(X < -0.4) = 0.15$

$\therefore \Pr(-0.4 < X < 3) = 0.70$



SECTION 2

B Q1a $(3,0)$ and $(0,p)$ are the x and y intercepts respectively.

$\frac{x}{3} + \frac{y}{p} = 1$, $\therefore y = -\frac{p}{3}(x-3)$

Q1b $y = (x-3)(x^2 + bx + c)$
 $(0,-3)$, $-3 = (-3)(c)$, $\therefore c = 1$
 $(-1,-4)$, $-4 = (-4)(1-b+1)$, $\therefore b = 1$

Q1ci Equation of the cubic function is $y = (x-3)(x^2 + x + 1)$.

Solve the above equation and $y = -\frac{p}{3}(x-3)$ simultaneously.

E $(x-3)(x^2 + x + 1) = -\frac{p}{3}(x-3)$, $(x-3)(x^2 + x + 1) + \frac{p}{3}(x-3) = 0$

$(x-3)\left(x^2 + x + 1 + \frac{p}{3}\right) = 0$

$x-3 = 0$, $\therefore x = 3$ is the x -coordinate of one of the intersections.

The other intersection(s) comes from $x^2 + x + 1 + \frac{p}{3} = 0$.

B One solution only: $b^2 - 4ac = 1^2 - 4\left(1 + \frac{p}{3}\right) = 0$, $\therefore p = -\frac{9}{4}$

Q1cii $x^2 + x + 1 + \frac{1}{3} \times \frac{-9}{4} = 0$, $x^2 + x + \frac{1}{4} = 0$, $\left(x + \frac{1}{2}\right)^2 = 0$,

$\therefore x = -\frac{1}{2}$, $y = -\frac{p}{3}(x-3) = -\frac{1}{3} \times \frac{-9}{4} \left(-\frac{1}{2} - 3\right) = -\frac{21}{8}$,

E $\left(-\frac{1}{2}, -\frac{21}{8}\right)$

E

Q1d $y = (x-3)(x^2 + x + 1) = x^3 - 2x^2 - 2x - 3$

A $\frac{dy}{dx} = 3x^2 - 4x - 2 = -\frac{p}{3}$ where $p = -\frac{9}{4}$

$\therefore 3x^2 - 4x - 2 = \frac{3}{4}$, $12x^2 - 16x - 11 = 0$, $(6x-11)(2x+1) = 0$

E $\therefore x = \frac{11}{6}$ and $y = -\frac{1561}{216}$, the point is $\left(\frac{11}{6}, -\frac{1561}{216}\right)$.

Q1ei $-3x + 18 = -x^3 + 4x^2 + 8x + 24$, $x = -1$ or 6 by CAS

Q1eii $-\frac{1}{2} \rightarrow -1$ and $3 \rightarrow 6$, the factor of dilation from the y -axis is 2 .

Q1fi and ii $\int_{-1}^6 (-x^3 + 4x^2 + 8x + 24 - (-3x + 18)) dx$

C $= \int_{-1}^6 (-x^3 + 4x^2 + 11x + 6) dx = \frac{2401}{12}$

Q1g Original line: y -intercept is $\left(0, -\frac{9}{4}\right)$; new line: y -intercept

is $(0,18)$. The factor of dilation from the x -axis is $\frac{18}{-\frac{9}{4}} = 8$.

Q2a

$$a \log_e(3+b) + c = 1 \dots\dots\dots(1)$$

$$a \log_e(2+b) + c = 0 \dots\dots\dots(2)$$

$$a \log_e(1.5+b) + c = -1 \dots\dots\dots(3)$$

Q2b

$$(1) - (2): a \log_e \frac{3+b}{2+b} = 1 \dots\dots\dots(4)$$

$$(2) - (3): a \log_e \frac{2+b}{1.5+b} = 1 \dots\dots\dots(5)$$

$$\therefore \log_e \frac{3+b}{2+b} = \log_e \frac{2+b}{1.5+b}, \therefore \frac{3+b}{2+b} = \frac{2+b}{1.5+b}, \therefore b = -1$$

Substitute $b = -1$ in (2): $c = 0$

$$\text{Substitute } b = -1 \text{ and } c = 0 \text{ in (1): } a = \frac{1}{\log_e 2}$$

$$\text{Q2c Wall A: } y = \frac{1}{\log_e 2} \log_e(x-1)$$

Reflection of $y = \frac{1}{\log_e 2} \log_e(x-1)$ in the line $y = x$ is

$$x = \frac{1}{\log_e 2} \log_e(y-1), \therefore (\log_e 2)x = \log_e(y-1), y-1 = (e^{\log_e 2})^x,$$

$$\therefore y = 2^x + 1$$

Q2d NE direction: $m = 1$

$$y = \frac{1}{\log_e 2} \log_e(x-1), \frac{dy}{dx} = \frac{1}{(\log_e 2)(x-1)} = 1, \therefore x = \frac{1}{\log_e 2} + 1$$

$$\therefore y = \frac{1}{\log_e 2} \log_e \left(\frac{1}{\log_e 2} \right) = -\frac{\log_e(\log_e 2)}{\log_e 2}$$

$$\left(\frac{1}{\log_e 2} + 1, -\frac{\log_e(\log_e 2)}{\log_e 2} \right)$$

Q2e The point on Wall B with a NE tangent is

$$\left(-\frac{\log_e(\log_e 2)}{\log_e 2}, \frac{1}{\log_e 2} + 1 \right).$$

The shortest distance is the distance between

$$\left(\frac{1}{\log_e 2} + 1, -\frac{\log_e(\log_e 2)}{\log_e 2} \right) \text{ and } \left(-\frac{\log_e(\log_e 2)}{\log_e 2}, \frac{1}{\log_e 2} + 1 \right).$$

$$\text{The shortest distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \approx 2.71 \text{ m}$$

$$\text{Q2f Area} = 3 \times 3 - 2 \int_2^3 \frac{1}{\log_e 2} \log_e(x-1) dx$$

$$\approx 9 - 2 \times 0.5573 \approx 7.89 \text{ m}^2 \text{ by CAS}$$

Q3a When the depth is h metres, $V = \pi r^2 h = \pi^3 h$, $\frac{dV}{dh} = \pi^3$.

$$\text{Related rates: } \frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}, \therefore \frac{dV}{dt} = \pi^3 \times \frac{dh}{dt},$$

$$\frac{dh}{dt} = \frac{1}{\pi^3} \left(\sin \frac{2t}{\pi} + \cos \frac{2t}{\pi} + \pi \right)$$

$$\text{Q3b } T = \frac{2\pi}{n} = \frac{2\pi}{\frac{2}{\pi}} = \pi^2 \text{ minutes}$$

$$\text{Q3ci Average over } n \text{ periods} = \frac{\int_0^{n\pi^2} \left(\sin \frac{2t}{\pi} + \cos \frac{2t}{\pi} + \pi \right) dt}{n\pi^2}$$

$$= \frac{\int_0^{n\pi^2} (\pi) dt}{n\pi^2} = \frac{[\pi t]_0^{n\pi^2}}{n\pi^2} = \frac{n\pi^3}{n\pi^2} = \pi \text{ m}^3 \text{ min}^{-1}$$

$$\text{Q3cii Volume of the tank} = 4\pi^3 \text{ m}^3, \text{ time} = \frac{4\pi^3}{\pi} = 4\pi^2 \text{ min},$$

$$\text{number of periods} = \frac{4\pi^2}{\pi^2} = 4$$

$$\text{Q3d Volume to be filled} = \pi r^2 h = \pi(3^2)(4) = 36\pi \text{ m}^3$$

Let τ minutes be the time to fill the second tank.

$$\frac{dV}{dt} = \sin \frac{2t}{\pi} + \cos \frac{2t}{\pi} + \pi$$

$$V = \int_0^\tau \left(\sin \frac{2t}{\pi} + \cos \frac{2t}{\pi} + \pi \right) dt = 36\pi$$

$\tau \approx 35.3$ minutes by CAS

Q3e Find the local maxima and minima of

$$\frac{dh}{dt} = \frac{1}{\pi^3} \left(\sin \frac{2t}{\pi} + \cos \frac{2t}{\pi} + \pi \right), \text{ let } \frac{d}{dt} \left(\frac{dh}{dt} \right) = 0$$

$$\therefore \frac{2}{\pi^4} \left(\cos \frac{2t}{\pi} - \sin \frac{2t}{\pi} \right) = 0, \sin \frac{2t}{\pi} = \cos \frac{2t}{\pi}, \tan \frac{2t}{\pi} = 1 \text{ and}$$

$$0 < t < 35.3, \text{ i.e. } 0 < \frac{2t}{\pi} < 22.4727$$

$$\therefore \frac{2t}{\pi} = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}, \frac{25\pi}{4}$$

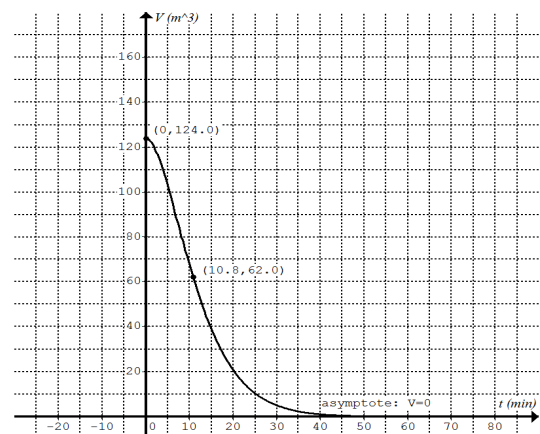
$$\therefore t = \frac{\pi^2}{8}, \frac{5\pi^2}{8}, \frac{9\pi^2}{8}, \frac{13\pi^2}{8}, \frac{17\pi^2}{8}, \frac{21\pi^2}{8}, \frac{25\pi^2}{8}$$

The greatest rate of increase occurs when

$$t = \frac{\pi^2}{8}, \frac{9\pi^2}{8}, \frac{17\pi^2}{8}, \frac{25\pi^2}{8}$$

$$\text{Q3fi } 9.164(t+10)^2 e^{-(0.2t+2)} = \frac{1}{2} \times 4\pi^3, t \approx 10.8 \text{ min by CAS}$$

Q3fii



Q4a $\Pr(A > 5)$, by CAS $\text{normalcdf}(5, e^{99}, 8.53, 5.23) \approx 0.75$

Q4b $\Pr(7.00 - 5.23 < A < 7.00 + 5.23) = \Pr(1.77 < A < 12.23) \approx 0.66$

Q4ci $\Pr(X = 7)$, by CAS $\text{binompdf}(10, 0.75, 7) \approx 0.25$

Q4cii $E(X) = np = 10 \times 0.75 \approx 8$

Q4d $\Pr(B > 5) = 0.6615$ and $\Pr(B < 6) = 0.4013$

$\therefore \Pr\left(Z > \frac{5 - \mu}{\sigma}\right) = 0.6615$, i.e. $\Pr\left(Z < \frac{5 - \mu}{\sigma}\right) = 0.3385$ and

$\Pr\left(Z < \frac{6 - \mu}{\sigma}\right) = 0.4013$

By CAS, $\text{invNorm}, \frac{5 - \mu}{\sigma} \approx -0.41656$ and $\frac{6 - \mu}{\sigma} \approx -0.24998$

$\therefore \mu = 7.50$ and $\sigma = 6.00$ minutes

Q4e $E(B) = \mu = 7.50$

Q4f $\Pr(B < 6 | B > 5) = \frac{\Pr(B < 6 \cap B > 5)}{\Pr(B > 5)} = \frac{\Pr(5 < B < 6)}{\Pr(B > 5)}$
 $\approx \frac{0.06283}{0.66154} \approx 0.095$

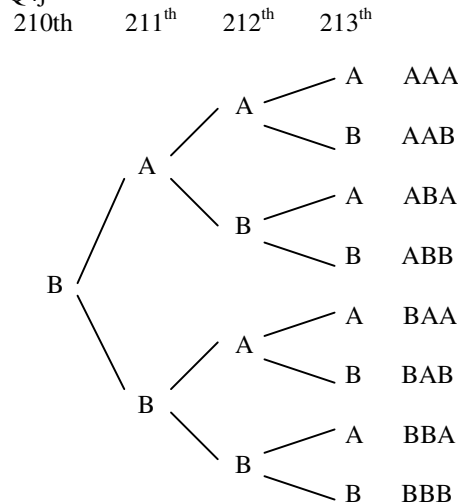
Q4g $A \rightarrow A, \frac{2}{5}; A \rightarrow B, \frac{3}{5}; B \rightarrow A, \frac{1}{4}; B \rightarrow B, \frac{3}{4}$
 $\therefore$ she shopped at Bestbuy last time.

Q4h $\Pr(AB \cup BA) = \Pr(AB) + \Pr(BA) = \frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{3}{4} = \frac{51}{80}$

Q4i Transition matrix = $\begin{bmatrix} \frac{2}{5} & \frac{3}{4} \\ \frac{3}{5} & \frac{1}{4} \end{bmatrix}$ $\begin{matrix} A \\ B \end{matrix}$

In the long run, probability of shopping at Bestbuy = $\frac{\frac{3}{5}}{\frac{3}{4} + \frac{3}{5}} = \frac{4}{9}$

Q4j



Let X be the number of times that Sofia goes to Bestbuy in her next three shopping trips.

$\Pr(AAA) = \frac{3}{4} \times \frac{2}{5} \times \frac{2}{5}, \Pr(AAB) = \frac{3}{4} \times \frac{2}{5} \times \frac{3}{5},$

$\Pr(ABA) = \frac{3}{4} \times \frac{3}{5} \times \frac{3}{4}, \Pr(ABB) = \frac{3}{4} \times \frac{3}{5} \times \frac{1}{4}$

$\Pr(BAA) = \frac{1}{4} \times \frac{3}{4} \times \frac{2}{5}, \Pr(BAB) = \frac{1}{4} \times \frac{3}{4} \times \frac{3}{5}$

$\Pr(BBA) = \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4}, \Pr(BBB) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

x	0	1	2	3
$\Pr(X = x)$	0.12	0.5925	0.271875	0.015625

$E(X) = 0 \times 0.12 + 1 \times 0.5925 + 2 \times 0.271875 + 3 \times 0.015625 \approx 1.18$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors