



Q1a $f(x) = ax^2 + bx + c$, $f'(x) = 2ax + b$

Q1b $f(g(x)) = a(g(x))^2 + b(g(x)) + c = a(2x+3)^2 + b(2x+3) + c$
 $= a(4x^2 + 12x + 9) + b(2x+3) + c$
 $= 4ax^2 + 2(6a+b)x + (9a+3b+c)$

Q1c $f(f'(x)) = f'(f(x))$

$a(2ax+b)^2 + b(2ax+b) + c = 2a(ax^2 + bx + c) + b$

$4a^3x^2 + 4a^2bx + ab^2 + b^2 + c = 2a^2x^2 + 2ac + b$

Compare the coefficients:

$4a^3 = 2a^2$, $4a^2b = 0$ and $ab^2 + b^2 + c = 2ac + b$

Since $f(x) = ax^2 + bx + c$ is a quadratic function,

$\therefore a \neq 0$, $\therefore b = 0$

$4a^3 = 2a^2$, $\therefore 4a^3 - 2a^2 = 0$, $2a^2(2a-1) = 0$, $\therefore a = \frac{1}{2}$

$ab^2 + b^2 + c = 2ac + b$, $\therefore 2ac = c$, $2ac - c = 0$, $c(2a-1) = 0$

Since $2a-1=0$, $\therefore c$ can be any real number.

Q2a Transition matrix $T = \begin{bmatrix} 0.30 & 0.60 \\ 0.70 & 0.40 \end{bmatrix}$, state matrix $S_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$S_5 = T^2 S_3 = \begin{bmatrix} 0.30 & 0.60 \\ 0.70 & 0.40 \end{bmatrix}^2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.42 \\ 0.58 \end{bmatrix}$ by hand

$\therefore \text{Pr}(\text{sugar in the fifth}) = 0.42$

Q2b $S_3 = T^2 S_1 = \begin{bmatrix} 0.30 & 0.60 \\ 0.70 & 0.40 \end{bmatrix}^2 \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.42 \\ 0.58 \end{bmatrix}$

$\therefore S_1 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ by recognition

$\therefore$ he does not have sugar in his first drink.

Q3a $f(x) = x^3 - 8x^2 - 4x + 32 = (x^3 - 8x^2) - (4x - 32)$
 $= x^2(x-8) - 4(x-8) = (x^2 - 4)(x-8) = (x-2)(x+2)(x-8)$

Q3b $2^{3y} - 2^{2y+3} - 2^{y+2} + 2^5 = 0$, $2^{3y} - 8 \times 2^{2y} - 4 \times 2^y + 32 = 0$,
 $(2^y)^3 - 8(2^y)^2 - 4(2^y) + 32 = 0$
 $\therefore (2^y - 2)(2^y + 2)(2^y - 8) = 0$

Since $2^y + 2 > 0$, $\therefore 2^y - 2 = 0$ or $2^y - 8 = 0$

$\therefore y = 1$ or 3

Q4a $y = \log_2 x = \frac{\log_e x}{\log_e 2}$, $\frac{dy}{dx} = \frac{1}{x \log_e 2}$

At $x = 2$, gradient of the tangent $m_t = \frac{1}{2 \log_e 2}$

Q4b At $x = 2$, $y = \log_2 2 = 1$

Equation of the tangent: $y - y_1 = m_t(x - x_1)$,

$y - 1 = \frac{1}{2 \log_e 2}(x - 2)$

y-intercept: Let $x = 0$, $y = 1 - \frac{1}{\log_e 2}$, $\therefore \left(0, 1 - \frac{1}{\log_e 2}\right)$

Q5 The line $y = x$ is a tangent to the graphs of $y = f^{-1}(x)$ and $y = f(x)$ at (a, a) .

$\therefore$ at (a, a) , the gradient of $y = f(x) = e^{x-a} + 1$ is 1.

The graph of $y = e^{x-a} + 1$ is the horizontal translation of the graph $y = e^x + 1$. At $(0, 2)$, the gradient of the graph $y = e^x + 1$

is $\frac{dy}{dx} = e^x = e^0 = 1$, $\therefore a = 2$

Q6a $\cos\left(\frac{\pi}{12} - \frac{3\pi}{4}\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

Q6b $\sin\left(\frac{\pi}{2} - 3x\right) + \frac{1}{2} = 0$, $\frac{\pi}{6} < x < \frac{\pi}{2}$

$\cos(3x) + \frac{1}{2} = 0$, $\frac{\pi}{2} < 3x < \frac{3\pi}{2}$

$\therefore \cos(3x) = -\frac{1}{2}$, $3x = \frac{2\pi}{3}$, $\frac{4\pi}{3}$

$\therefore x = \frac{2\pi}{9}$, $\frac{4\pi}{9}$

Q7a $\int_0^a \frac{x+1}{4} dx = 1$, $\left[\frac{x^2}{8} + \frac{x}{4}\right]_0^a = 1$, $\frac{a^2}{8} + \frac{a}{4} = 1$

$a^2 + 2a - 8 = 0$, $(a-2)(a+4) = 0$

Since $a > 0$, $\therefore a = 2$

Q7b Average value of $f(x) = \frac{\int_0^2 \frac{x+1}{4} dx}{2-0} = \frac{1}{2}$ in $0 \leq x \leq 2$.

Q7c Average value of $X = \int_0^2 x \left(\frac{x+1}{4}\right) dx = \int_0^2 \frac{x^2 + x}{4} dx$

$= \left[\frac{x^3}{12} + \frac{x^2}{8}\right]_0^2 = \frac{7}{6}$

Q8a The outcomes of distinct probability values are:

3 reds, 3 blues, 3 greens, 2 reds and 1 blue, 2 reds and 1 green, 2 blues and 1 red, 2 blues and 1 green, 2 greens and 1 red, 2 greens and 1 blue, 1 of each colour.

$\therefore$ 10 distinct probability values.

Q8b There are six possible outcomes consisting of different colours.

$$\Pr(\text{different colours}) = 6 \times \frac{1}{6} \times \frac{2}{6} \times \frac{3}{6} = \frac{1}{6}$$

Q8c Let X be the random variable of number of blue faces.

X has binomial distribution: $n = 3$, $p = \frac{2}{6} = \frac{1}{3}$

$$\Pr(X=1) = {}^3C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Q9a $f(x) = \frac{1}{5} \sin(5x-1)$, $f'(x) = \cos(5x-1)$, $f'(0.2) = \cos 0 = 1$

Q9b $f(a+h) \approx f(a) + h \times f'(a)$

$$f(0.21) = f(0.20 + 0.01)$$

$$\approx f(0.20) + 0.01 \times f'(0.20) = 0 + 0.01 \times 1 = 0.01$$

Q10a $y = 2 - f(1-x) \xrightarrow{(1)} x = 2 - f(1-y)$

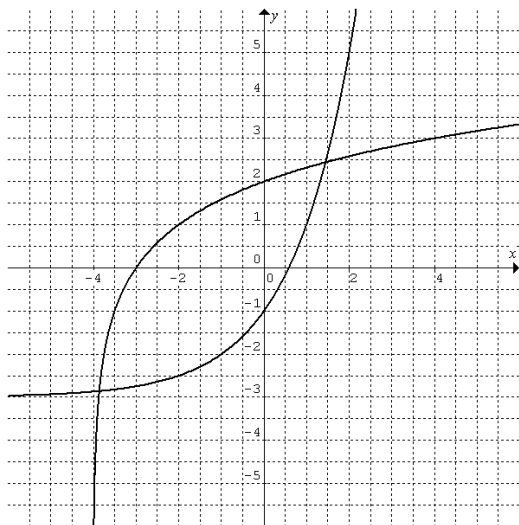
$$\xrightarrow{(2)} x = 2 - f(2-y)$$

$$\xrightarrow{(3)} x = 1 - f(2-y)$$

A sequence: (1) Reflection in the line $y = x$ (2) Upward translation by 1 unit (3) Translation to the left by 1 unit.

There are other possible sequences.

Q10b



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