



Victorian Certificate of Education 2012

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS (CAS)

Written examination 1

Wednesday 7 November 2012

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 10 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

a. If $y = (x^2 - 5x)^4$, find $\frac{dy}{dx}$.

1 mark

b. If $f(x) = \frac{x}{\sin(x)}$, find $f'\left(\frac{\pi}{2}\right)$.

2 marks

TURN OVER

Question 2

Find an anti-derivative of $\frac{1}{(2x-1)^3}$ with respect to x .

2 marks

Question 3

The rule for function h is $h(x) = 2x^3 + 1$. Find the rule for the inverse function h^{-1} .

2 marks

Question 4

On any given day, the number X of telephone calls that Daniel receives is a random variable with probability distribution given by

x	0	1	2	3
$\Pr(X = x)$	0.2	0.2	0.5	0.1

- a. Find the mean of X .

2 marks

- b. What is the probability that Daniel receives only one telephone call on each of three consecutive days?

1 mark

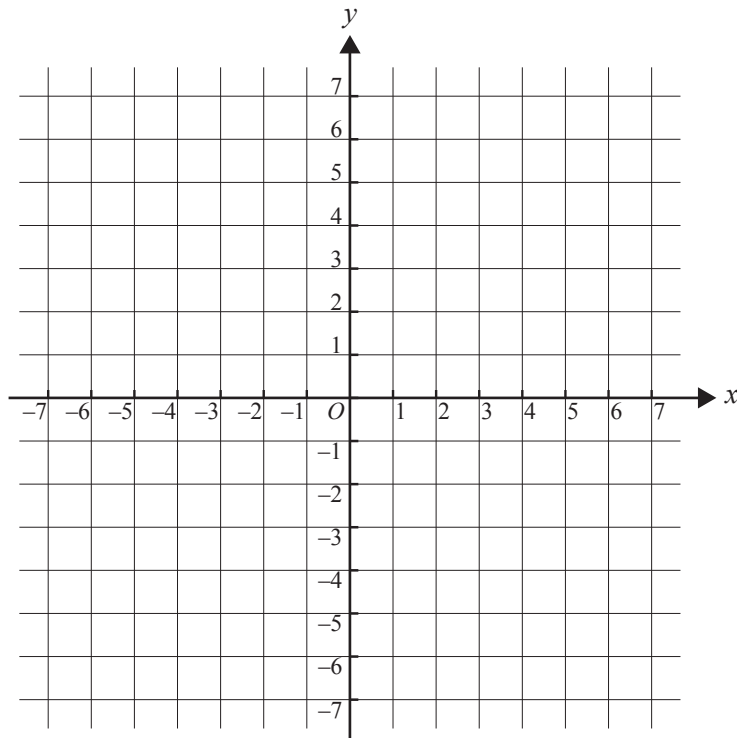
- c. Daniel receives telephone calls on both Monday and Tuesday.
What is the probability that Daniel receives a total of four calls over these two days?

3 marks

TURN OVER

Question 5

- a. Sketch the graph of $f: [0, 5] \rightarrow \mathbb{R}$, $f(x) = -|x - 3| + 2$. Label the axes intercepts and endpoints with their coordinates.



3 marks

- b. i. Find the coordinates of the image of the point $(3, 2)$ under a reflection in the x -axis, followed by a translation of 5 units in the positive direction of the x -axis.

- ii. Find the equation of the image of the graph of f under a reflection in the x -axis, followed by a translation of 5 units in the positive direction of the x -axis.

1 + 2 = 3 marks

Question 6

The graphs of $y = \cos(x)$ and $y = a \sin(x)$, where a is a real constant, have a point of intersection at $x = \frac{\pi}{3}$.

- a. Find the value of a .

2 marks

- b. If $x \in [0, 2\pi]$, find the x -coordinate of the other point of intersection of the two graphs.

1 mark

Question 7

Solve the equation $2 \log_e(x + 2) - \log_e(x) = \log_e(2x + 1)$, where $x > 0$, for x .

3 marks

TURN OVER

a. Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x \sin(x)$.
Find $f'(x)$.

b. Use the result of **part a.** to find the value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \cos(x) dx$ in the form $a\pi + b$.

[illegible]

TURN OVER

Question 10

Let $f: R \rightarrow R$, $f(x) = e^{-mx} + 3x$, where m is a positive rational number.

- a.** **i.** Find, in terms of m , the x -coordinate of the stationary point of the graph of $y = f(x)$.

- ii.** State the values of m such that the x -coordinate of this stationary point is a positive number.

2 + 1 = 3 marks

- b.** For a particular value of m , the tangent to the graph of $y = f(x)$ at $x = -6$ passes through the origin.
Find this value of m .

3 marks

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods (CAS)

Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc \sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	transition matrices: $S_n = T^n \times S_0$
mean: $\mu = E(X)$	variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X=x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$