

Student Name:

MATHEMATICAL METHODS (CAS)

Unit 4

Targeted Evaluation Task for School-assessed Coursework 3



2012 Applications Analysis Task on Integration for Outcomes 1, 2 & 3

Recommended writing time*: 120 minutes

Total number of marks available: 40 marks

TASK BOOK

* The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers, bound summary booklet, approved CAS calculator.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.
- Show appropriate scales on the axes provided when sketching graphs.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

Any question worth more than 1 mark, relevant working must be shown.

Question 1

The rate of change of the velocity of a car is given by:

$$v'(t) = 9 - 3t + 2\sin t, \quad t \geq 0$$

- a.** Use calculus to find the rule for $v(t)$ if the initial velocity is 1 ms^{-1} .

3 marks

- b.** At what time (correct to 3 decimal places) does the velocity become zero?

1 mark

- c.** Use calculus to find the displacement of the car over the first 4 seconds. Give your answer correct to 3 decimal places.

3 marks

- d.** What feature of the graph of $v(t)$ represents the quantity you found in part c.

1 mark

Total 8 marks

- b.** Why is it impossible to use calculus to find the exact area bounded by $f(x)$ and the x -axis, between $x = 0$ and $x = 1$? How could an approximation to this area be found while still using calculus?

2 marks

Total 9 marks

Question 3

- a.** Use CAS to find the exact indefinite integral of $e^{-x}\left(\frac{1}{x} - \ln 5x\right)$.

2 marks

- b.** What other information is required to correctly find the area bounded by the graph of $y = e^{-x}\left(\frac{1}{x} - \ln 5x\right)$, the x -axis and the lines $x = 0.5$ and $x = 1.5$?

1 mark

- c.** Find this information correct to 4 decimal places.

1 mark

- d.** Write out the integral which needs to be evaluated to find the area defined in part **b**.

2 marks

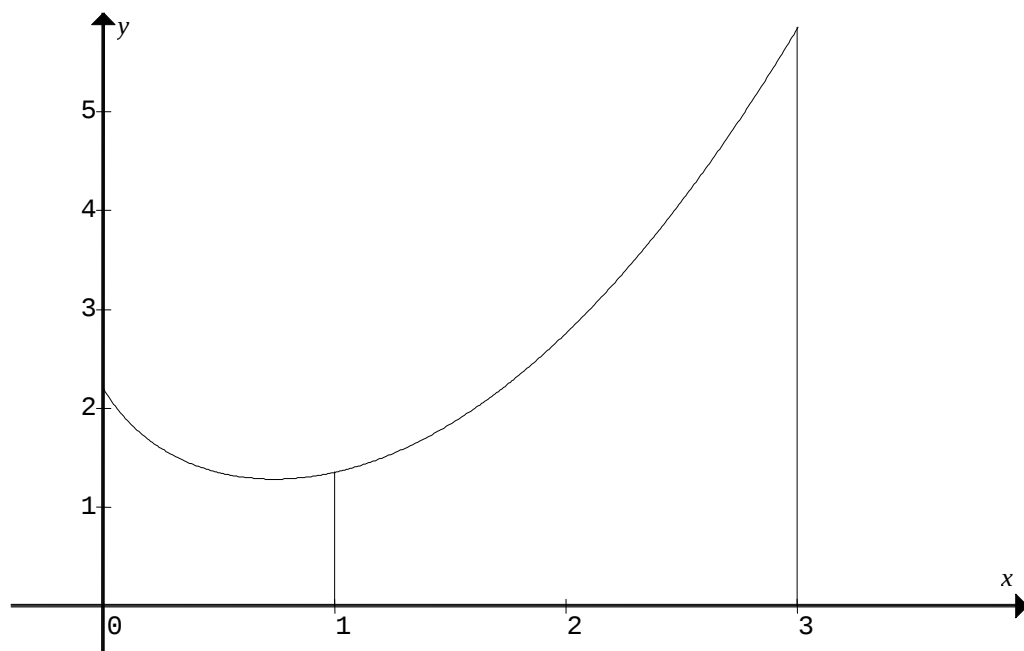
- e.** Hence find the required area correct to 3 decimal places.

1 mark

Total 7 marks

Question 4

A new building is being designed and one of the walls can be modelled by the graph below.



The base of the wall is the x -axis and the top of the wall is represented by the function

$$h(x) = \frac{1}{3x+1} + e^{-0.1x} + \frac{(2x-1)^2}{5}, \quad 0 \leq x \leq 3 \quad \text{where } x \text{ and } h \text{ are in metres.}$$

The section of the wall between $x=1$ and $x=3$ is to be constructed of concrete 0.25 m thick.

- a. Use the table function on your calculator to fill in the table below. Write in the values of $f(x)$ correct to 4 decimal places.

x	$f(x)$
1.0	
1.2	
1.4	
1.6	
1.8	
2.0	
2.2	
2.4	
2.6	
2.8	
3.0	

2 marks

- b.** Estimate the volume of concrete required (correct to 3 decimal places) to construct this section of the wall by dividing the required region of the wall into rectangles of width 0.2 m and finding the area of the:

- i.** Lower rectangles

2 marks

- ii.** Upper rectangles

2 marks

- c.** Use the two values found in part **b.** to calculate a better estimate of the amount of concrete required.

1 mark

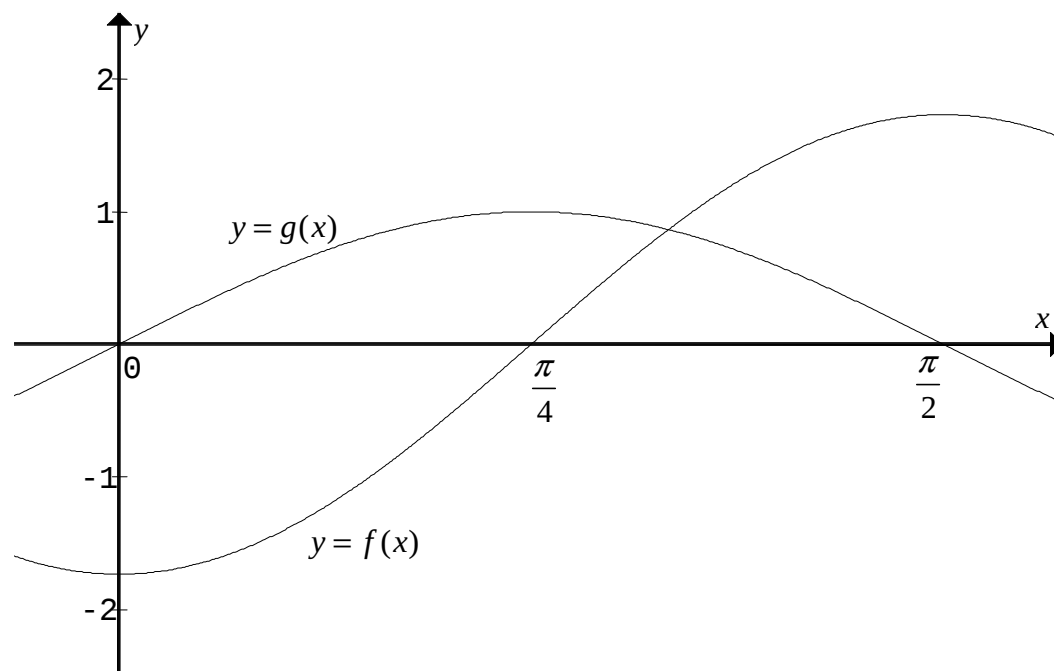
- d.** Use calculus to find the exact volume of concrete required.

4 marks

Total 11 marks

Question 5

The graphs of $f(x) = -\sqrt{3} \cos 2x$ and $g(x) = \sin 2x$ are shown on the set of axes below.



- a. Find the exact x -coordinate of the intersection of $y = f(x)$ and $y = g(x)$ for $0 \leq x \leq \frac{\pi}{2}$.

1 mark

- b.** Use calculus to find the area enclosed by $y = f(x)$, $y = g(x)$ and the lines

$$x = 0 \text{ and } x = \frac{\pi}{2}$$

4 marks

Total 5 marks

END TASK BOOK