

MATHEMATICAL METHODS (CAS)

Unit 4

Targeted Evaluation Task for School-assessed Coursework 2



2012 Modelling Analysis Task on Trigonometric Functions for

Outcomes 1, 2 & 3

SOLUTIONS & RESPONSE GUIDE

The marks given are allocated to the 3 outcomes according to the following:

A – Outcome 1, B – Outcome 2, C – Outcome 3

Question 1

- a. The amplitude of the variation in depth is $\frac{9.5-0.5}{2} = 4.5$ m so $a = 4.5$

The average depth of water is $\frac{9.5+0.5}{2} = 5$ m so $c = 5$

Now the period of the trigonometric function $T = \frac{2\pi}{b}$

$$\text{So } \frac{25}{2} = \frac{2\pi}{b}$$

$$b = \frac{4\pi}{25}$$

A3

- b. First maximum occurs when $\sin\left(\frac{4\pi t}{25}\right)$ first equals 1. That is, when $\frac{4\pi t}{25} = \frac{\pi}{2}$

$$t = \frac{25}{8} = 3.125 \text{ hrs or } 3 \text{ hrs } 7.5 \text{ min. Therefore the first maximum depth will occur at}$$

12.08 pm on Sept 2. (Accept 12.07 pm.)

A1

The first minimum occurs when $\sin\left(\frac{4\pi t}{25}\right)$ first equals -1 . That is, when $\frac{4\pi t}{25} = \frac{3\pi}{2}$

$$t = \frac{75}{8} = 9.375 \text{ hrs or } 9 \text{ hrs } 22.5 \text{ min. Therefore the first minimum depth will occur at}$$

6.23 pm on Sept 2. (Accept 6.22 pm.).

This may also be calculated as being 6 hrs 15 min (that is, half a period) after the time of the first maximum.

A1

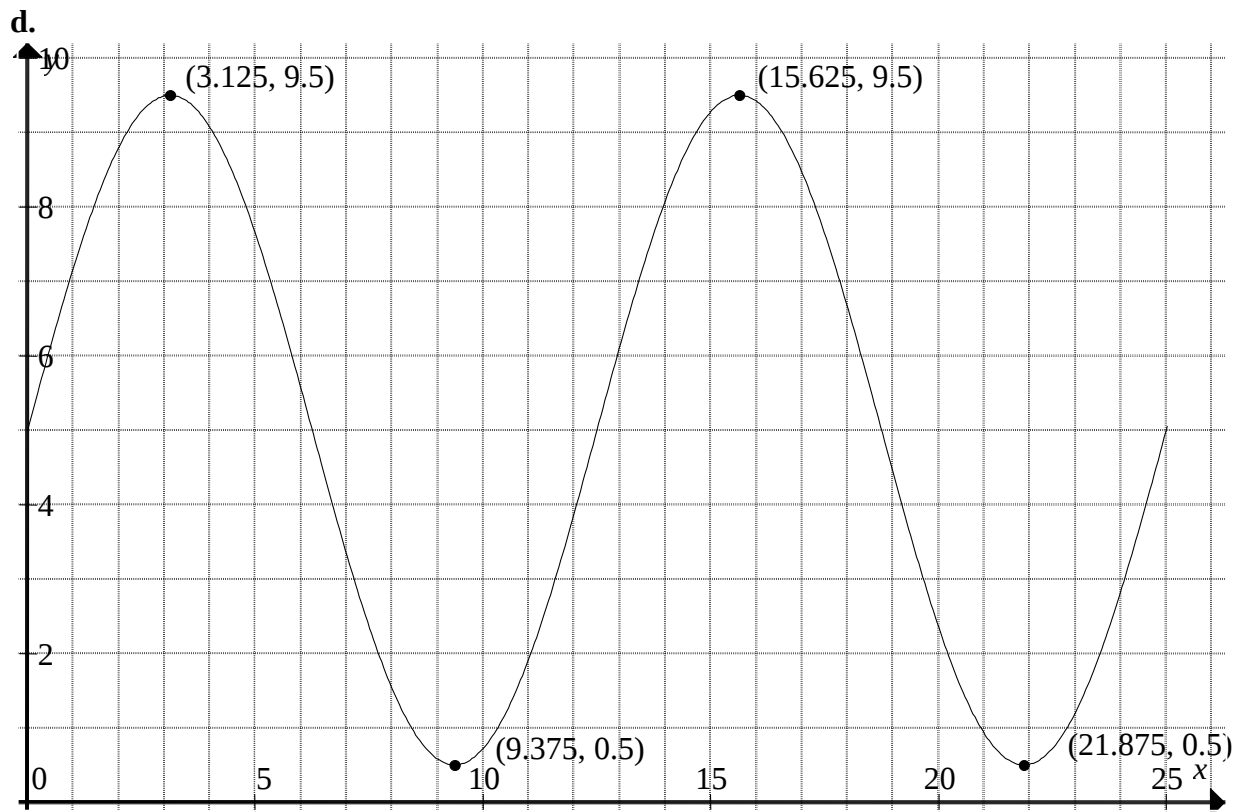
- c. This time is 45 hrs and 15 min after 9.00 am on Sept 2 therefore $t = 45.25$

B1

$$d_1(45.25) = 4.5 \sin\left(\frac{181\pi}{25}\right) + 5 = 1.91954$$

Depth is 192 cm at 6.15 am on Sept 4

A1



Correct graph and scale A2
Turning points correctly labelled B2

Question 2

a. $4.5 \sin\left(\frac{4\pi t}{25}\right) + 5 = 8.2$

A1

$$\frac{4\pi t}{25} = 0.7911, 2.3505, 7.0743, 8.6337$$

$$t = 1.5738, 4.6762, 14.0739, 17.1762$$

A1

Coordinates on graph; (1.5738, 8.2) (4.6762, 8.2) (14.0739, 8.2) (17.1762, 8.2)

B1

The depth will be 8.2 m at 10.34 am, 1.41 pm, and 11.04 pm on Sept 2 and 2.11 am on Sept 3.

B2

- b.** Graph $y = d(t)$ and $y = 3.5$ on a graphics calculator and find the t -values of the intersection points of the 2 graphs.
 $d_1(t) \geq 3.5$ from $t = 0$ to the first intersection point at $t = 6.9261$, that is for a total of 6.9261 hours.

C1

$d_1(t) \geq 3.5$ again between the second and third intersection points at $t = 11.8239$ and $t = 19.4261$ for a total of $19.4261 - 11.8239 = 7.6022$ hours.

C1

The cyclical nature of this trigonometric function means that there will be 2 more periods of 7.6022 hours in the first 48 hours for a total time of $6.9261 + 3 \times 7.6022 = 29.7327$ hours.

B1

This is a percentage time of $\frac{29.7327 \times 100}{48} = 61.9\%$

A1

Question 3

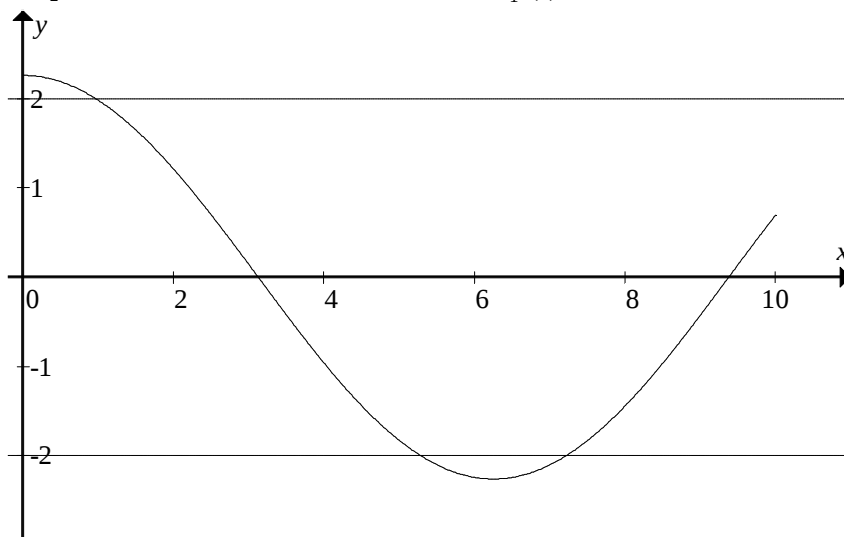
a. $d_1'(t) = \frac{18\pi}{25} \cos\left[\frac{4\pi t}{25}\right]$

A1

b. $\frac{18\pi}{25} = 2.26$ metres per hour

A1

- c.** Graph $y = d_1'(t)$, $y = 2$ and $y = -2$ as shown in the graph below and find the intersection points of the horizontal lines with $d_1'(t)$.



Intersection of $y = d_1'(t)$ and $y = 2$ at $t = 0.9669$

Intersection of $y = d_1'(t)$ and $y = -2$ at $t = 5.2831$ and $t = 7.2169$

C2

Therefore the ship can enter during the interval $t = 0.9669$ to $t = 5.2831$ which is a total of $5.2831 - 0.9669 = 4.3162$ hours as the depth is greater than 3.5 m during this time. Ship cannot enter between $t = 7.2169$ and $t = 10$ as depth is too shallow as found in **Question 2 b**. Thus the ship can enter the port during a time interval of 4 hours 19 minutes.

B2

Question 4

- a. The difference between successive high and low tides starts off quite large (almost 9 m) then decreases to almost zero and increases to about 9 m again before starting to decrease again.

C1, B1

- b. The period of this variation is determined by the cosine term.

$$\text{Period of } \cos\left(\frac{\pi t}{168}\right) \text{ is } \frac{2\pi}{\frac{\pi}{168}} = 2 \times 168 = 336 \text{ hours}$$

But the period of the variation in the difference between the heights of successive high and low tides is actually half of this value, that is 168 hours.

B2

- c. Using the maximum and minimum calculation functions of a graphics calculator and looking in the region between, say, $t = 80$ and $t = 90$ gives:
Lowest high tide of 5.12 metres at $t = 85.95$ hours and
Highest low tide of 4.93 metres at $t = 82.52$ hours.

C2

Question 5

- a. $d_2(t) = 4.5 \cos\left(\frac{\pi t}{168}\right) \sin\left(\frac{4\pi t}{25}\right) + 5$ and using the product rule

$$d_2'(t) = 4.5 \left[-\frac{\pi}{168} \sin\left(\frac{\pi t}{168}\right) \sin\left(\frac{4\pi t}{25}\right) + \cos\left(\frac{\pi t}{168}\right) \times \frac{4\pi}{25} \cos\left(\frac{4\pi t}{25}\right) \right]$$

$$d_2'(t) = 4.5 \left[\frac{4\pi}{25} \cos\left(\frac{\pi t}{168}\right) \cos\left(\frac{4\pi t}{25}\right) - \frac{\pi}{168} \sin\left(\frac{\pi t}{168}\right) \sin\left(\frac{4\pi t}{25}\right) \right]$$

B2

- b. i (6.233, -2.247)

C1

ii Find largest of the intersection points between $y = d_2'(t)$, $y = 2$ and $y = -2$.

$T = 25.2098$ which gives a time of 10.13 am Sept 3.

Care must be taken to use the correct region of the graph and to find the largest t -value as there is another intersection point (at $t = 24.6420$) which is quite close to the correct one.

C2

c. $d_2(t)$ will have turning points when $d_2'(t) = 0$ that is when

$$\frac{4\pi}{25} \cos\left(\frac{\pi t}{168}\right) \cos\left(\frac{4\pi t}{25}\right) - \frac{\pi}{168} \sin\left(\frac{\pi t}{168}\right) \sin\left(\frac{4\pi t}{25}\right) = 0$$

A1

$$\frac{4\pi}{25} - \frac{\pi}{168} \frac{\sin\left(\frac{\pi t}{168}\right) \sin\left(\frac{4\pi t}{25}\right)}{\cos\left(\frac{\pi t}{168}\right) \cos\left(\frac{4\pi t}{25}\right)} = 0$$

$$\frac{4\pi}{25} - \frac{\pi}{168} \tan\left(\frac{\pi t}{168}\right) \tan\left(\frac{4\pi t}{25}\right) = 0$$

$$\tan\left(\frac{\pi t}{168}\right) \tan\left(\frac{4\pi t}{25}\right) = \frac{4\pi}{25} \times \frac{168}{\pi}$$

$$\tan\left(\frac{\pi t}{168}\right) \tan\left(\frac{4\pi t}{25}\right) = \frac{672}{25}$$

B2

Summary of mark allocation per Outcome

Question	Part	Outcome 1	Outcome 2	Outcome 3
1	a	3		
	b	2		
	c	1	1	
	d	2	2	
2	a	2	3	
	b	1	1	2
3	a	1		
	b	1		
	c		2	2
4	a		1	1
	b		2	
	c			2
5	a		2	
	b i			1
	b ii			2
	c	1	2	
Raw Marks		14	16	10
Adjusted Marks		7	8	5