

MATHEMATICAL METHODS (CAS)

Unit 4

Targeted Evaluation Task for School-assessed Coursework 1



2012 Analysis Problems Task on Logs & Exponentials for

Outcomes 1, 2 & 3

SOLUTIONS & RESPONSE GUIDE

The marks given are allocated to the 3 outcomes according to the following:
A – Outcome 1, B – Outcome 2, C – Outcome 3

Question 1

a. $2^{x+1} = (2^3)^{3x} = 2^{9x}$

A1

Equating indices $\Rightarrow x+1=9x$

$$x = \frac{1}{8} \text{ or } x = 0.125$$

A1

b. $e^{-5x} = 12$

$$x = \frac{\ln(12)}{-5} = -0.4970$$

C1

Question 2

a. $\log_5 2x = \log_5 4^3 = \log_5 64$

$$2x = 64$$

$$x = 32$$

A1

b. $\text{LHS} = \log_7 x^2 + \log_7 x^{-6} - \log_7 3^4$

$$= \log_7 \left(\frac{x^2 x^{-6}}{3^4} \right)$$

A1

$$= \log_7 \left(\frac{1}{3^4 x^4} \right)$$

$$= \log_7 (3x)^{-4}$$

$$= -4 \log_7 3x \text{ as required}$$

B1

Question 3

a. Let $y = 16(1 - e^{-0.4x})$

Interchanging x and y

$$x = 16(1 - e^{-0.4y})$$

B1

$$\frac{x}{16} = 1 - e^{-0.4y}$$

$$e^{-0.4y} = 1 - \frac{x}{16}$$

$$y = \frac{1}{-0.4} \log\left(1 - \frac{x}{16}\right)$$

$$f^{-1}(x) = -2.5 \log\left(1 - \frac{x}{16}\right)$$

A1

- b.** $\text{dom } f^{-1} = \text{ran } f = [0, 16)$
 $\text{ran } f^{-1} = \text{dom } f = [0, \infty)$

B1

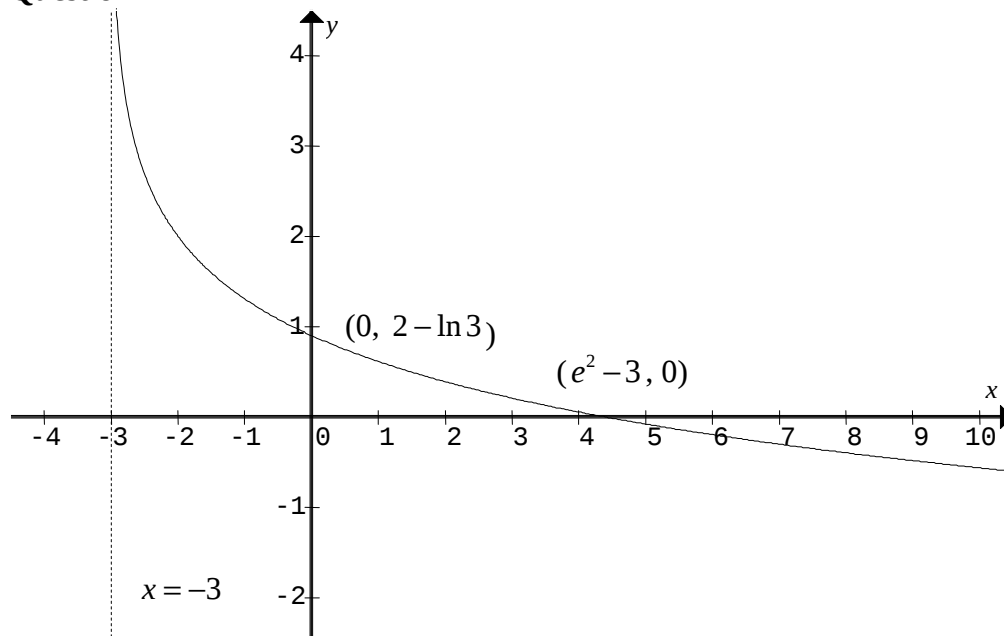
- c.** Graph both functions on a graphics calculator. Using the intersection function gives the intersection point as (15.973, 15.973)

C1

- d.** The time elapsed from when the parachutist jumped as a function of the speed of the parachutist.

B1

Question 4



Correct graph shape and asymptote A2
 Correct intercepts B2

Question 5

At the stationary points $\frac{dy}{dx} = 0$

$$\therefore e^x + 6e^{-x} - 5 = 0$$

$$e^{2x} + 6 - 5e^x = 0$$

$$(e^x)^2 - 5e^x + 6 = 0$$

A1

Let $u = e^x$

$$u^2 - 5u + 6 = 0$$

$$(u - 2)(u - 3) = 0$$

$$u = 2 \text{ or } u = 3$$

$$e^x = 2 \text{ or } e^x = 3$$

$$x = \ln 2 \text{ or } x = \ln 3$$

A1

When $x = \ln 2$

$$y = 2 - \frac{6}{2} - 5 \ln 2 = -1 - 5 \ln 2$$

When $x = \ln 3$

$$y = 3 - \frac{6}{3} - 5 \ln 3 = 1 - 5 \ln 3$$

Therefore coordinates of stationary points are $(\ln 2, -1 - 5 \ln 2)$ and $(\ln 3, 1 - 5 \ln 3)$

A1

Question 6

a. If half the original amount remains then $m = \frac{A}{2}$ at $t = 20$ as A is the original amount.

$$0.5 = e^{-20k}$$

$$\ln(0.5) = -20k$$

$$k = -\frac{\ln(0.5)}{20} = 0.03466$$

B2

b. Initial amount = $A = \frac{m}{e^{-0.03466t}}$

$$A = \frac{9.48}{e^{-0.03466 \times 6.8}} = 11.9996 \approx 12$$

Therefore initial amount is 12 g

B1

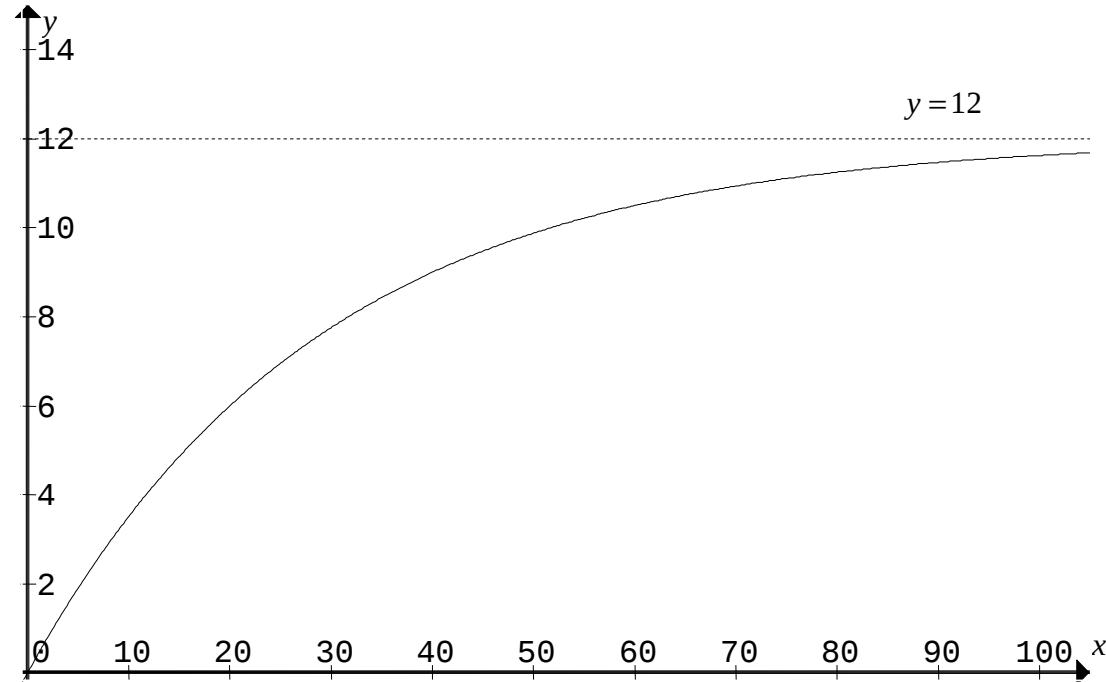
Question 7

- a. The mass of B will be the difference between the original mass of A and the amount of A remaining at any time t .

$$m_B = 12 - 12e^{-0.03466t}$$

$$m_B = 12(1 - e^{-0.03466t})$$

B1

b.

Note: Graph should pass through (20, 6)

Correct shape and asymptote A2

- c. The graph would start off like the graph in part **b** but would then gradually drop below it while still increasing for some time. The graph would then reach a maximum value and start to decrease. For larger values of t the graph would be very close to a typical radioactive decay curve.

B2

Question 8

- a. Substituting the given values into the equation gives:

$$250 = P_0 e^{-10k} \quad (1)$$

$$15 = P_0 e^{-30k} \quad (2)$$

$$\frac{(1)}{(2)} \Rightarrow 16.6667 = e^{20k}$$

$$k = \frac{\ln(16.6667)}{20} = 0.14067 \approx 0.1407 \text{ to four decimal places}$$

B1, C1

b. $P_0 = \frac{250}{e^{-0.1407 \times 10}} = 1020.92$

$$P_0 = 1021 \text{ to the nearest millibar}$$

A1, C1

- c. If $P = P_0 e^{-kh}$ then

$$h = -\frac{\ln\left(\frac{P}{P_0}\right)}{k} = \frac{\ln\left(\frac{65}{1021}\right)}{0.1407} = 19.5746 \text{ km}$$

therefore height is 19.575 km to the nearest metre

B1, C1

Question 9

Draw a graph of $y = 2xe^{-0.08x} + 0.0017x^2$ on a graphics calculator.

- a. Use the maximum and minimum functions to find:

Maximum at (13.3218, 9.4797) and minimum at (43.9605, 5.8959)

C2

- b. Either use the x-Cal function to find the values when $y = 7.8$ or find the intersections of the graph of $y = 2xe^{-0.08x} + 0.0017x^2$ with $y = 7.8$. Correct set of values is:
(6.4990, 25.1578) $\cup$ (64.4235, ∞).

Correct intersection points C2

Correct range of values C1

Question 10**a.** Using the product and chain rules:

$$f'(x) = 2(x-a)e^{-bx} - b(x-a)^2 e^{-bx}$$

A1

And factorising gives:

$$f'(x) = (x-a)[2-b(x-a)]e^{-bx}$$

B1

b. Let $(x-a)[2-b(x-a)]e^{-bx} = 0$ $e^{-bx} \neq 0$ therefore

$$(x-a) = 0 \text{ or } 2-b(x-a) = 0$$

$$x = a \text{ or } x = \frac{2}{b} + a$$

B1

Summary of mark allocation by Outcome

Question	Part	Outcome 1	Outcome 2	Outcome 3
1	a	2		
	b			1
2	a	1		
	b	1	1	
3	a	1	1	
	b		1	
	c			1
	d		1	
4		2	2	
5		3		
6	a		2	
	b		1	
7	a		1	
	b	2		
	c		2	
8	a		1	1
	b	1		1
	c		1	1
9	a			2
	b			3
10	a	1	1	
	b		1	
Raw marks		14	16	10
Adjusted marks		7	8	5