MATHEMATICAL METHODS (CAS)

Unit 3
Targeted Evaluation Task for School-assessed Coursework 3

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2012 Application task on Functions and Calculus for Outcomes 1, 2 & 3

<u>SOLUTIONS & RESPONSE GUIDE</u>

The marks given are allocated to the 3 outcomes according to the following:
A – Outcome 1, B – Outcome 2, C – Outcome 3

SECTION 1

Question 1

a. Starting point (0, 35) Minimum point (30, 10)

A2

b. In this form of a quadratic equation the minimum is at (-B, C), $\therefore$ B = - 30 and C = 10

A2

c. At the starting point

$$f(0) = A(0-30)^2 + 10 = 35$$

$$900A = 25$$

A1

$$A = \frac{1}{36}$$

A1

d. $$f_1(x) = \frac{1}{36}(x-30)^2 + 10$$

$$f_1(x) = \frac{1}{36}(x^2 - 60x + 900) + 10$$

$$f_1(x) = \frac{x^2}{36} - \frac{5x}{3} + 35$$

A1

e. $$f_1'(x) = \frac{x}{18} - \frac{5}{3}$$

A1

$$f_1'(0) = 0 - \frac{5}{3}$$

Initial gradient = $-\frac{5}{3}$

A1

Question 2

a. $$f_2'(x) = 2ax + b$$

$$f_2'(0) = b = -2$$

A1

b. $f_2'(30) = 60a - 2 = 0$

$a = \frac{2}{60} = \frac{1}{30}$

A1

$f_2(30) = 30 - 60 + c = 10$

$c = 40$

A1

$f_2(x) = \frac{x^2}{30} - 2x + 40$

A1

c. At B, $f_2'(x) = \frac{x}{15} - 2 = 1$

B1

$x = 45$

$f_2(45) = 17.5$

B = (45, 17.5)

A1

Domain of $f_2(x)$ is [0,45]

A1

Question 3

a. $g(x) = ax^2 + bx + c$ and $g'(x) = 2ax + b$

$g(45) = 2025a + 45b + c = 17.5$ (1)

$g(75) = 5625a + 75b + c = 25$ (2)

$g'(45) = 90a + b = 1$ (3)

B1

$(2) - (1) \Rightarrow 3600a + 30b = 7.5$ (4)

$30 \times (3) \Rightarrow 2700a + 30b = 30$ (5)

$(4) - (5) \Rightarrow 900a = -22.5$

$a = -\frac{1}{40}$

A1

Substituting in (3)

$-\frac{9}{4} + b = 1$

$b = \frac{13}{4}$

A1

Substituting in (1)

$$-\frac{2025}{40}+\frac{584}{4}+c=\frac{35}{2}$$

$$c=-\frac{625}{8}$$

A1

$$g(x)=-\frac{x^2}{40}+\frac{13x}{4}-\frac{625}{8}$$

A1

b. Let $-\frac{x^2}{40}+\frac{13x}{4}-\frac{625}{8}=5$

Remove the fractions by multiplying both sides by 40

$$-x^2+130x-3125=200$$

$$x^2-130x+3325=0$$

A1

$$x=\frac{130\pm\sqrt{130^2-4\times 3325}}{2\times 1}$$

$$x=\frac{130\pm 60}{2}$$

$x=35$ or $x=95$

A1

But the second section only starts at x = 45, so x = 95 is the only solution.

B1

Thus the second section ends at (95, 5) and therefore the domain of $g(x)$ is (45, 95)

B1

c. $g'(x)=-\frac{x}{20}+\frac{13}{4}=0$ at the maximum

$$-\frac{x}{20}=-\frac{13}{4}$$

$$x=65$$

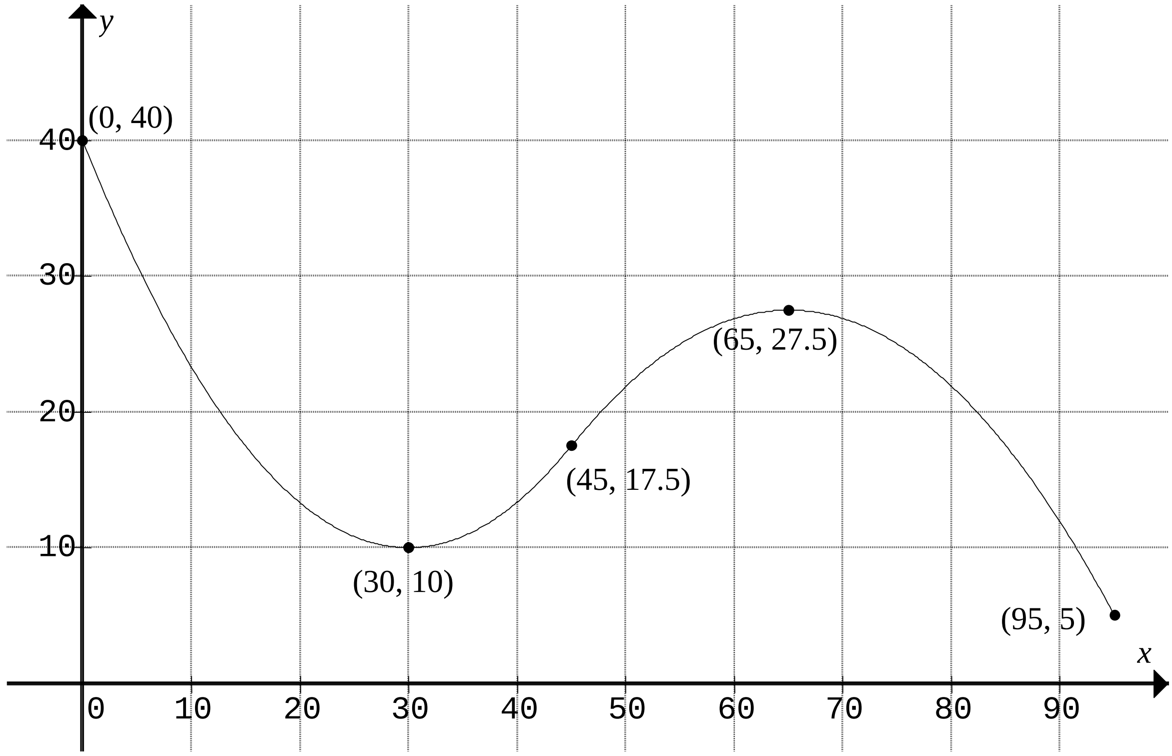
A1

$g(65)=27.5$

Maximum at $(65, 27.5)$

B1

d.



A1, B1 correct graphsA1, B1 correct labelling

SECTION 2

Question 1

a. $h_1'(x) = 3ax^2 + 2bx + c$

$h'(0) = 0 + 0 + c = -2$

$c = -2$

A1

b. $2700a + 60b = 2$

$27000a + 900b + d = 70$

$857375a + 9025b + d = 195$

B3

c.

$$\begin{bmatrix} 2700 & 60 & 0 \\ 27000 & 900 & 1 \\ 857375 & 9025 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 70 \\ 195 \end{bmatrix}$$

B2

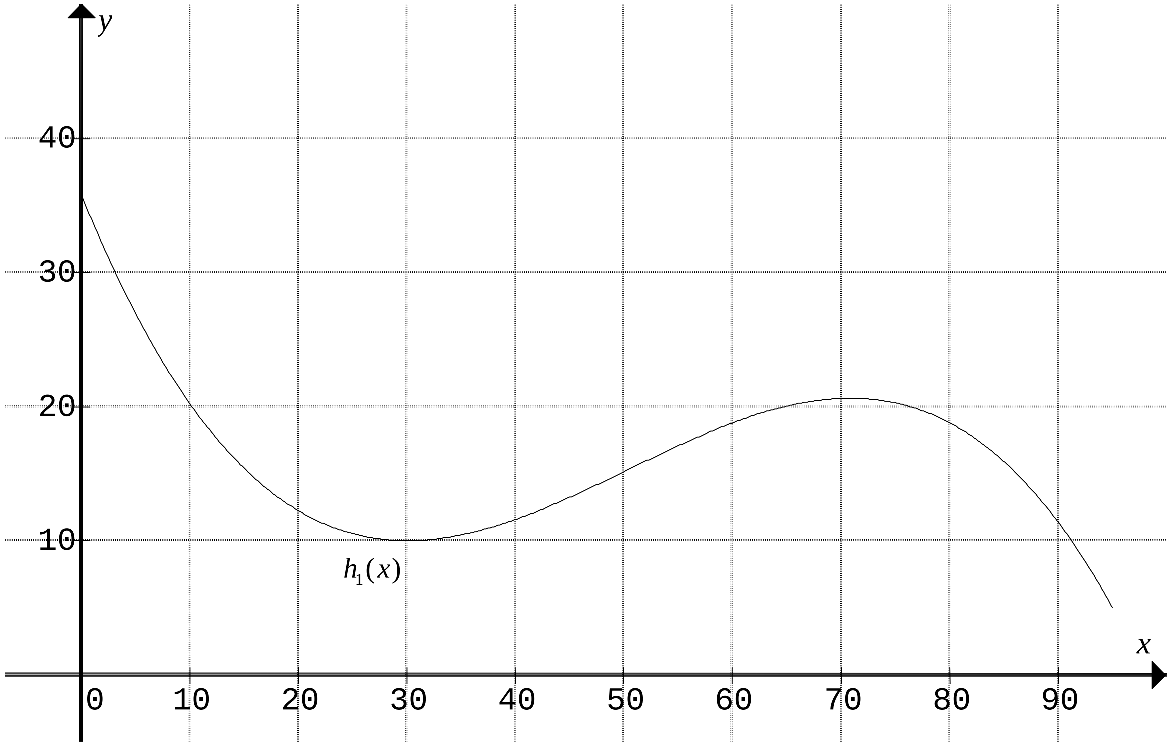
d. X = A^{-1}B

B1

e. $h_1(x) = -0.0003138x^3 + 0.04745x^2 - 2x + 35.76$

C2

f.



A1, B1

g. (70.8, 20.6)

C1

Question 2

a. Very little or no effect for small values of *x* but as *x* increases above about 40, the y-values start to decrease significantly and the decrease becomes larger as *x* increases. The maximum also moves downwards and to the left. If *a* is decreased sufficiently the minimum and maximum merge to form a point of inflection.

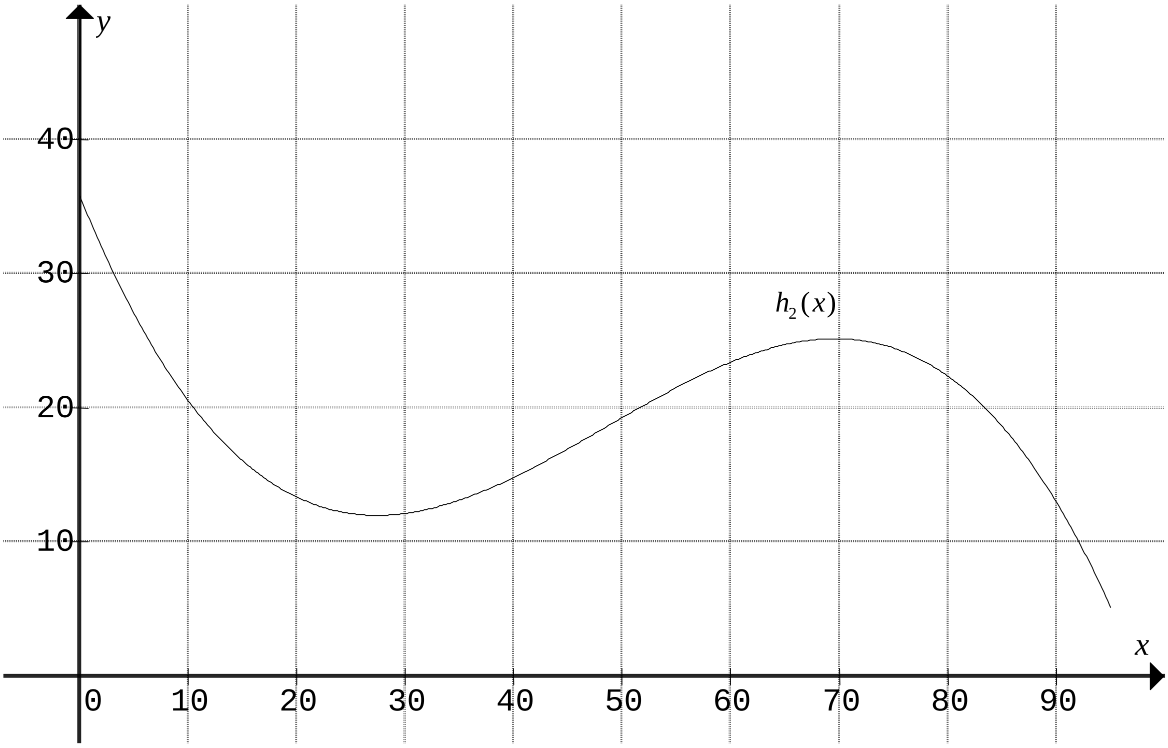
B2, C1

b. Very little or no effect for small values of *x* but as *x* increases above about 15, the y-values start to increase significantly and the increase becomes larger as *x* increases. The maximum also moves upwards and to the right.

B2, C1

c. $a = -0.00035$ and $b = 0.0509$ give the required features quite closely. Other solutions may be possible.

C2



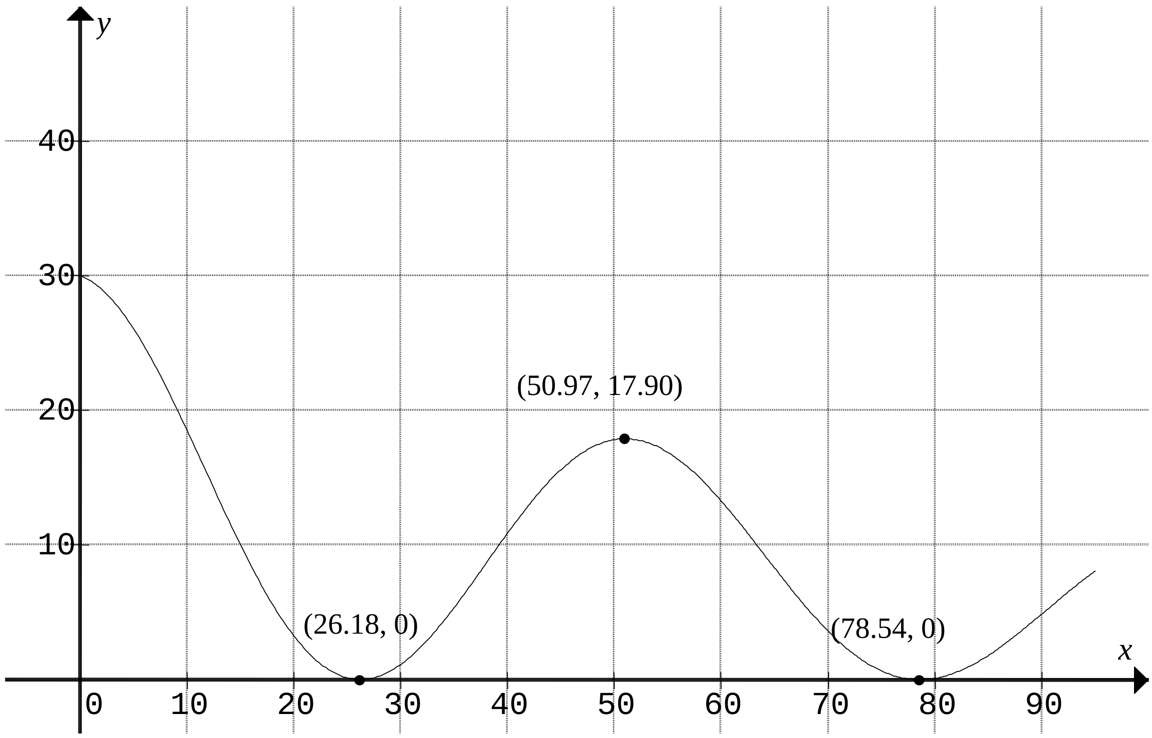
B2

d. The whole graph would be moved up or down parallel to the y-axis.

B1

SECTION 3

Question 1



B1, C2

Question 2

a. $j(x)$ is a product of 2 functions so need to use the product rule. However the first function is a composite function so it will have to be differentiated using the chain rule.

Let $y = u^2$ where $u = \cos(bx)$

$$\frac{dy}{dx} = 2u \times -b\sin(bx) = -2b\cos(bx)\sin(bx)$$

$$j'(x) = a\cos^2(bx) \times -ce^{-cx} - 2ab\cos(bx)\sin(bx) \times e^{-cx}$$

$$j'(x) = -a\cos(bx)e^{-cx}(c\cos(bx) + 2b\sin(bx))$$

B3

b. Let $j'(x)=0$

$-a\cos(bx)e^{-cx}(c\cos(bx)+2b\sin(bx))=0$

A1

Now $e^{-cx}\neq 0$, $\therefore \cos(bx)=0$ or $c\cos(bx)+2b\sin(bx)=0$

B1

If $\cos(bx)=0$

$bx=\dfrac{\pi}{2},\ \dfrac{3\pi}{2}$

$x=\dfrac{\pi}{2b},\ \dfrac{3\pi}{2b}$

B1

These 2 solutions occur when $\cos(bx)=0$ which means that $j(x)=0$ as well. Considering the graph from **Question 1** of this section the minima occur when $j(x)=0$ so these solutions give the minima.

B1

If $c\cos(bx)+2b\sin(bx)=0$

$c+\dfrac{2b\sin(bx)}{\cos(bx)}=0$

$c+2b\tan(bx)=0$

$\tan(bx)=-\dfrac{c}{2b}$

$bx=\tan^{-1}\left(-\dfrac{c}{2b}\right)$

B1

Now since b and c are positive, $-\dfrac{c}{2b}<0$ and hence $\tan^{-1}\left(-\dfrac{c}{2b}\right)<0$

B1

The first two positive solutions will be obtained for

$bx=\tan^{-1}\left(-\dfrac{c}{2b}\right)+\pi$ and $bx=\tan^{-1}\left(-\dfrac{c}{2b}\right)+2\pi$

$x=\dfrac{1}{b}\left[\tan^{-1}\left(-\dfrac{c}{2b}\right)+\pi\right]$ and $\dfrac{1}{b}\left[\tan^{-1}\left(-\dfrac{c}{2b}\right)+2\pi\right]$

B2

As the first pair of solutions give the minima this second pair of solutions must give the maxima.

B1

Question 3

a. 40

A1

b. From **b.** the second minimum occurs when $x = \frac{3\pi}{2b}$

A1

If $95 = \frac{3\pi}{2b}$

$b = \frac{3\pi}{190}$

A1

c. c =0.0075 and x = 61.81

C2

Question 4

a. Because of the trigonometric term in the rule of $j(x)$, extending the function over a larger domain will automatically generate more maxima and minima thus continuing the up and down nature of the track.

B2

b. The track would have an "amplitude" of 35 – 10 = 25 m so a = 25. Also as the minimums have been raised by 10 m there would have to be a constant term of 10 added to the rule of $j(x)$.

B2

c. The x-values of the turning points of $j(x)$, as found in **Question 2 b.** above, are independent of a so changing a will have no effect. Also the derivative of a constant term is zero so the constant term will not affect the horizontal position of the turning points.

B2

Summary of mark allocation per Outcome

Section	Question	Part	Outcome 1	Outcome 2	Outcome 3
1	**1**	**a**	2		
		b	2		
		c	2		
		d	1		
		e	2		
	2	**a**	1		
		b	3		
		c	2	1	
	3	**a**	4	1	
		b	2	2	
		c	1	1	
		d	2	2	
2	**1**	**a**	1		
		b		3	
		c		2	
		d		1	
		e			2
		f	1	1	
		g			1
	2	**a**		2	1
		b		2	1
		c		2	2
		d		1	
3	**1**			2	1
	2	**a**		3	
		b	1	8	
	3	**a**	1		
		b	2		
		c			2
	4	**a**		2	
		b		2	
		c		2	
		Raw Marks	30	40	10
		Adjusted Marks	15	20	5