Student Name: _____

MATHEMATICAL METHODS (CAS)

Unit 3
Targeted Evaluation Task for School-assessed Coursework 1

2012 Test (multiple choice, short answer, extended response) on Functions for Outcomes 1 & 3

Recommended writing time*: 60 minutes

Total number of marks available: 40 marks

TASK BOOK

* The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers, an approved CAS calculator.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.
- An approved CAS calculator may be used for sections 2 and 3 – Multiple Choice and Analysis Task Questions
- No calculator is permitted for use in section 1 –Short Answer Questions

Materials supplied

- Question and answer book of 11 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

SECTION 1- Short-answer Questions

Instructions for Section 1

For any question worth more than 1 mark, relevant working must be shown.

Question 1

a. Describe the transformations required to transform the graph of $y = \sin\left(x - \frac{\pi}{6}\right)$ to the graph of $y = 4\sin 2\left(x - \frac{2\pi}{3}\right) + 5$.

2 marks

b. Find the exact solutions of $4\sin 2\left(x - \frac{2\pi}{3}\right) + 5 = 7$ over the domain $[0, \pi]$

4 marks

Question 2

Show that $2\log_a 2x + \log_a 4x - 5\log_a x = -2\log_a\left(\frac{x}{4}\right)$

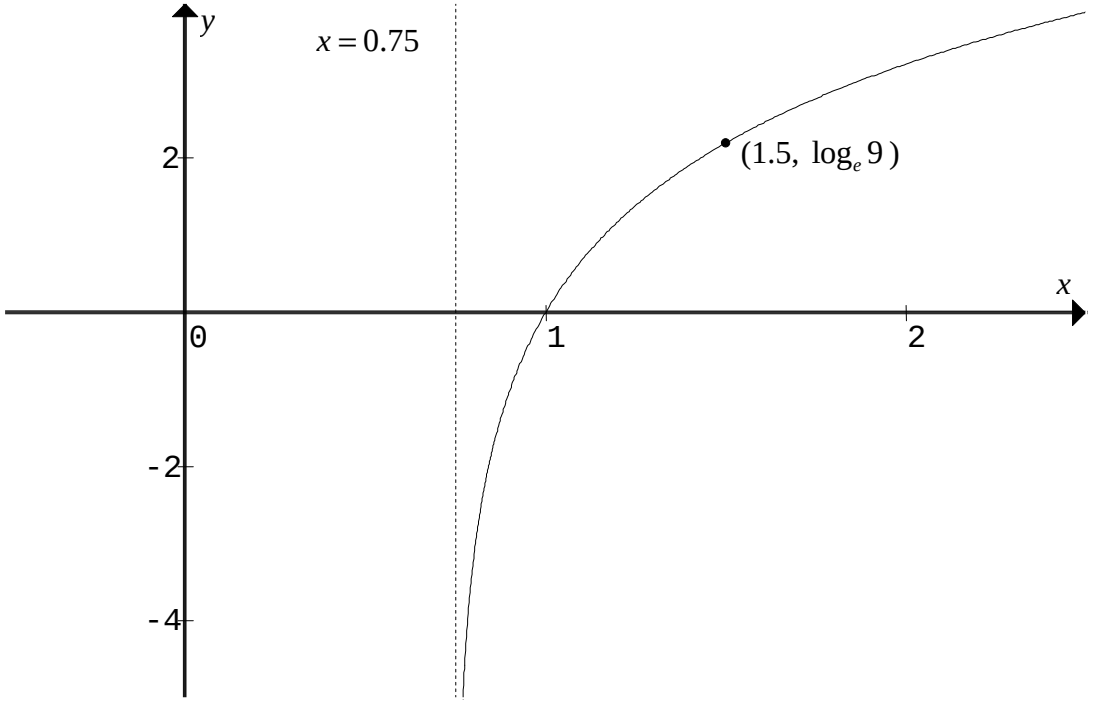
2 marks

Question 3

Find the exact *x*-intercepts of the graph of $y = 2x^4 + 5x^3 + x^2$.

3 marks

Question 4



A graph of a function of the form $f(x) = a\log_e(bx + c)$ is shown above. The graph has a vertical asymptote at $x = 0.75$, an x-intercept of 1 and passes through the point $(1.5, \log_e 9)$.

a. Find the values of b and c.

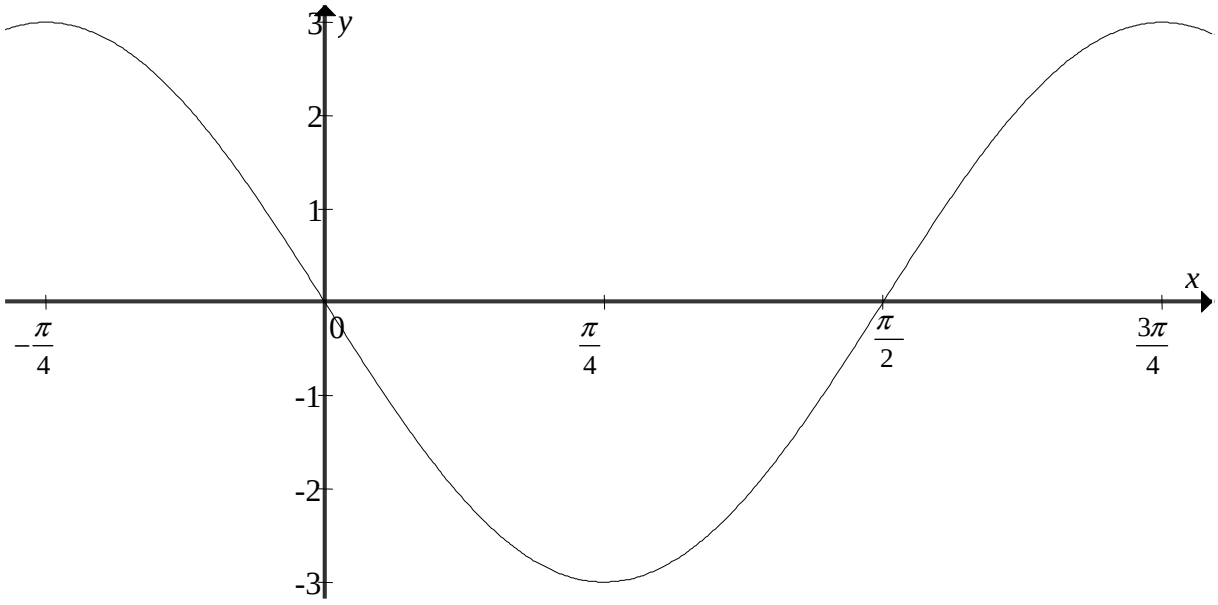
3 marks

b. Find the value of a.

1 mark

Section 2: Multiple Choice Questions

Question 1
The equation of the following graph could be:



A. $y = -3\sin x$

B. $y = -\frac{\pi}{2}\sin 3x$

C. $y = 3\sin\left(x - \frac{\pi}{2}\right)$

D. $y = 3\cos 2\left(x + \frac{\pi}{4}\right)$

E. $y = 3\cos 2\left(x - \frac{\pi}{4}\right)$

Question 2
Which of the following functions **does not** have an inverse function?

A. $f(x) = 3x - 4,\ x \in R$

B. $f(x) = (x+1)^2 - 2,\ x \le 0$

C. $f(x) = \cos\left(\frac{x}{2}\right),\ 0 \le x \le 2\pi$

D. $f(x) = 2e^{-3x},\ x \ge 0$

E. $f(x) = x^3,\ x \in R$

The following information relates to Questions 3 and 4.

Given $f(x) = \sqrt{x-4}$ and $g(x) = x^2$.

Question 3

$g[f(7)] =$

A. $\sqrt{3}$
B. $\sqrt{45}$
C. 49
D. 9
E. 3

Question 4

The maximal implied domain of $f[g(x)]$ is:

A. $\{x : x \leq -4\} \cup \{x : x \geq 4\}$
B. $\{x : x \geq 2\}$
C. $\{x : x \leq -2\} \cup \{x : x \geq 2\}$
D. $\{x : x \geq 4\}$
E. $\{x : x \geq 0\}$

Question 5

The rule for the inverse function of $f(x) = \sqrt{x+5} - 2$ would be:

A. $f^{-1}(x) = (x-2)^2 - 5$
B. $f^{-1}(x) = (x+5)^2 - 2$
C. $f^{-1}(x) = (x+2)^2 - 5$
D. $f^{-1}(x) = (x-5)^2 + 2$
E. $f^{-1}(x) = -2(x+5)^2$

Question 6

If $f(x) = (x-2)^2 + 3,\ 0 \leq x \leq 5$, the domain of $f^{-1}(x)$ would be:

A. [0, 5]
B. [2, 3]
C. [3, 5]
D. [3, 12]
E. [7, 12]

Question 7

The x-intercept of $y = \frac{1}{2}\log_e(x-1)+3$ is:

A. $e^{-6}+1$
B. $e^{-\frac{3}{2}}+1$
C. 1
D. $2e^{-\frac{1}{3}}$
E. $2e^{-3}+1$

Question 8

If the function $f(x) = \log_e(x+3)$ is dilated away from the x-axis, which of the following would remain unchanged?

A. The x-intercept only
B. The y-intercept only
C. The asymptote only
D. The x-intercept and the y-intercept
E. The x-intercept and the asymptote

Question 9

If the graph of $y = e^x + 3$ is

- Reflected about the x-axis
- Translated -2 units parallel to the x-axis
- Translated 1 unit parallel to the y-axis

in that order, the equation of the resulting graph would be:

A. $y = e^{-(x-2)}+4$
B. $y = -e^{(x+2)}-2$
C. $y = -e^{(x+2)}+4$
D. $y = -e^{(x-1)}-5$
E. $y = e^{-(x-1)}+1$

Question 10

The graph of $y = \left|(x-2)(x-4)\right|+3,\ 0 \le x \le 5$ will have:

A. A minimum value at (3, -1)
B. A minimum value at (3, 2)
C. A range of [0, 1]
D. Minimum values at (2, 3) and (4, 3) and a range of [3, 4]
E. Minimum values at (2, 3) and (4, 3) and a range of [3, 11]

SECTION 3- Analysis Questions

Instructions for Section 3

For any question worth more than 1 mark, relevant working must be shown.

Question 1

a. A population of bacteria is increasing according to the function $f_1(t) = Ae^{kt}$, $t \geq 0$, where t is the time in hours after 9 am.

At 11 am there are 5000 bacteria and at 2 pm there are 12500 bacteria.

i. Find the value of k correct to 4 decimal places.

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2 marks

ii. Use your value of k above to find the value of A correct to the nearest whole number.

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_____

1 mark

iii. Using your values for A and k, find the time, to the nearest minute, when the bacteria population will be 30000.

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_____

_____

_____

_____

2 marks

b. A second bacteria population grows according to the function
$f_2(t) = Be^{nt}$, $t \geq 0$, where t is the time in hours after 9 am.

i. Write down the value of B if, at 9 am, the number of bacteria in this population is one third of number of bacteria in the population in part **a.**

1 mark

ii. Write down the value of n if this population has the same initial population as the population in part **a.** but grows at twice the rate of the population in part **a.**

1 mark

c. Two other populations of bacteria are growing according to the functions
$g(x) = 2e^{2t} + 5,\ t \geq 0$ and
$h(x) = 11e^{t},\ t \geq 0$
Find the exact time (or times), t, when the two population are equal.

5 marks

d. An antibiotic solution is introduced into one of the bacteria populations. The bacteria start decreasing according to the function, $P = 16000e^{-0.2t} + 500,\ t \geq 0$

i. Find the rule of the function that gives t in terms of P.

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_____

1 mark

ii. Find the time, t, when the population of the bacteria is one tenth of its initial value. Give your answer in hours correct to 2 decimal places.

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2 marks

END OF TASK BOOK