

MATHEMATICAL METHODS (CAS)

Unit 3

Targeted Evaluation Task for School-assessed Coursework 1



**2012 Test (multiple choice, short answer, extended response) on
Functions for Outcomes 1 & 3**

SOLUTIONS & RESPONSE GUIDE

SECTION 1- Short-answer Questions**Question 1****a.**

- Translation of $\frac{\pi}{2}$ units parallel to the x -axis.
- Dilation by a factor of $\frac{1}{2}$ away from the y -axis.
- Dilation by a factor of 4 away from the x -axis.
- Translation of 5 units parallel to the y -axis.

 $(\frac{1}{2}$ mark for each correct transformation)

b. $4 \sin 2\left(x - \frac{2\pi}{3}\right) = 2$

$$\sin 2\left(x - \frac{2\pi}{3}\right) = \frac{1}{2}, \quad 0 \leq x \leq \pi$$

(1 mark)

$$\sin 2\left(x - \frac{2\pi}{3}\right) = \frac{1}{2}, \quad -\frac{2\pi}{3} \leq x - \frac{2\pi}{3} \leq \frac{\pi}{3}$$

$$\sin 2\left(x - \frac{2\pi}{3}\right) = \frac{1}{2}, \quad -\frac{4\pi}{3} \leq 2\left(x - \frac{2\pi}{3}\right) \leq \frac{2\pi}{3}$$

(1 mark)

$$2\left(x - \frac{2\pi}{3}\right) = -\frac{7\pi}{6}, \quad \frac{\pi}{6}$$

(1 mark)

$$x = \frac{\pi}{12}, \quad \frac{3\pi}{4}$$

(1 mark)

Question 2

$$\text{LHS} = \log_a 4x^2 + \log_a 4x - \log_a x^5$$

$$= \log_a \left(\frac{16x^3}{x^5} \right) = \log_a \left(\frac{16}{x^2} \right)$$

(1 mark)

$$= \log_a \left(\frac{4}{x} \right)^2$$

$$= \log_a \left(\frac{x}{4} \right)^{-2}$$

$$= -2 \log_a \left(\frac{x}{4} \right) = \text{RHS}$$

(1 mark)

Question 3

$$2x^4 + 5x^3 + x^2 = 0$$

$$x^2(2x^2 + 5x + 1) = 0$$

$x = 0$ is one solution and $2x^2 + 5x + 1 = 0$ will give the other solutions

(1 mark)

Using the quadratic rule on $2x^2 + 5x + 1 = 0$

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2 \times 2} = \frac{-5 \pm \sqrt{17}}{4}$$

$\therefore$ the other two solutions are

$$x = \frac{-5 - \sqrt{17}}{4} \text{ and } \frac{-5 + \sqrt{17}}{4}$$

(2 mark)

Question 4

a. The asymptote at $x = 0.75$ means that

$$0.75b + c = 0$$

$$c = -0.75b \quad (1)$$

(1 mark)

The x -intercept at 1 means that

$$b + c = 1 \quad (2)$$

(1 mark)

And combining equations (1) and (2)

$$b - 0.75b = 1$$

$$b = 4$$

$$c = -3$$

(1 mark)

b. $y = a \log_e(4x - 3)$ using values from part **a.**

Substituting in the given point values

$$a \log_e 3 = \log_e 9 = \log_e 3^2 = 2 \log_e 3$$

$$a = 2$$

(1 mark)

SECTION 2 Multiple-choice questions (1 mark each)**Question 1***Answer:* D*Explanation*

Graph can be considered a cosine graph of the form $y = a \cos n(x + b)$. $a = \text{amplitude} = 3$.

Period, $T = \pi$ and $n = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2$. There is a horizontal shift of $-\frac{\pi}{4}$ units, so $b = \frac{\pi}{4}$

Question 2*Answer:* B*Explanation*

A function will have an inverse if and only if it is a one-to-one function. All the functions are one-to-one over their given domains except for B.

Question 3*Answer:* E*Explanation*

$$g[f(x)] = (\sqrt{x-4})^2 = x-4$$

$$\therefore g[f(7)] = 7-4 = 3$$

Question 4*Answer:* C*Explanation*

$$f[g(x)] = \sqrt{x^2-4} \text{ which is defined for } x^2-4 \geq 0$$

$$\therefore x^2 \geq 4$$

$$x \geq |2|$$

$$x \leq -2 \text{ or } x \geq 2$$

Question 5*Answer:* C*Explanation*

Let $y = \sqrt{x+5} - 2$ and interchange x and y .

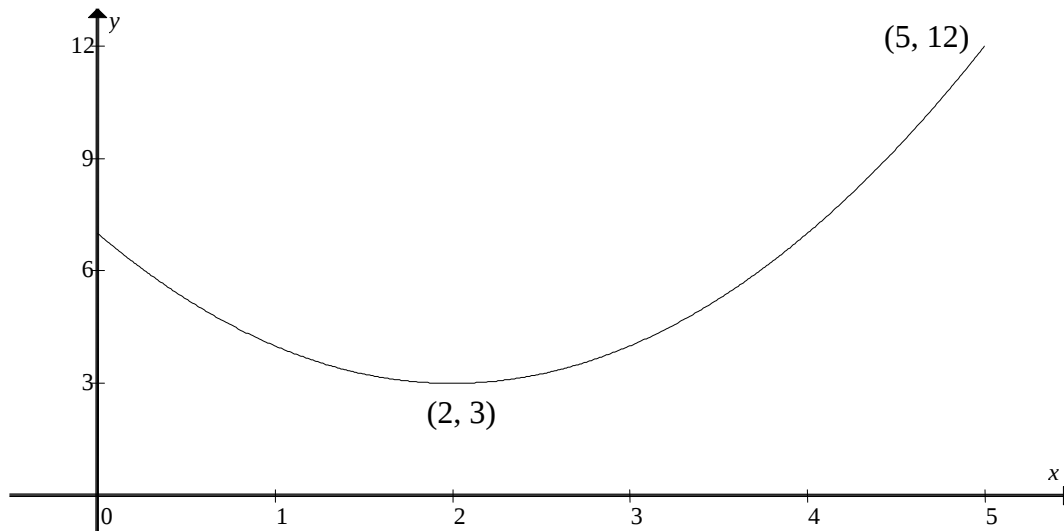
$$x = \sqrt{y+5} - 2$$

$$(x+2)^2 = y+5$$

$$y = (x+2)^2 - 5$$

Question 6*Answer:* D*Explanation*

$\text{dom } f^{-1}(x) = \text{ran } f(x) = [3, 12]$ from the graph of $y = f(x)$ below.

**Question 7***Answer:* A*Explanation*

$$\text{Let } 0 = \frac{1}{2} \log_e(x-1) + 3$$

$$-6 = \log_e(x-1)$$

$$e^{-6} = x-1$$

$$x = e^{-6} + 1$$

Question 8*Answer:* E*Explanation*

The y-intercept will change but the other two will remain the same.

Question 9*Answer:* B*Explanation*

$$\text{Reflection gives } y = -(e^x + 3) = -e^x - 3$$

$$\text{First translation gives } y = -e^{(x+2)} - 3$$

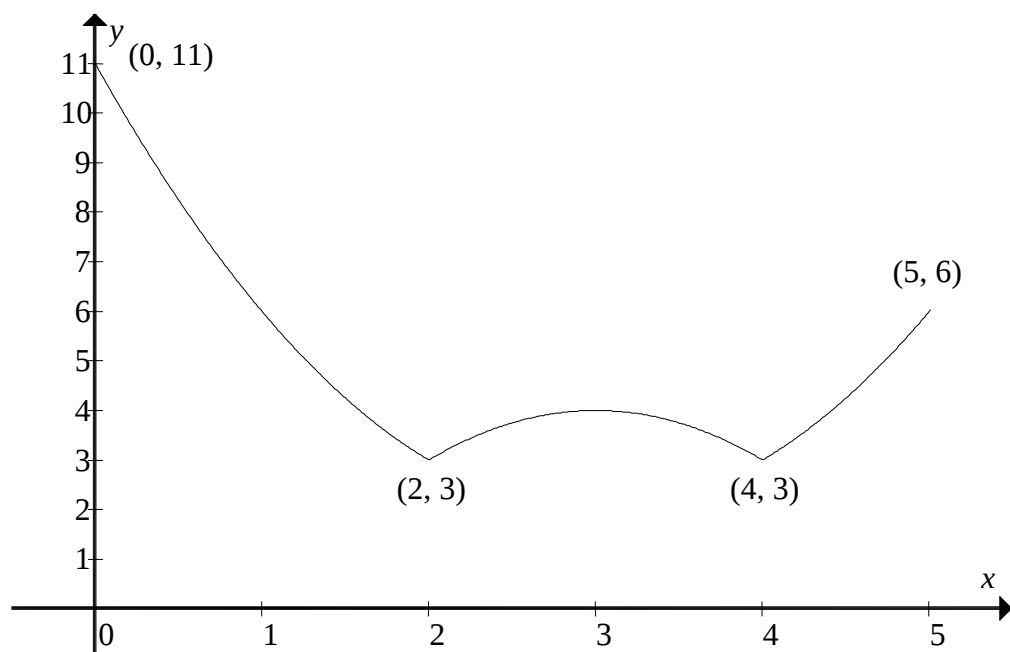
$$\text{Second translation gives } y = -e^{(x+2)} - 2$$

Question 10

Answer: E

Explanation

The graph of $y = |(x-2)(x-4)| + 3$, $0 \leq x \leq 5$ is shown below.



The y-values range from 3 to 11.

SECTION 3- Analysis Questions**a.**

- i.**
- Substituting the given values into the function

$$Ae^{2k} = 5000 \quad (1)$$

$$Ae^{5k} = 12500 \quad (2)$$

(1 mark)

Dividing (2) by (1)

$$e^{3k} = 2.5$$

$$k = \frac{\log_e 2.5}{3} = 0.3054$$

(1 mark)

- ii.**
- Substituting for
- k
- in equation (1) and rearranging

$$A = \frac{5000}{e^{0.6108}} = 2715$$

(1 mark)

- iii.**
- $2715e^{0.3054t} = 30000$

$$0.3054t = \log_e \left(\frac{30000}{2715} \right)$$

$$t = \frac{\log_e \left(\frac{30000}{2715} \right)}{0.3054} = 7.866 \text{ hrs}$$

(1 mark)

$$t = 7 \text{ hrs } 52 \text{ min}$$

Time will be 4.52 pm

(1 mark)

b.

$$\text{i. } B = \frac{A}{3} = 905$$

(1 mark)

$$\text{ii. } n = 2k = 0.6108$$

(1 mark)

- c.**
- Let
- $g(x) = h(x)$

$$2e^{2t} + 5 = 11e^t$$

$$2e^{2t} - 11e^t + 5 = 0$$

(1 mark)

$$\text{Let } e^t = x$$

$$2x^2 - 11x + 5 = 0$$

(1 mark)

$$(2x-1)(x-5) = 0$$

$$x = \frac{1}{2} \text{ or } 5$$

(1 mark)

$$e^t = \frac{1}{2} \text{ or } 5$$

$$t = \log_e \frac{1}{2} \text{ or } \log_e 5$$

(1 mark)

But $\log_e \frac{1}{2} < 0$ and $t \geq 0$

$\therefore t = \log_e 5$ is the only solution

(1 mark)

d.

i. $\frac{P-500}{16000} = e^{-0.2t}$

$$t = -5 \log_e \left(\frac{P-500}{16000} \right)$$

(1 mark)

ii. Initial population = $16000 + 500 = 16500$ and $\frac{16500}{10} = 1650$

(1 mark)

$$t = -5 \log_e \left(\frac{1650-500}{16000} \right) = 13.16 \text{ hrs}$$

(1 mark)