TSSM
Creating VCE Success

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2012 Trial Examination

STUDENT NUMBER

										Letter
Figures										
Words										

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION & ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 9 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = \log_e(x + 3)$and $g(x) = x^4 + 2$.

a. Write down a rule for $f\big(g(x)\big)$

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_____

_____

_____

2 marks

b. Find the derivative of $f\big(g(x)\big)$.

_____

_____

_____

_____

2 marks

c. Hence find $f'\big(g(-2)\big)$

_____

_____

_____

1 mark

Question 2

a. Find a given that $\int_0^a (5x - 10)\, dx = 0$, $a \neq 0$.

2 marks

b. Find the area between the curve $y = cos\left(\frac{x}{2}\right)$, the x-axis and the lines $x = 0$ and $x = \pi$.

2 marks

TURN OVER

Question 3

Consider the function $f: R \to R, f(x) = -4e^{\frac{x}{2}} + 1$.

a. Write down a rule for $f^{-1}(x)$.

3 marks

b. Find the domain of $f^{-1}(x)$.

1 mark

c. Solve for $f^{-1}(x) = 0$.

2 marks

Question 4

Given $f: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \rightarrow R, f(x) = 3\cos\ 2x$

a. Find the exact values of x for which $f(x) = -\frac{3\sqrt{3}}{2}$.

_____

_____

_____

2 marks

b. Find the exact values of x for which $f'(x) = 0$.

_____

_____

_____

_____

2 marks

c. Write down the interval over which the rate of change is negative.

_____

_____

1 mark

TURN OVER

Question 5

A transformation is defined through the matrix equation $\mathbf{T}(\mathbf{X} + \mathbf{B}) = \mathbf{X}'$ where

$\mathbf{T} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$. Find the equation of the image of $y = |x + 2| - 3$ under this transformation.

_____

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4 marks

Question 6

A pile of sand forms a right circular cone. Sand is being added at a constant rate of $2\text{m}^3/\text{s}$.
The growing pile is shaped such that the radius of its base is equal to half of its height.

a. Find the volume of the pile when the height of the pile is 6 m.

_____

_____

_____

1 mark

b. At what rate is the height of the pile increasing when the pile is 6 m high?

_____

_____

_____

_____

2 marks

Question 7

Use the approximation formula $f(x+h) \approx f(x) + hf'(x)$ for a small positive value of h, to find an approximate value for $\sqrt{24.95}$

3 marks

Question 8

The time taken to complete a task (in hours) is a random variable X with probability density function:

$$f(x) = \begin{cases} k(x-1)(4-x) & 1 < x < 4 \\ 0 & \text{elsewhere} \end{cases}$$

where k is a constant, find

a. the value of k.

3 marks

Question 8 - continued

TURN OVER

b. $\Pr(X > 3)$

2 marks

Question 9

Sue makes handmade soaps. The mass of soaps$X$ is a normally distributed random variable with mean 220 grams and a standard deviation of 10 grams and let Z be the random variable with the standard normal distribution. Find

a. $\Pr(m < X < 240) \approx 0.95$, find the value of m.

1 mark

b. Sue randomly selects a handmade soap from a box containing several soaps. What is the probability that the soap's mass is less than 210 grams? (Use $\Pr(Z > 1) = 0.16$

2 marks

Question 9 - continued

c. Sue randomly selects a handmade soap that had a mass more than the mean mass. What is the probability that its mass is more than 230 grams?

_____

_____

_____

_____

_____

2 marks

END OF QUESTION AND ANSWER BOOK