

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1



2012 Trial Examination

SOLUTIONS

Question 1

a. $f(x^4 + 2) = \log_e(x^4 + 2 + 3) = \log_e(x^4 + 5)$

M1+A1
2 marks

b. $f'(g(x)) = \frac{1}{(x^4+5)} \times 4x^3 = \frac{4x^3}{(x^4+5)}$

M1+A1
2 marks

c. $f'(g(-2)) = \frac{4(-2)^3}{((-2)^4+5)} = \frac{-32}{21}$

A1
1 mark

Question 2

a. $\left(\frac{5x^2}{2} - 10x\right)_0^a = 0$

$$\frac{5a^2}{2} - 10a = 0$$

$$5a(a - 4) = 0$$

$$a = 0 \text{ or } a = 4$$

Since $a \neq 0$, $a = 4$

M1+A1
2 marks

b. $\int_0^\pi \cos\left(\frac{x}{2}\right) dx = 2\sin\left(\frac{x}{2}\right)_0^\pi = 2$

M1+A1

2 marks

Question 3

a. $x = -4e^{\frac{y}{2}} + 1$

$$\frac{x-1}{-4} = e^{\frac{y}{2}}$$

$$\frac{y}{2} = \log_e\left(\frac{1-x}{4}\right)$$

$$f^{-1}(x) = 2\log_e\left(\frac{1-x}{4}\right)$$

M2+A1

3 marks

b. $1 - x > 0$

Domain of $f^{-1}(x)$ is $(-\infty, 1)$

M1

1 mark

c. $2\log_e\left(\frac{1-x}{4}\right) = 0$

$$\left(\frac{1-x}{4}\right) = 1$$

$$\frac{1-x}{4} = 1$$

$$x = -3$$

M1+A1

2 marks

Question 4

a. $3\cos(2x) = -\frac{3\sqrt{3}}{2}$

$$\cos(2x) = -\frac{\sqrt{3}}{2} \text{ for } -\pi \leq 2x \leq \pi$$

$$2x = \frac{5\pi}{6}, -\frac{5\pi}{6}$$

$$x = \frac{5\pi}{12}, -\frac{5\pi}{12}$$

M1+A1

2 marks

b. $-3 \sin(2x) \times 2 = 0$

$$2x = 0, \pi, -\pi$$

$$x = 0, \frac{\pi}{2}, -\frac{\pi}{2}$$

M1+A1

2 marks

c. $(0, \frac{\pi}{2})$

M1

1 mark

Question 5

$$\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 5 \end{bmatrix} \right) = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x-1 \\ y+5 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{bmatrix} -x+1 \\ -2y-10 \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$-x+1 = x' \text{ and } -2y-10 = y'$$

$$x = -x' + 1 \text{ and } y = \frac{-y' - 10}{2}$$

$$\frac{-y' - 10}{2} = |-x' + 1 + 2| - 3$$

$$y' + 10 = -2|x' - 3| + 6$$

$$y = -2|x - 3| - 4$$

M2+A2

4 marks

Question 6

a. $V = \frac{1}{3}\pi r^2 h = 18\pi$

M1

1 mark

b. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 \times h = \frac{\pi}{12}h^3$

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{4}{\pi h^2} \times 2 = \frac{2}{9\pi} \text{ m/s}$$

M1+A1

2 marks

Question 7

$$f(x) = \sqrt{x}, \quad f'(x) = \frac{1}{2\sqrt{x}}$$

$$\sqrt{24.95} \approx \sqrt{25} - 0.05 \times \frac{1}{2\sqrt{25}}$$

$$\approx 5 - \frac{0.05}{10} \approx 4.995$$

M1+A2
3 marks**Question 8**

a. $\int_1^4 k(-x^2 + 5x - 4)dx = 1$

$$k \left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right)_1^4 = 1$$

which gives $k = \frac{2}{9}$

M1+A2
3 marks

b. $\Pr(X > 3) = \frac{2}{9} \int_3^4 (-x^2 + 5x - 4)dx$

$$= \frac{2}{9} \left(-\frac{64}{3} + 40 - 16 + \frac{27}{3} - \frac{45}{2} + 12 \right)$$

$$= \frac{7}{27}$$

A2
2 marks**Question 9**

a. $m = 200$ (95% means 2 standard deviations from the mean)

M1
1 mark

b. $\Pr(X < 210) = \Pr \left(Z < \frac{210-220}{10} \right)$

$$= \Pr(Z < -1) = \Pr(Z > 1) = 0.16$$

M1+A1
2 marks

c. $\Pr(X > 230 | X > 220) = \frac{\Pr(X > 230)}{\Pr(X > 220)} = \frac{\Pr(Z > 1)}{\Pr(Z > 0)} = \frac{0.16}{0.5} = \frac{16}{50} = \frac{8}{25} \text{ or } 0.32$

M1+A1
2 marks