

**‘2016 Examination Package’ -
Trial Examination 2 of 5**

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

(TSSM’s 2012 trial exam updated for the current study design)

Reading time: 15 minutes

Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 22 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 –Multiple-choice questions

Instructions for Section 1

Answer all questions on the answer sheet provided for multiple choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

The amplitude and period of the graph $y = -2\cos 3\left(x - \frac{\pi}{3}\right), 0 \leq x \leq 2\pi$ is:

- A. 3 and π
- B. 2 and $\frac{2\pi}{3}$
- C. 3 and $\frac{2\pi}{3}$
- D. -2 and $\frac{2\pi}{3}$
- E. $\frac{1}{2}$ and $\frac{\pi}{3}$

Question 2

Let $f: R \setminus \{-1\} \rightarrow R, f(x) = \frac{kx-4}{x+1}$. The positive value of k for which there is only one value of x satisfying $f(x) = x$ is:

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

Question 3

The implied domain of the inverse of the function $f: (3, \infty) \rightarrow R, f(x) = \log_e(x - 3)$ is:

- A. $[3, \infty)$
- B. $(3, \infty)$
- C. $(-3, \infty)$
- D. $(0, \infty)$
- E. R

SECTION 1 - continued

Question 4

If $f(x) = e^x - e^{-x}$, $x \in R$, the function $(f(x))^3$ is the same as:

- A. $f(x^3)$
- B. $f(3x) - 3f(x)$
- C. $3f(x)$
- D. $f(3x) + f(x)$
- E. $f(x) - 3f(x)$

Question 5

If $\frac{f(1+h)-f(1)}{h} = 4h + 8$, then $f'(1)$ equals:

- A. 6
- B. 10
- C. 12
- D. 0
- E. 8

Question 6

If $y = ax^2 + bx$ has a maximum at $(2, 3)$, the values of a and b are:

- A. $\frac{3}{4}$ and -3
- B. $-\frac{3}{4}$ and -3
- C. $\frac{3}{4}$ and 3
- D. $-\frac{3}{4}$ and 3
- E. 3 and $-\frac{3}{4}$

Question 7

If $y = F(x)$ and $\frac{dy}{dx} = f(x)$, then $\int_2^3 f(x)dx$ equals:

- A. $F(3) - F(2)$
- B. $f(3) - f(2)$
- C. $f(3) - F(2)$
- D. $F(3) - f(2)$
- E. $f(x) + c$, where c is a constant

SECTION 1 - continued
TURN OVER

Question 8

A line with gradient m , $m < 0$, passes through the point $(1, 2)$. The value of m for which the area enclosed by the line and the two axes is a minimum is:

- A. -6
- B. -5
- C. -4
- D. -3
- E. -2

Question 9

If $\int_0^a (3x - 6)dx = 0$, then the value(s) of a is (are):

- A. 0 and 4
- B. 0 only
- C. 4 only
- D. -3 and 4
- E. -3 only

Question 10

The scores on a test are normally distributed with mean μ and standard deviation σ . About 95% of the scores on the test are between 42 and 58. Which one of the following is true?

- A. $\mu = 46$ and $\sigma = 4$
- B. $\mu = 55$ and $\sigma = 5$
- C. $\mu = 50$ and $\sigma = 4$
- D. $\mu = 50$ and $\sigma = 5$
- E. $\mu = 52$ and $\sigma = 10$

Question 11

If $x = 5$ is a solution of the equation $\log_e(ax + 3) = 4$, then the exact value of a is:

- A. $\frac{e^4}{5} + 3$
- B. $\frac{e^4 - 3}{5}$
- C. $\frac{e^4}{5} - 3$
- D. $\frac{\log_a(4) - 3}{5}$
- E. $\frac{e}{3}$

Question 12

The continuous random variable X has a normal distribution with mean 5.8 and variance 1.69. The continuous random variable Z has the standard normal distribution. The probability that X is less than 4.5 is equal to

- A. $\Pr(Z > -1)$
- B. $\Pr(Z < 1)$
- C. $\Pr(Z > 1)$
- D. $1 - \Pr(Z < -1)$
- E. $\Pr(-1 < Z < 1)$

SECTION 1 – continued
TURN OVER

Question 13

The following transformations of the plane as given by

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \end{bmatrix} \text{ are:}$$

- A. dilation by factor 3 from x -axis, reflection in y -axis, translation 2 units to the right and 4 units down
- B. dilation by factor 3 from y -axis, reflection in x -axis, translation 2 units to the right and 4 units down
- C. dilation by factor $\frac{1}{3}$ from y -axis, reflection in x -axis, translation 2 units to the left and 4 units up
- D. dilation by factor $\frac{1}{3}$ from y -axis, reflection in x -axis, translation 2 units to the right and 4 units down
- E. dilation by factor 3 from x -axis, reflection in x -axis, translation 2 units to the left and 4 units down

Question 14

If $f(x) = e^x$ and $g(x) = \sin(x)$ and $h(x) = f(g(x))$, which of the following statements about $h(x)$ is correct?

- A. The maximal domain of h is R^+
- B. Local minimum turning points occur at $x = (2n - 1)\pi + \frac{\pi}{2}$, $n \in Z$
- C. $h(x) = \sin(e^x)$
- D. Range of h is $[0, e]$
- E. The minimum turning points are on the x axis.

Question 15

The average value of the function $y = \tan(x)$ for the interval $[\frac{\pi}{8}, \frac{\pi}{4}]$ is:

- A. $\frac{8}{\pi} \log_e\left(\frac{\sqrt{2}+2}{2}\right)$
- B. $\frac{1}{2} \log_e\left(\frac{\sqrt{2}+2}{2}\right)$
- C. $\frac{8}{\pi}$
- D. $\frac{4}{\pi} \log_e\left(\frac{\sqrt{2}+2}{2}\right)$
- E. ∞

Question 16

A graph has a rule $f(x) = -(x+2)(x-2)^3 + 2$. At $x = 2$, the graph has a:

- A. x-axis intercept
- B. stationary point of inflection
- C. y-axis intercept
- D. local maximum
- E. local minimum

Question 17

The sum of the solutions of $\tan(2x) = \sqrt{3}$ for $0 \leq x \leq \pi$ is:

- A. $\frac{5\pi}{3}$
- B. $\frac{11\pi}{3}$
- C. $\frac{\pi}{6}$
- D. $\frac{5\pi}{6}$
- E. $\frac{22\pi}{3}$

SECTION 1 – continued
TURN OVER

Question 18

The length of the wires sold at a store is a random variable X metres with a probability density function $f(x) = \begin{cases} 4e^{-4x}, & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases}$. The value of k such that 80% of wires are less than k metres is closest to:

- A. 0.056
- B. 0.402
- C. 0.438
- D. 0.893
- E. 1.609

Question 19

A binomial random variable has a mean of 80 and standard deviation of 4. The values of n and p respectively are:

- A. 100 and $\frac{1}{5}$
- B. 100 and $\frac{4}{5}$
- C. 200 and $\frac{2}{5}$
- D. 400 and $\frac{4}{5}$
- E. 1600 and $\frac{1}{20}$

Question 20

The coordinates of the turning point of $y = \log_e(2x) - 2x$ are:

- A. $\left(\frac{1}{4}, \log_e \frac{1}{2} - \frac{1}{2}\right)$
- B. $\left(\frac{1}{4}, 0\right)$
- C. $\left(\frac{1}{2}, -1\right)$
- D. $\left(\frac{1}{2}, \log_e 2\right)$
- E. $\left(-\frac{1}{2}, -1\right)$

Question 21

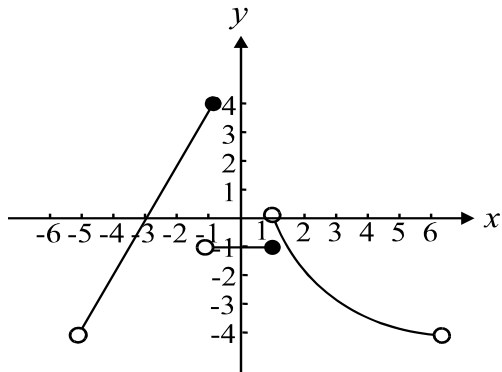
Given that $\log_a 5 + 2\log_a (2x+1) = \log_a 45$, then **all** the values of x which satisfy the equation are:

- A. 2
- B. 1, 2
- C. -2, 1
- D. 0, 1
- E. 1

SECTION 1 – continued
TURN OVER

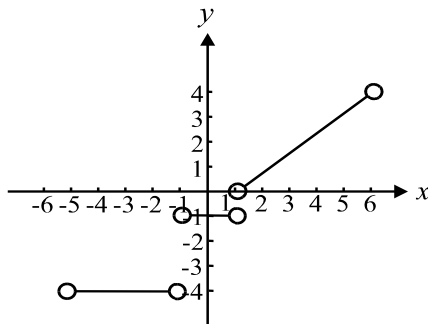
Question 22

The graph of the function g is shown below:

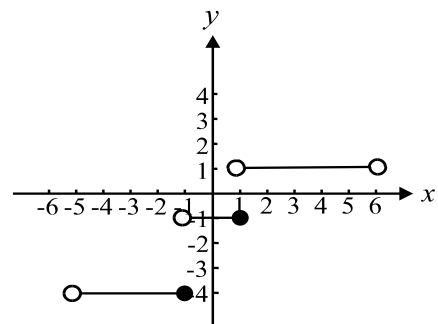


The gradient function of g is closest to:

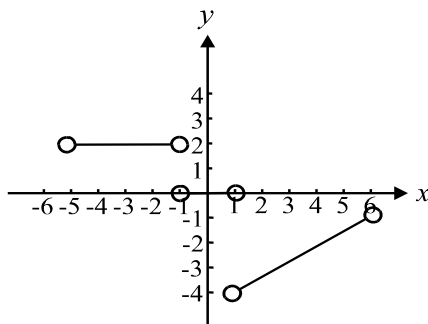
A.



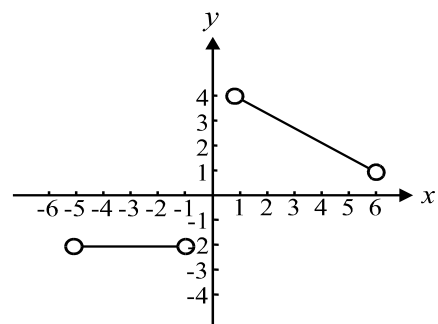
B.



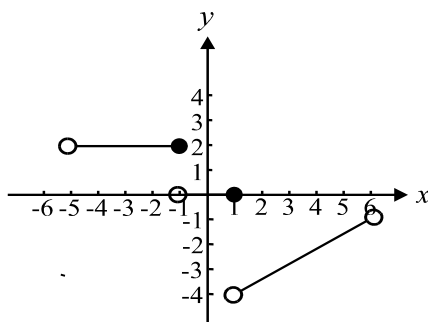
C.



D.



E.



END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

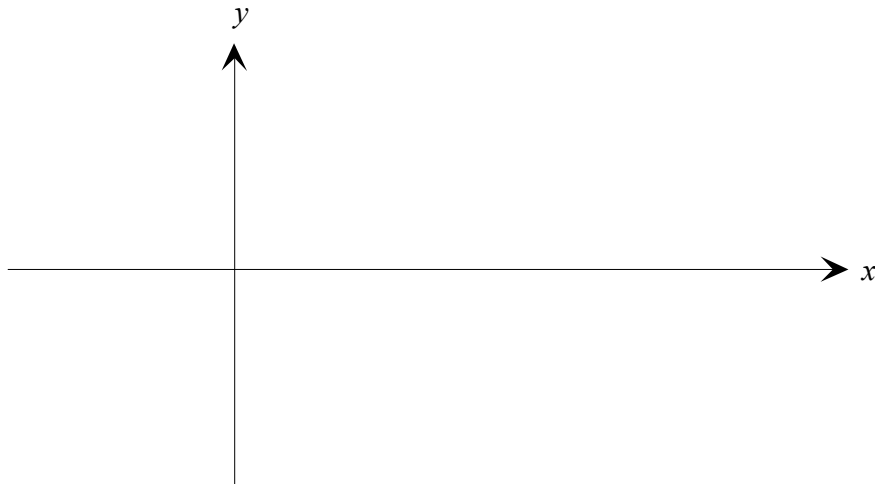
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider the function $f: R \rightarrow R$, $f(x) = \frac{1}{2}x^2(x - 8) + 12$

- a.** Sketch the graph of $y = f(x)$ on the axes below.

Label intersection points with the axes and stationary points with their exact coordinates.



3 marks

SECTION 2 – Question 1 - continued
TURN OVER

- b. Find the exact area of the region enclosed by the graph of $y = f(x)$ and the x axis.

2 marks

Given $g: R \rightarrow R$, $g(x) = ax^2(x - 8)$ where $a > 0$

- c. Show that the equation of the tangent to the curve $y = g(x)$, in terms of a , at the point where $x = 1$ is $y = 6a - 13ax$

3 marks

- d. Hence, find, in terms of a , the area of the region bounded by the curve $y = g(x)$ and the tangent to the curve, $y = 6a - 13ax$.

3 marks

- e. Find the value of a so that the tangent to the curve $y = g(x)$ at $(1, g(1))$ is also the normal to the curve at the second point of intersection of the tangent with the curve.

3 marks

Total 14 marks

SECTION 2 – continued
TURN OVER

Question 2

John keeps hens in his backyard. He regularly records the weights of the eggs that they lay and finds that the weights are normally distributed with a mean of 61 grams and a standard deviation of 8 grams.

One afternoon John checks to find a fresh laid egg in the hen coop.

- a. Calculate the probability, correct to four decimal places, that the egg weighs more than 67 grams.

1 mark

- b. Calculate the probability, correct to four decimal places, that the egg weighs more than 67 grams, given that he knows it weighs more than 61 grams.

2 marks

- c. Find the standardised value for the weight of an egg that is 59 grams.

1 mark

- d. Hence, calculate the probability, correct to four decimal places, that the egg weighs less than 59 grams.

1 mark

SECTION 2 – Question 2 - continued

The next morning, John finds 6 freshly laid eggs in the coop.

- e. Find the probability that at least two of the eggs weigh more than 67 grams.

1 mark

John's neighbors, Kath and Kim, also keep hens, and they lay eggs whose weights are normally distributed with a standard deviation of only 2 grams.

Kath and Kim brag that 98% of their eggs weigh more than 67 grams.

- f. Find the mean weight of Kath and Kim's eggs be? Give your answer correct to four decimal places.

2 marks

Total 8 marks

SECTION 2 – continued
TURN OVER

Question 3

Cooper is learning to swim. In his first 10 attempts at swimming 50m unassisted he is successful 7 times.

- a. Find the sample proportion of successfully completing the 50m swim.

1 mark

- b. In his next 4 attempts at swimming, Cooper improves his sample proportion of successfully completing the 50m to $\hat{p} = a$. If the sample proportion has an approximately normal distribution and the standard deviation is equal to 0.2, find the value of a .

2 marks

- c. Use your answer from part b. to state the 95% confidence interval for p .

1 mark

After more training the length of time Cooper can now swim follows a normal distribution, with a mean time of 55 minutes. The probability that he stays in the water for more than 80 minutes is 0.04.

- d. Find the standard deviation of the time he spends in water, correct to 2 decimal places.

2 marks

- e. Hence find the probability that he is in the water for less than one hour, given that he stays in at least 30 minutes, correct to 2 decimal places.

2 marks

Total 8 marks

Question 4

The number of mice, M , during a 12 week period in a farm can be modeled by the following rule

$$M(t) = 120 + 40\sin\left(\frac{\pi}{3}\left(t - \frac{3}{2}\right)\right)$$

Where t is the number of weeks from when the mice were first counted.

- a. State the minimum and maximum number of mice in the 12-week period.

2 marks

- b. How many mice are there after 5 weeks?

1 mark

SECTION 2 – Question 4 - continued
TURN OVER

- c. At what time(s) are there 140 mice? Show all working out.

3 marks

The farmer decides to travel during the weeks when the mice numbers are 150 or more.

- d. For how long does he stay away in total during the 12-week period, correct to two decimal places?

2 marks

- e. Consider the two-week period when the mice numbers are at their lowest level. What is the maximum number of mice on the farm during this period?

2 marks

- f. i. Show that the derivative can be expressed as $M'(t) = \frac{40\pi}{3} \sin\left(\frac{\pi t}{3}\right)$.

1 mark

- ii. Hence find the times when the mice are increasing at a rate of $\frac{20\pi}{3}$ mice per week.

2 marks

Total 13 marks

Question 5

A cuboid has dimensions x metres, h metres and $4x$ metres. The cuboid is made of 240 metres of wire.

- a. Find h in terms of x .

2 marks

- b. Find the volume, $V\text{m}^3$, of the cuboid in terms of x .

1 mark

SECTION 2 – Question 5 - continued
TURN OVER

- c. Find the volume when $x = 11$.

2 marks

- d. Find the possible values of x for which the cuboid will exist.

2 marks

- e. Find the possible values of x when $V = 1620$, correct to two decimal places.

1 mark

- f. Find the maximum volume of the cuboid and the value of x when this exists.

2 marks

Total 10 marks

Question 6

- a. Find the points of intersection of $y = 2\cos(3x)$ and $y = 1$ for $x \in [0, \pi]$.

2 marks

- b. Determine the value(s) of k for which the straight line with equation $y = x - 2k$ ($k \in \mathbb{R}$) doesn't intersect the parabola with equation $y = x^2 - 2x + 1$.

3 marks

Total 5 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name: _____

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E