

MATHS METHODS 2012

UNIT 3 SAC 2 : TEST

PART 1 SOLUTIONS

1. (a) ignoring the translations

$$y = 4 \sin(2x)$$

$$\Rightarrow y = 4 \sin\left(\frac{x}{\frac{1}{2}}\right)$$

dilation of factor 4 from x-axis (1)

dilation of factor $\frac{1}{2}$ from y-axis

translation $\frac{2\pi}{3}$ units in positive x direction

translation 5 units in positive y direction

(1)

(dilations must be stated before translations).

$$(b) \text{ range} = [5-4, 5+4]$$

$$= [1, 9] \quad (1)$$

$$\text{period} = \frac{2\pi}{\frac{1}{2}}$$

$$= \frac{2\pi}{1} \quad (1)$$

$$(c) \quad 0 \leq x \leq \pi$$

$$-\frac{2\pi}{3} \leq x - \frac{2\pi}{3} \leq \frac{\pi}{3}$$

$$-\frac{4\pi}{3} \leq 2\left(x - \frac{2\pi}{3}\right) \leq \frac{2\pi}{3}$$

start to look for solutions between $-\frac{4\pi}{3}$ and $\frac{2\pi}{3}$

$$\sin 2\left(x - \frac{2\pi}{3}\right) = \frac{1}{2}$$

$$2\left(x - \frac{2\pi}{3}\right) = -\frac{7\pi}{6}, \frac{\pi}{6}$$

(1) for $\frac{\pi}{6}$ as basic angle

$$x - \frac{2\pi}{3} = -\frac{7\pi}{12}, \frac{\pi}{12}$$

$$x = -\frac{7\pi}{12} + \frac{2\pi}{3}, \frac{\pi}{12} + \frac{2\pi}{3}$$

$$x = \frac{\pi}{12}, \frac{9\pi}{12}$$

$$x = \frac{\pi}{12}, \frac{3\pi}{4} \quad (1)$$

2. (a) vertical asymptote

at $x = 0.75$

$$\Rightarrow b(x+c) = 0$$

$$\frac{3b+c}{4} = 0 \quad (\times 4)$$

$$3b + 4c = 0$$

at $(1, 0)$

$$0 = a \log_e(b+c)$$

$$0 = \log_e(b+c)$$

$$e^0 = b+c$$

$$b+c = 1 \quad (\times -3)$$

$$3b + 4c = 0$$

$$-3b - 3c = -3$$

$$c = -3$$

$$\Rightarrow b = 4$$

$$(b) \log_e 9 = a \log_e (4 \times 1.5 - 3)$$

$$\log_e 9 = a \log_e 3$$

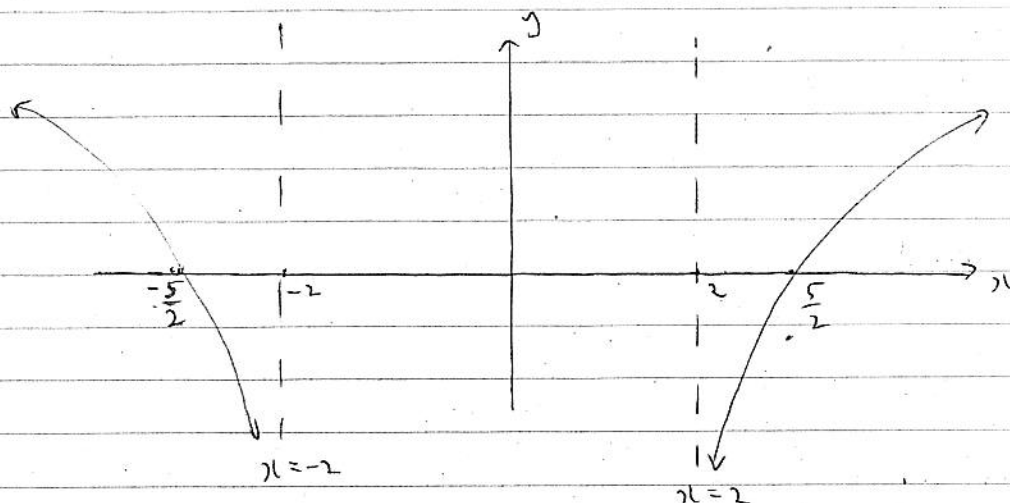
$$2 \log_e 3 = a \log_e 3$$

$$a = 2$$

(1)

3. $y = \log_e (2|x| - 4)$

(is a reflection in the y-axis of $y = \log_e (2x - 4)$
as well as the graph of
 $y = \log_e (2x - 4)$)



x-intercept, $0 = \log_e (2x - 4)$

$$e^0 = 2x - 4$$

$$2x = 5$$

$$x = \frac{5}{2}$$

vertical asymptote, where $2x - 4 = 0$
 $x = 2$

4. Let $a = 2^x$

$$2 \times a - 5 = \frac{12}{a} \quad (\times a)$$

$$2a^2 - 5a = 12$$

$$2a^2 - 5a - 12 = 0$$

$$(2a+3)(a-4) = 0$$

$$a = -\frac{3}{2} \quad \text{or} \quad a = 4$$

$$2^x = -\frac{3}{2} \quad \text{or} \quad 2^x = 4$$

no solution

$$\therefore x = 2$$

as $2^x > 0$

(1)

	f	g
dom	$\mathbb{R}$	$(0, \infty)$
ran	$[-1, 1]$	$\mathbb{R}$

$$\text{ran } g \subseteq \text{dom } f$$

$$\therefore f(g(x)) \text{ defined}$$

$$\text{dom } f(g(x)) = \text{dom } g(x) \quad (1)$$

$$= (0, \infty) \quad (1)$$

MULTI. CHOICE

1. period = π
 $\pi = \frac{2\pi}{n}$

$n = 2$

(not A or B)

AMPLITUDE = 3

when $x = 0$

C, D, E are true

BY INSPECTION

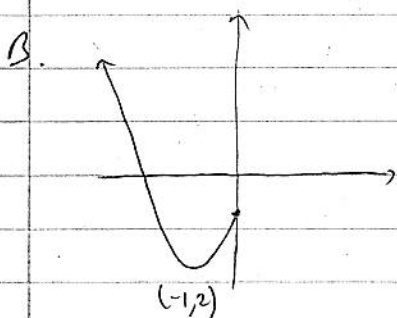
it's a cos graph translated

$\frac{\pi}{4}$ left, $\therefore$ D

(or a sin graph $\frac{\pi}{2}$ RIGHT) or C

2. must be one-to-one

A. is one to one



not one-to-one

3. $0 = \frac{1}{2} \log_e (x-1) + 3$

use solve,

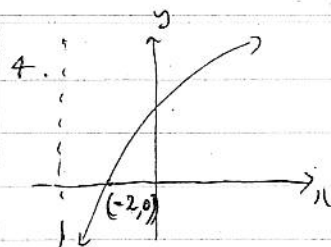
or

$-6 = \log_e (x-1)$

$e^{-6} = x-1$

$x = \frac{1}{e^6} + 1$

(A)



$x = -3$

y values change,

x values stay the same

but $(-2, 0)$ would

stay as $(-2, 0)$

and asymptote would not change

(E)

5. $y = \log_e (x+2) + 3$

then

$y = \log_e (x+2) + 3 + 1$

then

$y = -(\log_e (x+2) + 4)$

$y = -\log_e (x+2) - 4$

(D)

6. period = $\frac{\pi}{\frac{2\pi}{5}}$

$= \frac{\pi}{1} \times \frac{5}{2\pi}$

$= \frac{5}{2}$

(C)

7. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -x \\ 2y \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix}$
 $= \begin{bmatrix} -x-2 \\ 2y+2 \end{bmatrix}$

$x' = -x-2$ and $y' = 2y+2$

$x = -x'-2$ and $y = \frac{y'-2}{2}$

equation of image
 $\frac{y'-2}{2} = e^{-x'-2}$

$$y' - 2 = 2e^{-x-2}$$

$$y = 2e^{-x-2} + 2 \quad (C)$$

8. on calculator,

$$\text{Solve } (10 \cos(3x) = 5, x) \mid 0 \leq x \leq \pi$$

$$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$$

$$\frac{\pi}{9} + \frac{5\pi}{9} + \frac{7\pi}{9} = \frac{13\pi}{9} \quad (D)$$

9. inverse

$$x = 2 \log_e(y+2) - 1$$

Solve for y on calculator,

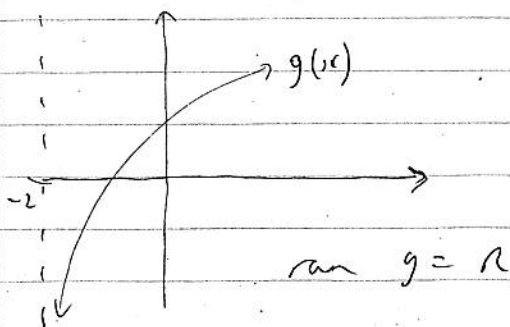
OR

$$\frac{x+1}{2} = \log_e(y+2)$$

$$y+2 = e^{\frac{x+1}{2}}$$

$$y = e^{\frac{x+1}{2}} - 2$$

dom $g^{-1} = \text{ran } g$



$$g^{-1}: \mathbb{R} \rightarrow \mathbb{R}, g^{-1}(x) = e^{\frac{x+1}{2}} - 2$$

(D)

10. Solve $\sin(2x) = -1$
on calculator

$$x = \frac{(4n-1)\pi}{4}, n \in \mathbb{Z}$$

$$x = \frac{4n\pi}{4} - \frac{\pi}{4}$$

$$= n\pi - \frac{\pi}{4} \quad (A)$$

$$11. \text{ period} = \frac{2\pi}{2}$$

use calculator to solve

$$\text{for e.g. } |2 \cos(2x)| = 2$$

for given domain

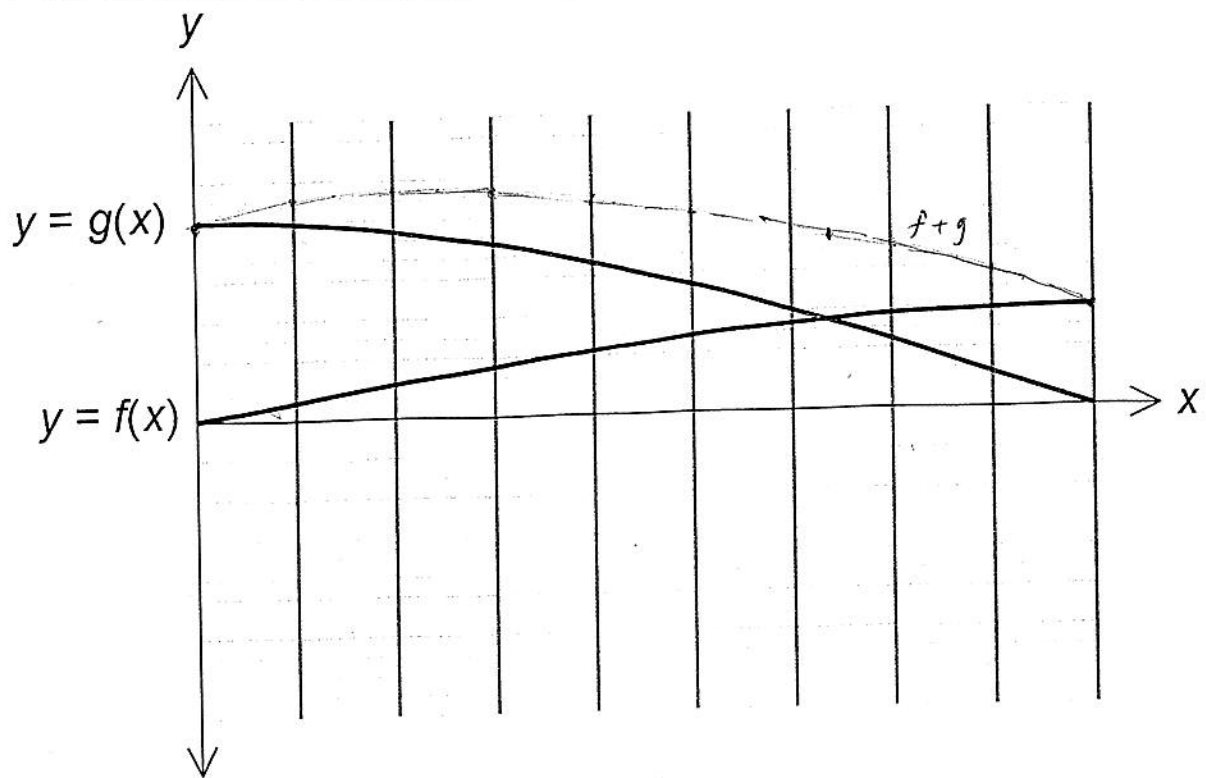
gives 9 solutions

(or draw or picture graph)

(E)

$$12. \mathbb{R} \setminus \{2\}$$

(C)



2 marks

1 mark for endpoints placed correctly
1 mark for correctly placed shape

2. (a) $h(0) = -1.0 \text{ m.}$

(b) maximum height = 8.2 m.
at $t = 25.4 \text{ seconds}$

(c) $4 = -0.5 + e^{0.07t} \sin\left(\frac{\pi t - 554}{2}\right) + 0.11t$

$t = 16.9 \text{ s.}$