

Trial Examination 2012

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

Question and answer booklet of 10 pages, with detachable sheet of miscellaneous formulas in the centerfold.

Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this book during reading time.

Write your **name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2012 VCE Mathematical Methods (CAS) Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

If $h(x) = x \sin^2(x)$, find $h'\left(\frac{\pi}{6}\right)$.

3 marks

Question 2

Find the value of k given that $\int_{\sqrt{2}}^2 \frac{1}{3-x} dx = \log_e(k)$.

2 marks

Question 3

Functions f and g are given by

$$f: \mathbb{R} \rightarrow \mathbb{R} \text{ where } f(x) = x^2$$

and

$$g: \mathbb{R} \setminus \{-2\} \rightarrow \mathbb{R} \text{ where } g(x) = \frac{1}{x+2}.$$

- a. Solve the equation $f(g(x)) = 4$.

2 marks

- b. The inverse of g is g^{-1} .

Find $g^{-1}(x)$, stating its domain and range.

3 marks

Total 5 marks

Question 4

Find the smallest positive value of x that satisfies the equation $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$.

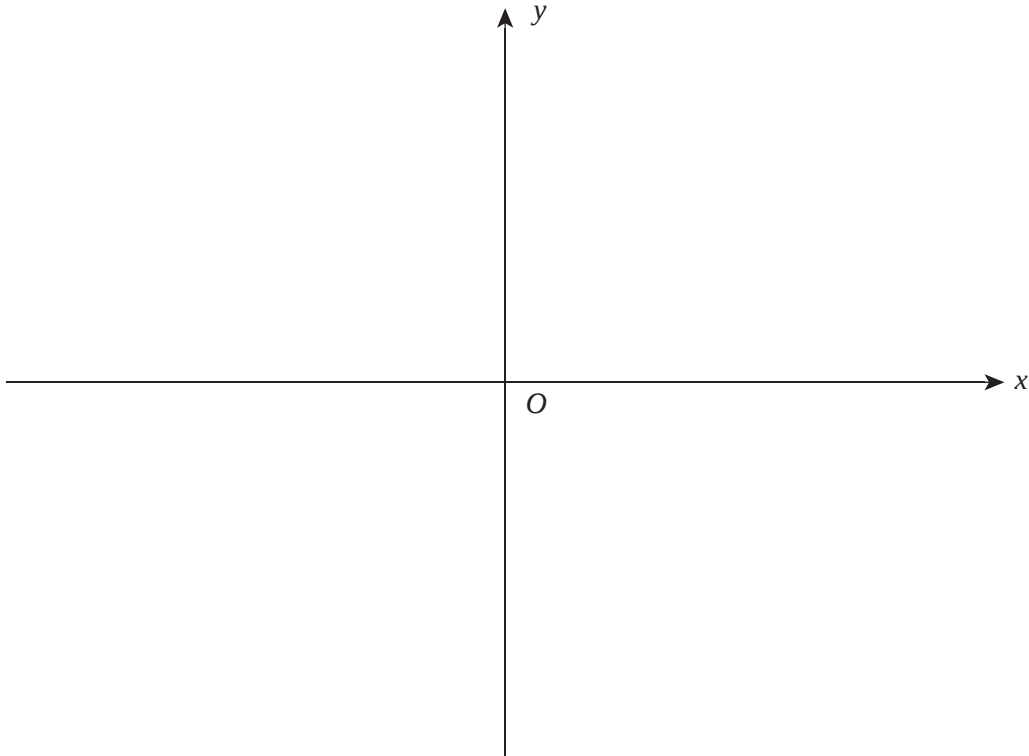
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4 marks

Question 5

- a. Sketch the curve with equation $y = \frac{2x - 1}{x + 1}$.

State the coordinates of the points where the curve intersects the coordinate axes, and the equations of any asymptotes.



4 marks

- b. Hence, or otherwise, solve the inequality $0 < \frac{2x - 1}{x + 1} < 2$.

1 mark

Total 5 marks

Question 6

A continuous random variable X has probability density function

$$f(x) = \begin{cases} ax + b & \text{if } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

The mean of X is $\frac{7}{12}$.

Find the values of a and b .

4 marks

Question 7

If two events, A and B , are such that $\Pr(A) = \frac{2}{5}$, $\Pr(B) = \frac{1}{5}$ and $\Pr(A|B) = \frac{3}{5}$, find $\Pr(B|A)$.

3 marks

Question 8

The discrete random variable X has the probability distribution

x	0	1	2	3
$\Pr(X = x)$	$2p$	$2p^2$	$p^2 + p$	$2p^2 + p$

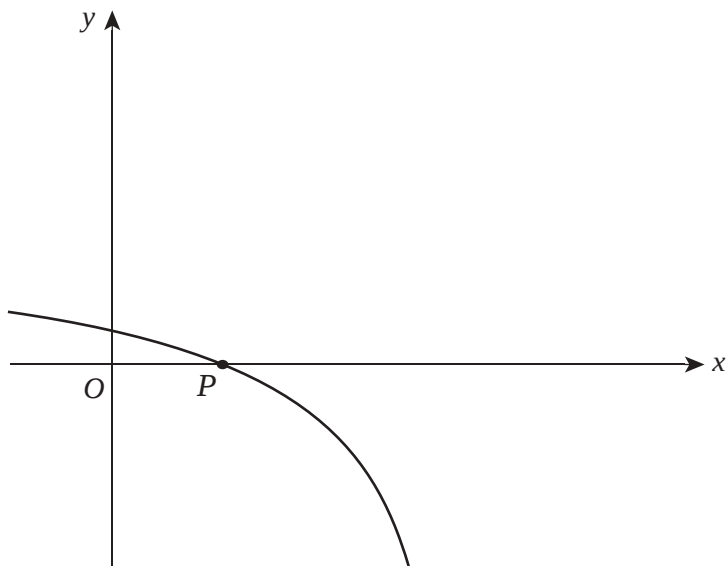
Find the value of p .

2 marks

Question 9

Part of the graph of $y = 4 - e^{2x}$ is shown below.

The curve crosses the x -axis at P . The coordinates of P are $(\log_e(2), 0)$.



- a. Show that the equation of the normal to the curve $y = 4 - e^{2x}$ at the point P is given

by $y = \frac{1}{8}x - \frac{1}{8}\log_e(2)$.

2 marks

- b. Find the area of the region enclosed by the curve $y = 4 - e^{2x}$, the normal to the curve at P and the y -axis.

3 marks
Total 5 marks

Question 10

Consider the function $f: R \rightarrow R, f(x) = x + \sqrt{x^2 + 1}$.

- a. Show that $f'(x) = 1 + \frac{x}{\sqrt{x^2 + 1}}$.

2 marks

- b. Given that $g(x) = \log_e(f(x))$, show that $g'(x) = \frac{1}{\sqrt{x^2 + 1}}$.

3 marks

- c. Hence, evaluate $\int_0^1 \frac{1}{\sqrt{x^2 + 1}} dx$.

2 marks

Total 7 marks

END OF QUESTION AND ANSWER BOOKLET