



Trial Examination 2012

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1

- a. Using the product rule, i.e. $u = x$, $\frac{du}{dx} = 1$, $v = \sin^2(x)$ and $\frac{dv}{dx} = 2 \sin(x) \cos(x)$. M1

Hence $h'(x) = 2x \sin(x) \cos(x) + \sin^2(x)$. A1

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \text{ and } \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

$$h'\left(\frac{\pi}{6}\right) = 2 \times \frac{\pi}{6} \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + \left(\frac{1}{2}\right)^2$$

$$h'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}\pi}{12} + \frac{1}{4}$$
 A1

Question 2

$$\int_{\sqrt{2}}^2 \frac{1}{3-x} dx = [-\log_e |3-x|]_{\sqrt{2}}^2$$
 A1

$$= -\log_e(1) + \log_e(3 - \sqrt{2})$$

$$= \log_e(3 - \sqrt{2})$$

So, $\log_e(3 - \sqrt{2}) = \log_e(k)$ and hence $k = 3 - \sqrt{2}$. A1

Question 3

a. $f(g(x)) = \frac{1}{(x+2)^2}$ A1

$$\frac{1}{(x+2)^2} = 4 \text{ and so } (x+2)^2 = \frac{1}{4}.$$

$$x+2 = \pm \frac{1}{2}$$

Hence $x = -\frac{5}{2}, -\frac{3}{2}$. A1

b. Interchanging x and y we obtain $x = \frac{1}{y+2}$.

Solving for y we obtain $y = \frac{1}{x} - 2$, i.e. $g^{-1}(x) = \frac{1}{x} - 2$. A1

The domain of g^{-1} is $\mathbb{R} \setminus \{0\}$ and the range of g^{-1} is $\mathbb{R} \setminus \{-2\}$. A1

Question 4

Using $\cos\left(\pm\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$. A1

More generally, $\cos\left(\pm\frac{\pi}{4} + 2k\pi\right) = \frac{1}{\sqrt{2}}$. M1

Since $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$

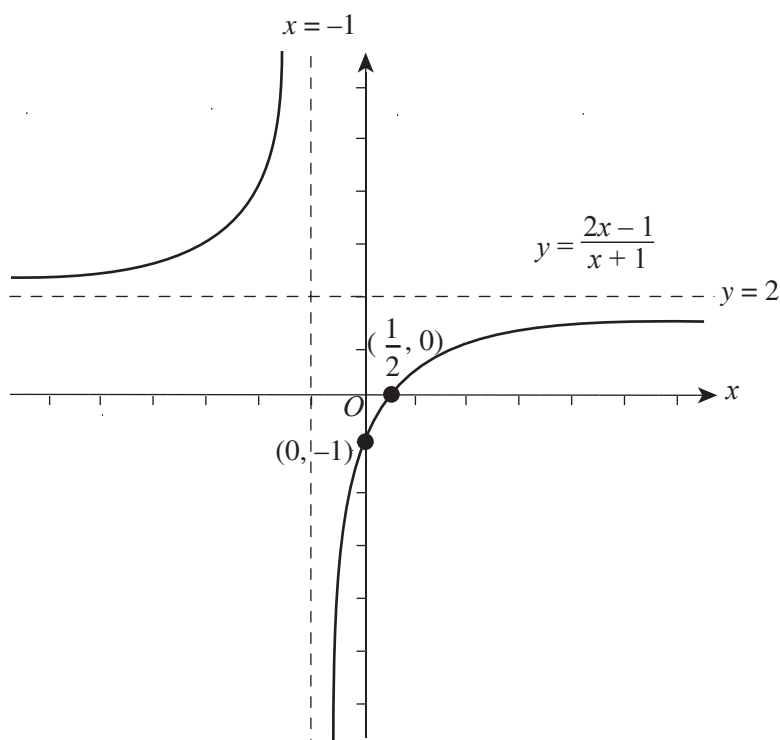
$$\frac{x}{2} + \frac{\pi}{3} = \pm\frac{\pi}{4} + 2k\pi$$

$$\text{So } x = -\frac{2\pi}{3} \pm \frac{\pi}{2} + 4k\pi \quad \text{M1}$$

Substituting $k = 1$ gives $x = \frac{17\pi}{6}$. A1

Question 5

a.



Attempting to express $y = \frac{2x-1}{x+1}$ as $y = 2 - \frac{3}{x+1}$. M1

Asymptotes $x = -1$, $y = 2$. A1

Axes intercepts $(0, -1)$, $(\frac{1}{2}, 0)$. A1

Two correct branches. A1

b. Referring to the graph, $0 < y < 2$ for $x > \frac{1}{2}$ A1

Question 6

From $\int_0^1 (ax + b)dx = 1$ we obtain $\left[\frac{ax^2}{2} + bx\right]_0^1 = 1$.

Hence $\frac{a}{2} + b = 1$. (1) A1

From $\int_0^1 x(ax + b)dx = \frac{7}{12}$ we obtain $\left[\frac{ax^3}{3} + \frac{bx^2}{2}\right]_0^1 = \frac{7}{12}$.

Hence $\frac{a}{3} + \frac{b}{2} = \frac{7}{12}$. (2) A1

Attempting elimination or substitution, for example $2 \times (2) - (1)$ gives $\frac{a}{6} = \frac{1}{6}$. M1

Hence $a = 1$ and $b = \frac{1}{2}$. A1

Question 7

Using $\Pr(A \cap B) = \Pr(A|B) \times \Pr(B)$ we obtain $\Pr(A \cap B) = \frac{3}{5} \times \frac{1}{5}$ i.e. $\Pr(A \cap B) = \frac{3}{25}$. M1

Now $\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)}$ and so $\Pr(B|A) = \frac{\frac{3}{25}}{\frac{2}{5}}$, i.e. $\Pr(B|A) = \frac{3}{10}$. M1 A1

Question 8

Given that $\sum \Pr(X = x) = 1$ we obtain $5p^2 + 4p - 1 = 0$. M1

Factorising the LHS we obtain $(p + 1)(5p - 1) = 0$.

We reject $p = -1$ since $0 < p < 1$, and so $p = \frac{1}{5}$. A1

Question 9

- a. Given $y = 4 - e^{2x}$ we obtain $\frac{dy}{dx} = -2e^{2x}$.

At $x = \log_e(2)$, $\frac{dy}{dx} = -2e^{2\log_e(2)}$ i.e. $\frac{dy}{dx} = -8$.

So the gradient of the normal is $\frac{1}{8}$.

M1

Using $y - y_1 = m(x - x_1)$ we obtain $y = \frac{1}{8}(x - \log_e(2))$.

Hence the equation of the normal is $y = \frac{1}{8}x - \frac{1}{8}\log_e(2)$.

A1

- b. The area bounded by the curve and the two axes is given by

$$\int_0^{\log_e(2)} (4 - e^{2x}) dx = \left[4x - \frac{1}{2}e^{2x} \right]_0^{\log_e(2)}$$

$$= \left[4\log_e(2) - \frac{1}{2}e^{2\log_e(2)} \right] - \left[0 - \frac{1}{2}e^0 \right]$$

M1

$$= 4\log_e(2) - 2 + \frac{1}{2}$$

$$= 4\log_e(2) - \frac{3}{2}$$

The area of the triangle bounded by the normal and the two axes can be calculated as follows:

When $x = 0$, $y = -\frac{1}{8}\log_e(2)$.

The area of the triangle is $\frac{1}{2} \times \frac{1}{8}\log_e(2) \times \log_e(2)$, i.e. $\frac{1}{16}(\log_e(2))^2$.

So the total area is $4\log_e(2) - \frac{3}{2} + \frac{1}{16}(\log_e(2))^2$ (square units).

A1 A1

Question 10

- a. Given $f(x) = x + \sqrt{x^2 + 1}$,

$$= x + (x^2 + 1)^{\frac{1}{2}}$$

$$f'(x) = 1 + \frac{1}{2}(2x)(x^2 + 1)^{-\frac{1}{2}}$$

M1

$$f'(x) = 1 + \frac{x}{\sqrt{x^2 + 1}}$$

A1

- b. Given that $g(x) = \log_e(f(x))$, $g'(x) = \frac{f'(x)}{f(x)}$.

$$\text{So, } g'(x) = \frac{1 + \frac{x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} \quad \text{M1}$$

$$= \frac{\frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}}}{x + \sqrt{x^2 + 1}} \quad \text{A1}$$

$$= \frac{\sqrt{x^2 + 1} + x}{\sqrt{x^2 + 1}} \times \frac{1}{x + \sqrt{x^2 + 1}}$$

$$= \frac{1}{\sqrt{x^2 + 1}}$$

$$\text{Hence } g'(x) = \frac{1}{\sqrt{x^2 + 1}}. \quad \text{A1}$$

- c. As $g(x) = \log_e(f(x))$, $\int g'(x) dx = \log_e(f(x)) + c$.

$$\text{So, } \int_0^1 \frac{1}{\sqrt{x^2 + 1}} dx = [\log_e(x + \sqrt{x^2 + 1})]_0^1 \quad \text{A1}$$

$$= \log_e(1 + \sqrt{2}) \quad \text{A1}$$