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YEAR 12 Trial Exam Paper

2012

MATHEMATICAL METHODS (CAS)

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 1 hour

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring notes of any kind, sheets of paper, white out liquid/tape or a calculator into the examination.

Materials provided

- The question and answer book of 11 pages, with a separate sheet of miscellaneous formulas.
- Working space is provided throughout the question book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Let $f(x) = \sqrt{3-2x}$. Write down the rule for $f(\cos x)$.

1 mark

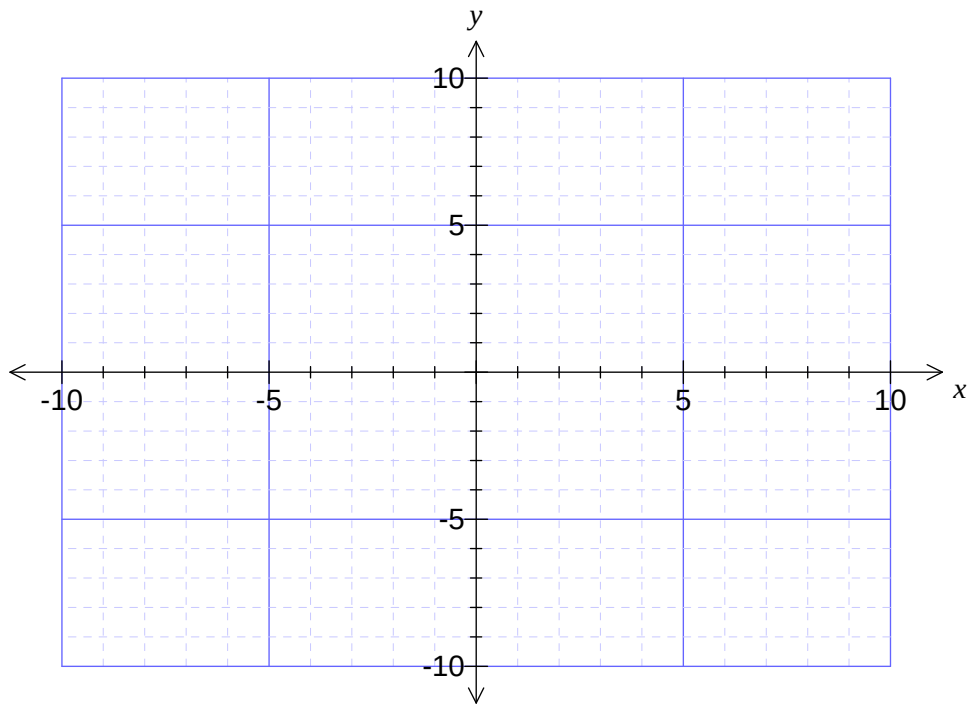
Question 2

For the function $f : (-1, \infty) \rightarrow \mathbb{R}$, $f(x) = \frac{1}{3} \log_e \left(\frac{x+1}{2} \right)$

a. Find the rule for the inverse function, f^{-1} .

2 marks

- b. Sketch the graph of $y = f^{-1}(f(x))$ on the axes below.



1 mark

- c. The function $f(x)$ undergoes a transformation as defined by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.
State the new equation.

1 mark

End of Question 2
TURN OVER

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Question 3

a. Let $y = \frac{e^{2x}}{x}$. Find $\frac{dy}{dx}$.

2 marks

b. Let $f(x) = \sqrt{\sin(2x)}$. Find $f'\left(\frac{\pi}{4}\right)$.

3 marks

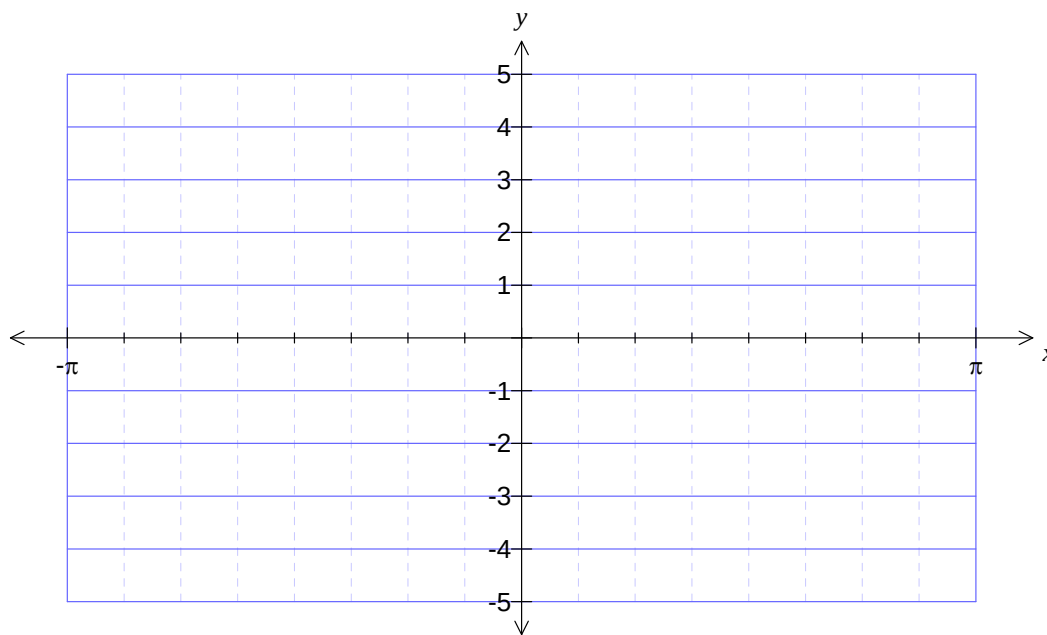
End of Question 3
TURN OVER

Question 4

The graph of $y = \cos(x)$ undergoes the following transformations:

- a dilation of factor $\frac{1}{2}$ from the y-axis
- a translation of +3 units up.

- a.** Sketch the transformed graph over the domain $[-\pi, \pi]$ on the axes below.
Label all intercepts and endpoints as co-ordinates.



2 marks

- b.** State the equation of the normal to the graph in part **a** at $x = \frac{\pi}{2}$ and sketch the normal on the axes above.

2 marks

End of Question 4

Question 5

Voicefone, a telephone company, currently has 30% of the market for a new type of home phone system. There are no contracts and customers simply take out a plan for a month at a time. Of the current customers, 80% will still be customers in the next month.

Assume that, for this type of system, $a\%$ of the rest of the market switches to this telephone company from one month to the next.

- a. i.** Write a transition matrix for this situation.

- ii.** Find the value of a needed in order for Voicefone to maintain its market share.

3 marks

Voicefone runs an advertising campaign and hopes to eventually hold 60% of the total market, for this type of system.

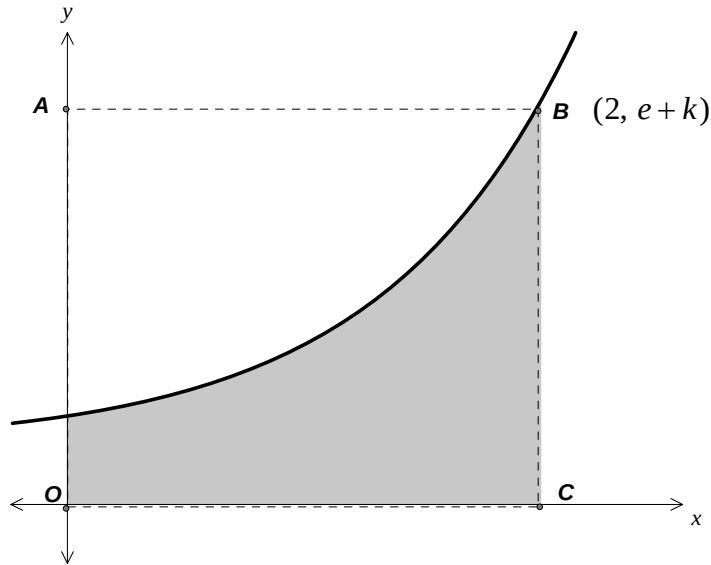
- b.** Find the value of a for this situation.

1 mark

**End of Question 5
TURN OVER**

Question 6

Consider the graph of $y = e^{\frac{x}{2}} + k$. $OABC$ is a rectangle, as shown in the diagram below. If the shaded and unshaded regions are equal in area, find k .



3 marks

Question 7

- a. Find the general solution of $\sqrt{2} \cos(3x) = -1$.

3 marks

- b. Find the average value of the function $y = 2 \cos(2x)$ over the interval $\left[0, \frac{\pi}{12}\right]$.

3 marks

Question 8

A spherical balloon is being inflated. Its volume is increasing at the rate of 4 cm^3 per second. Find the rate, in cm s^{-1} , at which the radius of the balloon is increasing when the radius is 2 cm.

3 marks

End of Question 8
TURN OVER

Question 9

Let X be a random variable with a normal distribution with mean 6 and variance 4, and let Z be a random variable with the standard normal distribution. If $\Pr(Z > 1) = 0.16$,

- a.** Find $\Pr(X > 8)$.

1 mark

- b.** Find $\Pr(X > 8 | X > 6)$.

2 marks

- c.** Find a such that $\Pr(Z > a) = \Pr(X < 5)$.

2 marks

End of Question 9

Question 10

- a. Using the linear approximation $f(x+h) \approx f(x) + hf'(x)$, where h is 0.03, and $f(x) = \sqrt{x}$, find an approximate value of $\sqrt{16.03}$.

2 marks

- b. Explain why your answer to part a overestimates the value of $\sqrt{16.03}$.

1 mark

Question 11

If $f(x) = 3x^2$, show that $f(u+v) + f(u-v) = 2(f(u) + f(v))$.

2 marks

END OF QUESTION AND ANSWER BOOK