
Question 1

a. $y = \frac{e^{2x}}{x^2 + 3}$
 $\frac{dy}{dx} = \frac{(x^2 + 3) \times 2e^{2x} - 2xe^{2x}}{(x^2 + 3)^2}$ (quotient rule)

(1 mark)

Avoid “further engagement” errors by leaving this fraction as it is. No like terms are obtained by expanding the brackets in the numerator.

b. $f(x) = 2x \tan\left(\frac{x}{3}\right)$
 $f'(x) = 2x \times \frac{1}{3} \sec^2\left(\frac{x}{3}\right) + 2 \tan\left(\frac{x}{3}\right)$ (product rule)

$$f'(\pi) = \frac{2\pi}{3} \sec^2\left(\frac{\pi}{3}\right) + 2 \tan\left(\frac{\pi}{3}\right) \quad \textbf{(1 mark)}$$

$$= \frac{2\pi}{3} \times \frac{1}{\cos^2\left(\frac{\pi}{3}\right)} + 2 \tan\left(\frac{\pi}{3}\right)$$

$$= \frac{2\pi}{3} \times \frac{1}{\left(\frac{1}{2}\right)^2} + 2\sqrt{3}$$

$$= \frac{2\pi}{3} \times 1 \div \frac{1}{4} + 2\sqrt{3}$$

$$= \frac{2\pi}{3} \times 1 \times \frac{4}{1} + 2\sqrt{3}$$

$$= \frac{8\pi}{3} + 2\sqrt{3}$$

(1 mark)

Question 2

a.
$$\int \sin(3x-1) dx$$

$$= -\frac{1}{3} \cos(3x-1)$$

(1 mark)

Note, because we are looking for ‘an’ antiderivative, the constant of antidifferentiation c does not have to be included because “an” antiderivative could be the case where $c = 0$.

Marks would not be lost though if c was included.

b.
$$\int_0^2 \frac{1}{2x+1} dx = \frac{1}{2} \int_0^2 \frac{2}{2x+1} dx$$

$$= \left[\frac{1}{2} \log_e |2x+1| \right]_0^2$$

(1 mark)

$$= \frac{1}{2} \log_e |4+1| - \frac{1}{2} \log_e |1|$$

$$= \frac{1}{2} \log_e (5) \quad \text{since } \log_e (1) = 0$$

(1 mark)**Question 3**

a.
$$f(x) = \log_e(x), \quad x > 0$$

$$f\left(\frac{1}{u}\right) = \log_e\left(\frac{1}{u}\right), \quad u > 0$$

$$= \log_e(u^{-1})$$

$$= -1 \log_e(u)$$

$$= -f(u)$$

as required.

(1 mark)

b. Given $2f(u) = f(2v) + f(3v)$ and $u, v \in \mathbb{R}^+$,

$$2 \log_e(u) = \log_e(2v) + \log_e(3v)$$

(1 mark)

$$\log_e(u^2) = \log_e(6v^2)$$

$$u^2 = 6v^2$$

So, $u = \pm\sqrt{6}v$ but $u, v \in \mathbb{R}^+$

so $u = \sqrt{6}v$

(1 mark)

Question 4

a. $g: R \rightarrow R, g(x) = 3 + \sin\left(\frac{2x}{3}\right)$

$$\text{period} = \frac{2\pi}{n} \text{ where } n = \frac{2}{3}$$

$$= 2\pi \div \frac{2}{3}$$

$$= 2\pi \times \frac{3}{2}$$

$$= 3\pi$$

(1 mark)

The amplitude of the function is 1 and the graph of $y = \sin\left(\frac{2x}{3}\right)$ is translated 3 units

up to become the graph of $y = 3 + \sin\left(\frac{2x}{3}\right)$.

The range of g is therefore $[2, 4]$.

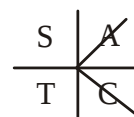
(1 mark)

b. $\cos(3x) = \frac{\sqrt{3}}{2}, \quad x \in R$

$$3x = -\frac{\pi}{6} + 2n\pi, \quad \frac{\pi}{6} + 2n\pi, \quad n \in Z$$

$$3x = 2n\pi \pm \frac{\pi}{6}$$

$$x = \frac{2n\pi}{3} \pm \frac{\pi}{18}$$

**(1 mark)** – correct base angles**(1 mark)** – correct use of $2n\pi$ **(1 mark)** – correct answer**Question 5**

x	0	1	2	3
$\Pr(X = x)$	0.2	0.1	0.4	p

a. $0.2 + 0.1 + 0.4 + p = 1$

$$p = 0.3$$

(1 mark)

b. $\Pr(X \leq 1 | X < 3) = \frac{0.2 + 0.1}{0.2 + 0.1 + 0.4}$

(1 mark)

$$= \frac{0.3}{0.7}$$

$$= \frac{3}{7}$$

(1 mark)

c. $E(X) = 0 \times 0.2 + 1 \times 0.1 + 2 \times 0.4 + 3 \times 0.3$

(1 mark)

$$= 0.1 + 0.8 + 0.9$$

$$= 1.8$$

(1 mark)

Question 6

$$f: R \rightarrow R, f(x) = 1 - e^{2(x-1)}$$

Let $y = 1 - e^{2(x-1)}$

Swap x and y for inverse.

$$x = 1 - e^{2(y-1)}$$

(1 mark)

Rearrange

$$x - 1 = -e^{2(y-1)}$$

$$1 - x = e^{2(y-1)}$$

$$\log_e(1 - x) = 2(y - 1)$$

$$\frac{1}{2} \log_e(1 - x) = y - 1$$

$$y = 1 + \frac{1}{2} \log_e(1 - x)$$

So $f^{-1}(x) = 1 + \frac{1}{2} \log_e(1 - x)$

(1 mark)

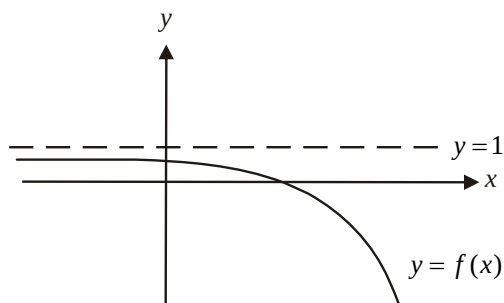
To find the domain of f^{-1} ,

Method 1

$$d_{f^{-1}} = r_f$$

Do a quick sketch of $y = f(x)$.

So $r_f = (-\infty, 1)$ and so $d_{f^{-1}} = (-\infty, 1)$

**(1 mark)**Method 2

Since $f^{-1}(x) = 1 + \frac{1}{2} \log_e(1 - x)$,

$$1 - x > 0 \text{ for the log function to be defined}$$

$$-x > -1$$

$$x < 1$$

So $d_{f^{-1}} = (-\infty, 1)$

(1 mark)

Note that to find the inverse function f^{-1} you must give the domain and the rule.

Question 7Method 1

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

where (x', y') is the image point

$$\begin{aligned} x' &= 2x + 0y + 1 & y' &= 0x + 4y - 3 \\ &= 2x + 1 & &= 4y - 3 \end{aligned}$$

(1 mark)

$$\begin{aligned} \text{Rearrange } x' - 1 &= 2x & y' + 3 &= 4y \\ x &= \frac{x' - 1}{2} & y &= \frac{y' + 3}{4} \end{aligned}$$

$$\begin{aligned} \text{Substitute into } y &= \frac{1}{x} \\ \frac{y' + 3}{4} &= 1 \div \frac{x' - 1}{2} \\ \frac{y' + 3}{4} &= 1 \times \frac{2}{x' - 1} \\ y' + 3 &= \frac{8}{x' - 1} \\ y' &= \frac{8}{x' - 1} - 3 \end{aligned}$$

$$\text{So the image equation is } y = \frac{8}{x - 1} - 3$$

(1 mark)

RE-READ THE QUESTION!

$$\text{Since } y = \frac{a}{x + b} + c, \quad a = 8, \quad b = -1 \text{ and } c = -3.$$

(1 mark)Method 2

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

We have a dilation by a factor 2 from the y-axis

$$\text{so } y = \frac{1}{x} \text{ becomes } y = \frac{1}{\frac{x}{2}} = \frac{2}{x}.$$

We have a dilation by a factor of 4 from the x-axis

$$\text{so } y = \frac{2}{x} \text{ becomes } \frac{y}{4} = \frac{2}{x} \text{ so } y = \frac{8}{x}.$$

(1 mark)

We have a translation of 1 unit to the right and 3 units down

$$\text{so } y = \frac{8}{x} \text{ becomes } y + 3 = \frac{8}{x - 1} \text{ so } y = \frac{8}{x - 1} - 3$$

$$\text{So the image equation is } y = \frac{8}{x - 1} - 3$$

(1 mark)

RE-READ THE QUESTION!

$$\text{Since } y = \frac{a}{x + b} + c, \quad a = 8, \quad b = -1 \text{ and } c = -3.$$

(1 mark)

Question 8

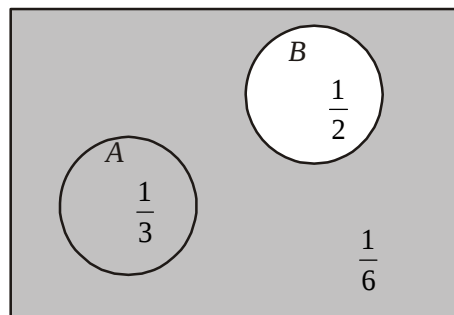
$$\Pr(A) = \frac{1}{3}, \quad \Pr(B) = \frac{1}{2}$$

- a. If A and B are mutually exclusive then $\Pr(A \cap B) = 0$

Method 1 – Venn diagram

$$\begin{aligned} \text{Note: } & \frac{1}{3} + \frac{1}{2} \\ &= \frac{2}{6} + \frac{3}{6} \\ &= \frac{5}{6} \end{aligned}$$

$\Pr(A \cup B')$ is shaded.



(1 mark)

$$\text{So } \Pr(A \cup B') = 1 - \frac{1}{2} = \frac{1}{2}$$

(1 mark)

Method 2 – Addition formula

$$\Pr(A \cup B') = \Pr(A) + \Pr(B') - \Pr(A \cap B')$$

$$\begin{aligned} &= \frac{1}{3} + \frac{1}{2} - \frac{1}{3} \\ &= \frac{1}{2} \end{aligned}$$

(1 mark)

(1 mark)

- b. If A and B are independent events then $\Pr(A) \times \Pr(B) = \Pr(A \cap B)$

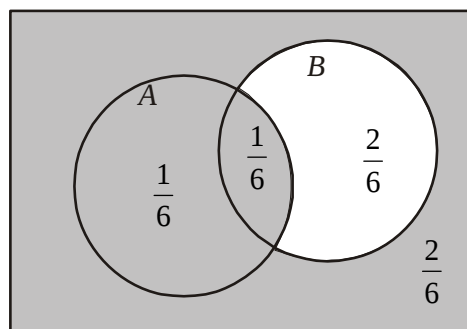
Method 1 – Venn diagram

$$\begin{aligned} \text{So } \Pr(A \cap B) &= \frac{1}{3} \times \frac{1}{2} \\ &= \frac{1}{6} \end{aligned}$$

$\Pr(A \cup B')$ is shaded.

$$\begin{aligned} \text{So } \Pr(A \cup B') &= \frac{1}{6} + \frac{1}{6} + \frac{2}{6} \\ &= \frac{2}{3} \end{aligned}$$

(1 mark)



(1 mark)

Method 2 – Addition formula

$$\Pr(A \cup B') = \Pr(A) + \Pr(B') - \Pr(A \cap B')$$

$$\begin{aligned} &= \frac{1}{3} + \frac{1}{2} - \frac{1}{6} \\ &= \frac{2}{6} + \frac{3}{6} - \frac{1}{6} \\ &= \frac{2}{3} \end{aligned}$$

(1 mark)

(1 mark)

Question 9

Pr(June shops locally exactly once in the coming 3 weeks)

$$= \Pr(L, M, M) + \Pr(M, L, M) + \Pr(M, M, L) \quad (1 \text{ mark})$$

$$= 0.4 \times 0.7 \times 0.6 + 0.6 \times 0.4 \times 0.7 + 0.6 \times 0.6 \times 0.4 \quad (1 \text{ mark})$$

$$= 0.168 + 0.168 + 0.144$$

$$= 0.48$$

(1 mark)

Question 10

$$y = x^{\frac{3}{5}} + c$$

$$\frac{dy}{dx} = \frac{3}{5} x^{-\frac{2}{5}}$$

At $x=1$,

$$\frac{dy}{dx} = \frac{3}{5} \times 1$$

$$= \frac{3}{5}$$

The gradient of the tangent at $x=1$ is $\frac{3}{5}$ so the gradient of the normal at $x=1$ is $-\frac{5}{3}$.

(1 mark)

The normal passes through the point $(a,0)$ which is its x -intercept and through the point of

normalcy which occurs at $x=1$. At $x=1$, $y = 1^{\frac{3}{5}} + c = 1 + c$.

So the point of normalcy is $(1, 1 + c)$.

The gradient of the normal is therefore given by

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 + c - 0}{1 - a}$$

$$= \frac{1 + c}{1 - a}$$

$$\text{So } \frac{-5}{3} = \frac{1 + c}{1 - a}$$

(1 mark)

$$-5(1 - a) = 3(1 + c)$$

$$-5 + 5a = 3 + 3c$$

$$5a = 8 + 3c$$

$$a = \frac{3c + 8}{5}$$

(1 mark)

Question 11

The graphs intersect when $2\sin(x) = 2\cos(x)$

$$\sin(x) = \cos(x)$$

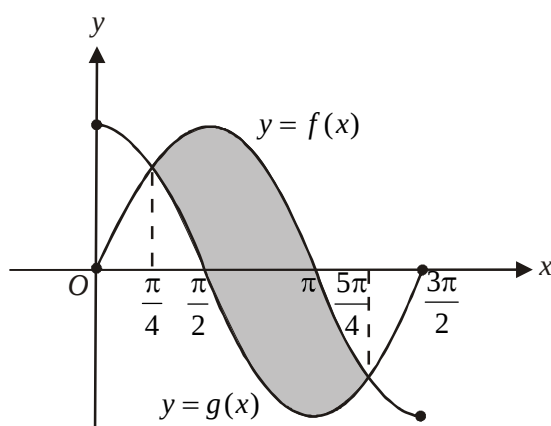
$$\frac{\sin(x)}{\cos(x)} = 1$$

$$\tan(x) = 1$$

$$x = \frac{\pi}{4}, \frac{5\pi}{4}$$



(1 mark)



$$\begin{aligned} \text{shaded area} &= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (f(x) - g(x)) dx \\ &= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (2\sin(x) - 2\cos(x)) dx \end{aligned}$$

(1 mark) for correct terminals of integration

(1 mark) for correct integrand

(1 mark)

$$\begin{aligned} &= \left[-2\cos(x) - 2\sin(x) \right]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \\ &= -2 \left\{ \left(\cos \frac{5\pi}{4} + \sin \frac{5\pi}{4} \right) - \left(\cos \left(\frac{\pi}{4} \right) + \sin \frac{\pi}{4} \right) \right\} \\ &= -2 \left\{ \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \right\} \\ &= -2 \left(\frac{-2}{\sqrt{2}} - \frac{2}{\sqrt{2}} \right) \\ &= -2 \times \frac{-4}{\sqrt{2}} \\ &= 4\sqrt{2} \text{ square units} \end{aligned}$$

(1 mark)