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Units 3 and 4 Maths Methods (CAS): Exam 1

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Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$$\begin{aligned}\frac{d}{dx}(x \log_e(x)) &= \frac{d}{dx}x \times \log_e(x) + x \times \frac{d}{dx}\log_e(x) \quad [1] \\ &= \log_e(x) + x \frac{1}{x} = 1 + \log_e(x) \quad [1]\end{aligned}$$

Question 1b

$$\begin{aligned}\int_1^2 \log_e(x) dx &= \int_1^2 [1 + \log_e(x)] dx - \int_1^2 (x) dx \quad [1] \\ &= [x \log_e(x)]_1^2 - [x]_1^2 = 2 \log_e(2) - 1 \quad [1]\end{aligned}$$

Question 2

$$\begin{aligned}\mu &= np = 5, \sigma^2 = np(1-p) = 4 \quad [1] \\ \frac{\sigma^2}{\mu} &= 1-p = \frac{4}{5} \Rightarrow p = \frac{1}{5} \quad [1] \\ n &= \frac{\mu}{p} = 25 \quad [1]\end{aligned}$$

Question 3

$$\begin{aligned}f(x) &= |x^2(x^2 - 1)| = \begin{cases} x^4 - x^2, & x < -1 \text{ or } x > 1 \\ x^2 - x^4, & -1 \leq x \leq 1 \end{cases} \\ f'(x) &= \begin{cases} 4x^3 - 2x, & x < -1 \text{ or } x > 1 \\ 2x - 4x^3, & -1 \leq x \leq 1 \end{cases} = 0 \text{ at stationary points} \quad [1]\end{aligned}$$

for $x < -1$ or $x > 1$, $(4x^2 - 2) = 0 \Rightarrow x = 0, \pm \frac{1}{\sqrt{2}}$ none of which lie within $x < -1$ or $x > 1$

for $-1 \leq x \leq 1$, $x(2 - 4x^2) = 0 \Rightarrow x = 0, \pm \frac{1}{\sqrt{2}} \quad [1]$

$\therefore$ we have stationary points at $(0,0), \left(\frac{1}{\sqrt{2}}, \frac{1}{4}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right) \quad [1]$

sign diagrams may be used to show that $(0,0)$ is a local minimum, others are local maxima [1]

Question 4a

$$\begin{aligned}\log_e 2 &= \log_2 x = \frac{\log_e x}{\log_e 2} \quad [1] \\ \Rightarrow \log_e x &= [\log_e 2]^2 \\ \Rightarrow x &= e^{[\log_e 2]^2} \quad [1]\end{aligned}$$

Question 4b

$$\begin{aligned}25^x - 5^{x+1} + 6 &= 0 \\ \Rightarrow (5^x)^2 - 5(5^x) + 6 &= 0 \quad [1] \\ \Rightarrow (5^x - 2)(5^x - 3) &= 0 \\ \Rightarrow 5^x &= 2 \text{ or } 3 \quad [1] \\ \therefore x &= \log_5 2 \text{ or } \log_5 3 \quad [1]\end{aligned}$$

Question 5

For infinite solutions, the lines described by the equations must be parallel [1]

$$\begin{aligned}\text{So: } \frac{3k}{3} &= \frac{k}{1} = \frac{6}{k-1} \quad [1] \\ \Rightarrow k(k-1) &= 6 \Rightarrow k^2 - k - 6 = 0 \Rightarrow (k-3)(k+2) = 0 \\ \therefore k &= -2 \text{ or } 3 \quad [2]\end{aligned}$$

Question 6a

$$2 \sin\left(x + \frac{\pi}{2}\right) + 1 = 0$$

$$\sin\left(x + \frac{\pi}{2}\right) = -\frac{1}{2} [1]$$

$$x + \frac{\pi}{2} = \frac{7\pi}{6} \text{ or } \frac{11\pi}{6}$$

$$x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3} [2]$$

Question 6b

$$x = \frac{2\pi}{3} - \frac{\pi}{3} \text{ or } \frac{4\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3} \text{ or } \pi [1]$$

Question 7

Curves intersect where $ax = x^2 \Rightarrow x(x - a) = 0 \Rightarrow x = 0, a$ [1]

$$\int_0^a ax - x^2 dx = \left[\frac{1}{2}ax^2 - \frac{1}{3}x^3 \right]_0^a = \frac{1}{2}a^3 - \frac{1}{3}a^3 = \frac{1}{6}a^3 = \frac{9}{2} [2]$$

$$\Rightarrow a^3 = 27$$

$$\Rightarrow a = 3 [1]$$

Question 8

$$\frac{dV}{dt} = 10 [1]$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2 = 100\pi \text{ cm}^3/\text{cm} \text{ when } r = 5 \text{ cm} [1]$$

$$\frac{dr}{dt} = \frac{dr}{dV} \frac{dV}{dt} [1]$$

$$= \frac{1}{100\pi} \frac{10}{1} = \frac{1}{10\pi} \text{ cm/s} [1]$$

Question 9

Shape of f and g correct (g is given by f reflected in both axes) [1]

Shape of $f + g$ similar to $y = x^3$ [1]

with intercept at (0,0) [1]

and no stationary points [1]

Question 10a

$$\text{area} \approx \frac{1}{2} \frac{1}{2^2} + \frac{1}{2} \frac{1}{2.5^2} [1]$$

$$= \frac{1}{2} \left(\frac{1}{4} + \frac{4}{25} \right) = \frac{1}{2} \left(\frac{25}{100} + \frac{16}{100} \right) = \frac{1}{2} \frac{41}{100} = 0.205 [1]$$

Question 10b

$$\int_2^3 \frac{1}{x^2} dx = \left[-\frac{1}{x} \right]_2^3 = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6} [1]$$

Question 10b

Larger: $\frac{1}{x^2}$ is a decreasing function of x over $[2,3]$ hence left rectangles will overestimate the area under the curve on this interval. [1]