



Victorian Certificate of Education 2011

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS (CAS)

Written examination 1

Tuesday 8 November 2011

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 13 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Differentiate $\sqrt{4-x}$ with respect to x .

1 mark

- b. If $g(x) = x^2 \sin(2x)$, find $g'\left(\frac{\pi}{6}\right)$.

2 marks

TURN OVER

a. Find an antiderivative of $\frac{1}{3x-4}$ with respect to x .

b. Solve the equation $4^x - 15 \times 2^x = 16$ for x .

[illegible]

3 marks

Question 3

- a.** State the range and period of the function

$$h: R \rightarrow R, h(x) = 4 + 3\cos\left(\frac{\pi x}{2}\right).$$

2 marks

- b.** Solve the equation

$$\sin\left(2x + \frac{\pi}{3}\right) = \frac{1}{2} \text{ for } x \in [0, \pi].$$

[illegible]

2 marks

Question 4

If the function f has the rule $f(x) = \sqrt{x^2 - 9}$ and the function g has the rule $g(x) = x + 5$

- a. find integers c and d such that $f(g(x)) = \sqrt{(x+c)(x+d)}$

2 marks

- b. state the maximal domain for which $f(g(x))$ is defined.

2 marks

Question 5

The probability distribution function for the continuous random variable X is given by

$$f(x) = \begin{cases} |3-x| & \text{if } 2 \leq x \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

- a.** Find $\Pr(X < 3.5)$.

2 marks

- b.** Find $\Pr(X < 2.5 \mid X < 3.5)$.

2 marks

Question 6

Consider the simultaneous linear equations

$$kx - 3y = k + 3$$

$$4x + (k + 7)y = 1$$

where k is a real constant.

- a.** Find the value of k for which there are infinitely many solutions.

3 marks

- b.** Find the values of k for which there is a unique solution.

1 mark

Question 7

A biased coin is tossed three times. The probability of a head from a toss of this coin is p .

- a.** Find, in terms of p , the probability of obtaining
- i.** three heads from the three tosses

- ii.** two heads and a tail from the three tosses.

1 + 1 = 2 marks

- b.** If the probability of obtaining three heads equals the probability of obtaining two heads and a tail, find p .

2 marks

Question 8

Two events, A and B , are such that $\Pr(A) = \frac{3}{5}$ and $\Pr(B) = \frac{1}{4}$.

If A' denotes the complement of A , calculate $\Pr(A' \cap B)$ when

a. $\Pr(A \cup B) = \frac{3}{4}$

2 marks

b. A and B are mutually exclusive.

1 mark

Question 9

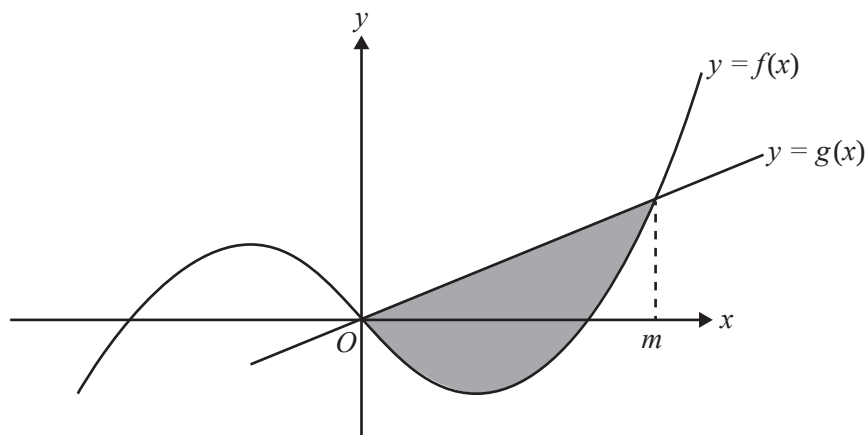
Parts of the graphs of the functions

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 - ax \quad a > 0$$

$$g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = ax \quad a > 0$$

are shown in the diagram below.

The graphs intersect when $x = 0$ and when $x = m$.



[illegible]

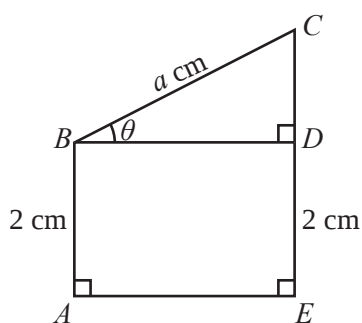
TURN OVER

Question 10

The figure shown represents a wire frame where $ABCE$ is a convex quadrilateral. The point D is on line segment EC with $AB = ED = 2$ cm and $BC = a$ cm, where a is a positive constant.

$$\angle BAE = \angle CEA = \frac{\pi}{2}$$

Let $\angle CBD = \theta$ where $0 < \theta < \frac{\pi}{2}$.



- a. Find BD and CD in terms of a and θ .

2 marks

- b. Find the length, L cm, of the wire in the frame, including length BD , in terms of a and θ .

1 mark

- c. Find $\frac{dL}{d\theta}$, and **hence** show that $\frac{dL}{d\theta} = 0$ when $BD = 2CD$.

2 marks

- d. Find the maximum value of L if $a = 3\sqrt{5}$.

1 mark

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods (CAS)

Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc \sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	transition matrices: $S_n = T^n \times S_0$
mean: $\mu = E(X)$	variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$