



2011 Trial Examination

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40
		Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 8 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Differentiate $f(x) = \log_e(2 - x)^4$ with respect to x .

2 marks

- b. Evaluate $f^{-1}(0)$ where f^{-1} is the inverse function of f .

3 marks

- c. For $g(x) = x^2 e^{\sin(x)}$, find $g'\left(\frac{\pi}{2}\right)$

2 marks

Question 2

- a. Find an anti-derivative of $\sqrt{x} - 2 \sin\left(\frac{\pi x}{4}\right)$ with respect to x .

2 marks

- b. Find m given that $\int_0^2 (2x - 1)^3 dx = e^{\log_e m}$

2 marks

TURN OVER

Question 3

- a. Write down the amplitude and the period of the function $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = -3 \cos\left(\frac{\pi-x}{5}\right)$.

2 marks

- b. Solve the equation $-\cos(2x) = \sin(2x)$ for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

3 marks

- c. Sketch the graph of $y = \tan(2x)$ for $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, showing the exact values of the axial intercepts and asymptotes.

2 marks

Question 4

The transformations $T: R^2 \rightarrow R^2$ is defined by $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}$. The image of the curve $y = 2 \log_e(4x) + 1$ under the transformation T has equation $y = a \log_e(bx + c) + d$. Find the values of a , b , c , and d .

4 marks

Question 5

Soft drink is leaking from a bottle at a constant rate to form a circular puddle on the bench. The soft drink is added to the puddle at a rate of 12 mm^3 per minute causing the puddle to spread out evenly, which has a height that is a sixth of its radius.

Find the exact rate at which the radius of the puddle is increasing when the radius is 20 mm.

3 marks

TURN OVER

Question 6

A square piece of paper is cut from a larger piece of paper. Use the relationship

$f(x + h) \approx f(x) + hf'(x)$ for a small positive value of h , to find an approximate value for the side length of the square if the area of the square piece of paper is 9.01 cm.

4 marks

Question 7

The continuous random variable X has a distribution with probability density function given by

$$f(x) = \begin{cases} m \cos(x) & x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right] \\ 0 & \text{elsewhere} \end{cases}$$

where m is a constant, find

a. the value of m .

3 marks

b. $Pr(X > \pi)$

2 marks

Question 8

Let X be a normally distributed random variable with mean 11 and variance 4 and let Z be the random variable with the standard normal distribution. Find

a. $Pr(X < 7)$

1 mark

b. m such that $Pr(X < 8) = Pr(Z > m)$

2 marks

TURN OVER

Question 9

- a. Show that $\frac{3-2x}{x+2} = \frac{7}{x+2} - 2$

2 marks

- b. Hence, find $\int \frac{3-2x}{x+2} dx$.

1 mark

END OF QUESTION AND ANSWER BOOK