

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1



2011 Trial Examination

SOLUTIONS

Question 1

a. $f'(x) = \frac{-4(2-x)^3}{(2-x)^4} = \frac{4}{x-2}$

M1+A1
2 marks

b. $x = \log_e(2-y)^4$
 $x = 4\log_e(2-y)$
 $\frac{x}{4} = \log_e(2-y)$
 $e^{\frac{x}{4}} = 2-y$
 $y = 2 - e^{\frac{x}{4}}$
 $f^{-1}(0) = 1$

M2+A1
3 marks

c. $g'(x) = 2xe^{\sin(x)} + x^2 \cos(x)e^{\sin(x)}$
 $g'(x) = 2 \times \frac{\pi}{2} \times e^{\sin(\frac{\pi}{2})} + \left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) e^{\sin(\frac{\pi}{2})} = \pi e$

M1+A1
2 marks

Question 2

a. $\int \sqrt{x} - 2 \sin\left(\frac{\pi x}{4}\right) dx = \frac{2x^{\frac{3}{2}}}{3} + \frac{8}{\pi} \cos\left(\frac{\pi x}{4}\right) + c$ where c is a real constant (but not necessary)

M1+A1
2 marks

b. $\left[\frac{(2x-1)^4}{2 \times 4} \right]_0^2 = e^{\log_e m}$
 $\frac{81}{8} - \frac{1}{8} = m$
 $m = 10$

M1+A1
2 marks

Question 3

a. Amplitude = 3

Period = $\frac{2\pi}{n} = 2\pi \times 5 = 10\pi$

A2
2 marks

b. **Error! Bookmark not defined.** $\frac{-\cos(2x)}{\cos(2x)} = \frac{\sin(2x)}{\cos(2x)}$

$\tan(2x) = -1, -\pi \leq x \leq \pi$

Reference $\angle: \frac{\pi}{4}$

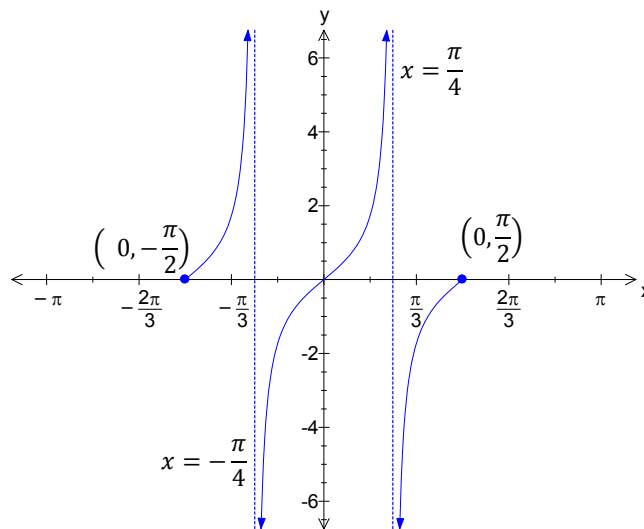
$2x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ or $2x = \frac{3\pi}{4} - \pi = -\frac{\pi}{4}$

$x = \frac{3\pi}{8}$

$x = -\frac{\pi}{8}$

M2+A1
3 marks

c.



A2
2 marks

Question 4

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4x + 3 \\ 2y - 1 \end{bmatrix}$$

$$x' = -4x + 3 \text{ and } y' = 2y - 1$$

$$x = \frac{3-x'}{4} \text{ and } y = \frac{y'+1}{2}$$

$$\frac{y'+1}{2} = 2 \log_e(3 - x') + 1$$

$$y' = 4 \log_e(3 - x') + 1$$

$$a = 4, b = -1, c = 3 \text{ and } d = 1$$

M2+A2

4 marks

Question 5

$$\frac{dV}{dt} = 12 \text{ mm}^3/\text{minute}, h = \frac{r}{6} \text{ and } \frac{dV}{dr} = \frac{\pi r^2}{2}$$

$$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$12 = \frac{\pi r^2}{2} \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{12 \times 2}{400\pi} = \frac{3}{50\pi} \text{ mm/min}$$

M2+A1

3 marks

Question 6

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}, \quad f'(x) = \frac{1}{2}x^{-\frac{1}{2}}, \quad x = 9, \quad h = 0.01$$

$$f(x+h) \approx f(x) + hf'(x)$$

$$\sqrt{9.01} \approx \sqrt{9} + 0.01 \times \frac{1}{2}(9)^{-\frac{1}{2}}$$

$$\sqrt{9.01} \approx 3 + 0.01 \times \frac{1}{6}$$

$$\sqrt{9.01} \approx 3 + \frac{1}{600} \approx 3 \frac{1}{600}$$

M2+A2

4 marks

Question 7

$$\text{a. } \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} m \cos(x) dx = 1$$

$$m[\sin(x)]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = 1$$

$$m(-1 - 1) = 1$$

$$m = \frac{-1}{2}$$

M2+A1

3 marks

b. $Pr(X > \pi) = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -0.5 \cos(x) dx$
 $-0.5[\sin(x)]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = -0.5(-1) = 0.5$

M1+A1
2 marks

Question 8

a. $Pr(X < 7) = 0.025$
 $z = \frac{x-\mu}{\sigma} = \frac{7-11}{2} = -2 \Rightarrow 2 \text{ standard deviations below the mean}$
 $Pr(Z < -2) = Pr(X < 7) = 0.025$

A1
1 mark

b. $Pr(X < 8) = Pr(X > 14) = Pr\left(Z > \frac{14-11}{2}\right) = Pr\left(Z > \frac{3}{2}\right), m = \frac{3}{2}$

M1+A1
2 marks

Question 9

a.
$$\begin{array}{r} -2 \\ x+2 \overline{) -2x+3} \\ \underline{-2x-4} \\ 7 \end{array}$$

 $\frac{3-2x}{x+2} = \frac{7}{x+2} - 2$

M1+A1
2 marks

b. $\int \frac{3-2x}{x+2} dx = \int \frac{7}{x+2} - 2 dx = 7 \log_e |x+2| - 2x + c$

A1
1 mark