

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 2

Suggested Solutions

SECTION 1

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E

12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

SECTION 1**Question 1 A**

Using CAS, define $f(x) = 4 - 2x - x^2$ and $g(x) = |x - 7|$.

$f(g(5))$ is found directly on CAS or alternatively:

$$f(g(x)) = f(|x - 7|) = 4 - 2|x - 7| - (|x - 7|)^2$$

There is no need to simplify this. Let $x = 5$ to directly give:

$$\begin{aligned} f(g(5)) &= 4 - 2|5 - 7| - (|5 - 7|)^2 \\ &= 4 - 4 - 4 \\ &= -4 \end{aligned}$$

Question 2 A

$$\lim_{h \rightarrow 0} \frac{\log_e(e + h) - 1}{h} = \lim_{h \rightarrow 0} \frac{\log_e(e + h) - \log_e(e)}{h}$$

This is the first principle's definition of $f'(e)$, where $f(x) = \log_e(x)$.

Question 3 B

$$\begin{aligned} g(x) &= f(1 - 3x) + 2 \\ &= f\left(-3\left(x - \frac{1}{3}\right)\right) + 2 \end{aligned}$$

This means f undergoes a reflection in the y -axis: $f(x) \rightarrow f(-x)$; $(3, -11) \rightarrow (-3, -11)$,

followed by a dilation away from the y -axis of scale factor $\frac{1}{3}$: $f(-x) \rightarrow f(-3x)$; $(-3, -11) \rightarrow (-1, -11)$,

followed by a translation of $\frac{1}{3}$ to the right:

$$f(-3x) \rightarrow f\left(-3\left(x - \frac{1}{3}\right)\right), \text{ i.e. } f(1 - 3x); (-1, -11) \rightarrow \left(-\frac{2}{3}, -11\right).$$

Finally, it is translated 2 units vertically up: $f(1 - 3x) \rightarrow f(1 - 3x) + 2$; $\left(-\frac{2}{3}, -11\right) \rightarrow \left(-\frac{2}{3}, -9\right)$.

Question 4 D

The graph of $f(x) = 1 - 5\cos(2x - \pi)$ clearly shows an amplitude of 5, a range of $[-4, 6]$ and the period is π , so **A**, **B** and **C** are correct.

Consider alternative **D**. If $g(x) = 5\cos(2x - \pi)$, then a vertical translation of 1 unit up means

$g(x) = 5\cos(2x - \pi) + 1$. If we now reflect in the x -axis, we get

$$f(x) = -(5\cos(2x - \pi) + 1) = -5\cos(2x - \pi) - 1 \text{ which is incorrect.}$$

Alternative **E** shows $h(x) = -5\cos(2x)$ shifted $\frac{\pi}{2}$ units right giving

$$h(x) = -5\cos\left(2\left(x - \frac{\pi}{2}\right)\right) = -5\cos(2x - \pi). \text{ Translating this 1 unit up results in } f(x) = 1 - 5\cos(2x - \pi).$$

Question 5 B

From the matrix equation $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2x + 1 \\ 4y - 2 \end{bmatrix}$

i.e. $x' = 2x + 1$ and $y' = 4y - 2$.

Therefore $x = \frac{x' - 1}{2}$ and $y = \frac{y' + 2}{4}$.

So $y = x^3$ becomes $\frac{y' + 2}{4} = \left(\frac{x' - 1}{2}\right)^3$

$y = 4\left(\frac{x - 1}{2}\right)^3 - 2 \Rightarrow y = \frac{1}{2}(x - 1)^3 - 2$

Question 6 B

During the first second, the position of the body changes (in the negative direction) by an amount equal to the area of the triangle, i.e. $\frac{1}{2} \times 1 \times 4 = 2$.

Notice this means the body moves to the left by 2 m during this time.

From $1 < t < 2$, the body moves in the opposite direction (right) by $\frac{1}{2} \times 1 \times 3 = 1.5$. So the net result in the first 2 seconds is a move of 0.5 units to the left.

So $x(2) < x(0)$. Only alternatives **A** and **B** satisfy this requirement.

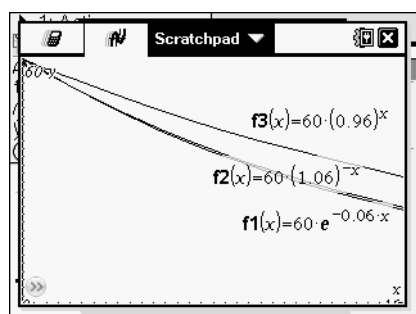
From $2 < t < 4$, the body moves further right by $\frac{1}{2}(2 + 1) \times 3 = 4.5$.

After this, the body moves back to the left once again. Therefore $x(6) < x(4)$. Combining these results gives $x(2) < x(0) < x(6) < x(4)$.

Question 7 D

There are many varied approaches which could be used, ranging from statistical regression to trial and error.

One quick approach is to graph the 3 exponential functions in an appropriate window using 0 to 16 for $x(n)$ and 0 to 60 for $y(N)$.



$N = 60(0.96)^n$ follows the data more closely over the domain than the others. Clearly the data is not negative so alternative **E** is rejected. Alternative **C** is quadratic. A quick look at its graph also shows it matches the data only for very small values of n .

Question 8 E

Given $p(x) = 5x^{2k+1} - 10x^{2k} + 3x^{2k-1} + 5$, the remainder when divided by $x + 1$ is $p(-1)$.

$$\text{Now } p(-1) = 5(-1)^{2k+1} - 10(-1)^{2k} + 3(-1)^{2k-1} + 5$$

As k is a positive integer, $2k \pm 1$ is odd and $2k$ is even.

$$\begin{aligned}\text{Thus } p(-1) &= 5(-1) - 10(1) + 3(-1) + 5 \\ &= -5 - 10 - 3 + 5 = -13\end{aligned}$$

Question 9 A

An inverse exists if the function is one-to-one.

It is tempting to think of cubics as many-to-one functions but both f and g are one-to-one.

A sketch of their graphs on CAS quickly shows this is the case. Alternatively, the derivatives of f and g are shown algebraically:

$$f'(x) = x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

$$g'(x) = 3x^2 + 2x + 1 = (x + 1)^2 + 2x^2$$

Clearly each of these derivatives is always positive. Thus f and g are increasing functions and hence one-to-one.

The graph of $h(x) = x^{-\frac{2}{3}}$ is symmetrical about the y -axis and hence the function is not one-to-one.

Question 10 C

A dilation from the x -axis by a factor of $\frac{1}{2}$ means $\cos(3x) \rightarrow \frac{1}{2}\cos(3x)$ and $\sin(3x) \rightarrow \frac{1}{2}\sin(3x)$.

A dilation from the y -axis by a factor of 3 means $x \rightarrow \frac{1}{3}x$ so we have new equations $y_1 = \frac{1}{2}\cos(x)$ and $y_2 = \frac{1}{2}\sin(x)$.

These graphs meet when $\frac{1}{2}\sin(x) = \frac{1}{2}\cos(x) \Rightarrow \tan(x) = 1$.

$x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ are the first two positive solutions with $y = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ and $\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

Thus the graphs intersect at $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{4}\right)$ and $\left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{4}\right)$ after the dilations have occurred.

Question 11 D

$$\int_p^3 \frac{1}{3}x^2 dx = 1$$

$$\left[\frac{1}{9}x^3\right]_p^3 = 1 \quad \text{Or using CAS: solve}\left(\int_p^3 \frac{1}{3}x^2 dx = 1, p\right)$$

$$3^3 - p^3 = 9$$

$$p^3 = 18$$

$$p = \sqrt[3]{18}$$

Question 12 C

X : number of goals scored in 30 attempts

$$X \sim \text{Bi}(n = 30, p = 0.7)$$

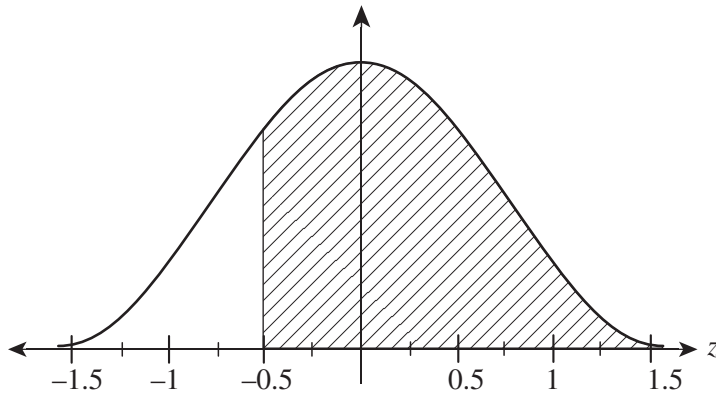
$$\mu = 30 \times 0.7 = 21$$

$$\sigma = \sqrt{30 \times 0.7 \times 0.3} = 2.51$$

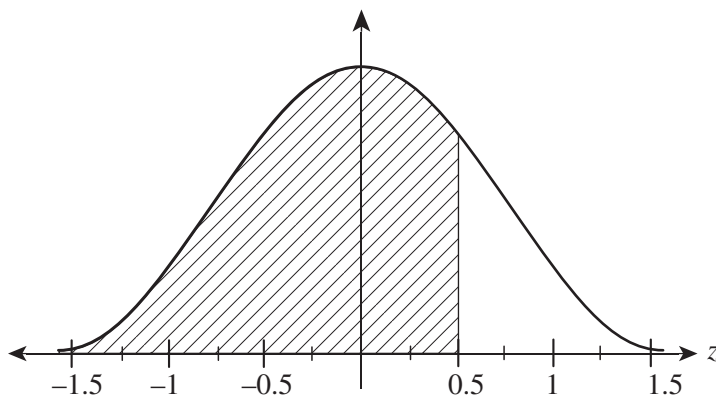
$$\begin{aligned} \Pr(\mu - \sigma < X < \mu + \sigma) &= \Pr(21 - 2.57 < X < 21 + 2.51) \\ &= \Pr(18.49 < X < 23.51) \\ &= \Pr(19 \leq X \leq 23) \\ &= 0.6812 \end{aligned}$$

Question 13 B

$$\begin{aligned} \Pr(47.5 < X < 57.5) &= \Pr\left(\frac{47.5 - 50}{5} < Z < \frac{57.5 - 50}{5}\right) \\ &= \Pr(-0.5 < Z < 1.5) \end{aligned}$$



is equivalent to:



Question 14 A

$$\begin{aligned}
\Pr(\text{at least 1 boy and 1 girl}) &= 1 - \Pr(\text{BBB}) - \Pr(\text{GGG}) \\
&= 1 - \left(\frac{4}{12} \cdot \frac{3}{11} \cdot \frac{2}{10}\right) - \left(\frac{8}{12} \cdot \frac{7}{11} \cdot \frac{6}{10}\right) \\
&= 1 - \frac{1}{55} - \frac{14}{55} \\
&= 1 - \frac{3}{11} \\
&= \frac{8}{11}
\end{aligned}$$

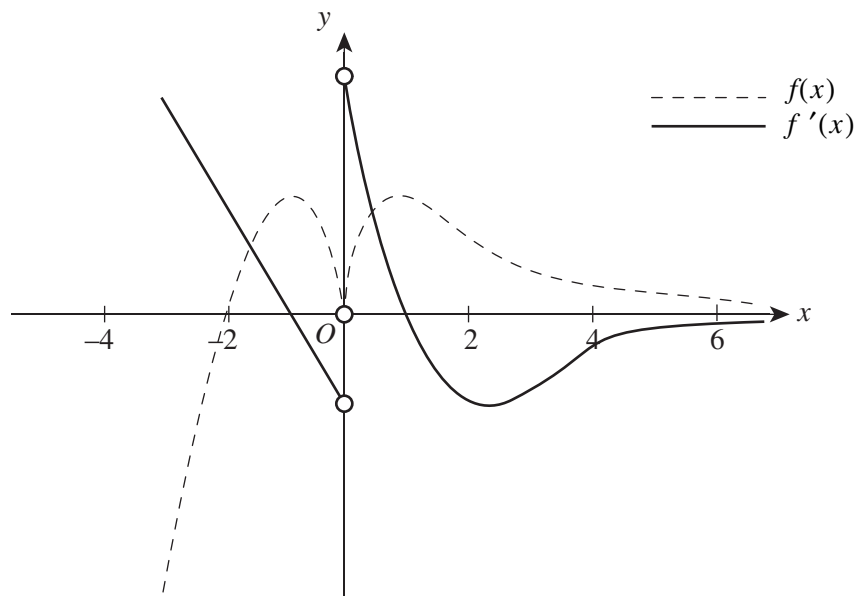
Question 15 D

$$\begin{aligned}
\Sigma \Pr(X = x) &= 1 \\
p + p + p + 2p + 3p + 2p &= 1 \\
10p &= 1 \\
p &= 0.1
\end{aligned}$$

x	1	2	4	6	8	10
$\Pr(X \leq x)$	0.1	0.2	0.3	0.5	0.8	1

As $\Pr(X \leq 6) = 0.5$, the median is halfway between 6 and the next value of X ; here 8.

$$\therefore \text{median} = \frac{6 + 8}{2} = 7$$

Question 16 A

For:

$x < -1$, $f'(x) > 0$ i.e. graph of $f'(x)$ is above the x -axis.

$x = -1$, $f(x)$ has a turning point, $\therefore f'(x) = 0$.

$-1 < x < 0$, $f'(x) < 0$ i.e. graph of $f'(x)$ is below the x -axis.

$x = 0$, $f'(x)$ does not exist.

$0 < x < 1$, $f'(x) > 0$ i.e. graph of $f'(x)$ is above the x -axis.

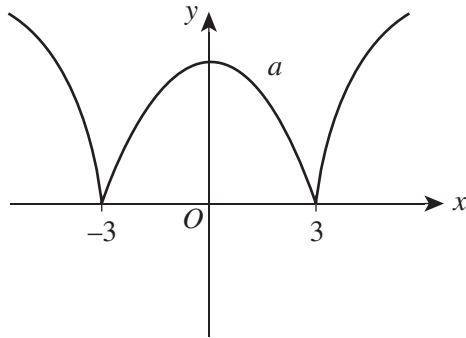
$x = 1$, $f(x)$ has a turning point $\therefore f'(x) = 0$.

$x > 1$, $f'(x) < 0$ i.e. graph of $f'(x)$ is below x -axis.

As $x \rightarrow \infty$, $f'(x) \rightarrow 0$.

Question 17 E

$f(x) = |9 - x^2|$ has graph:



Clearly at $x = \pm 3$ there are cusps, $\therefore f'(-3)$ and $f'(3)$ do not exist.

Alternatively using CAS:

$$f'(x) = 2x \operatorname{sign}(x^2 - 9)$$

$$\lim_{x \rightarrow 3^-} f'(x) = -6$$

$$\lim_{x \rightarrow 3^+} f'(x) = 6 \quad \therefore \text{the limit does not exist and } f'(3) \text{ does not exist.}$$

Note: $\operatorname{sign}(0)$ is undefined.

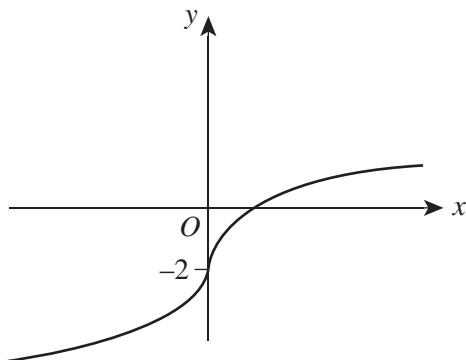
Question 18 C

Using a CAS calculator:

$\text{normalLine}\left(x^{\frac{3}{5}} - 2, x, 0\right)$ or $\text{normal}\left(x^{\frac{3}{5}} - 2, x, 0\right)$ gives -2 , which we interpret to mean $y = -2$.

OR

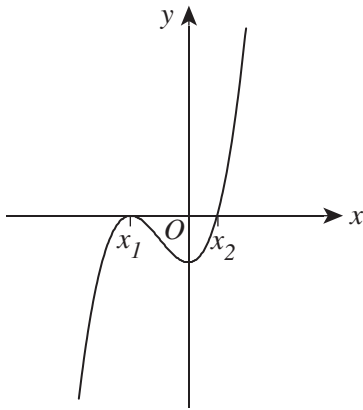
A graph of $y = f(x)$ is:



$\frac{d}{dx} f(x)$ at $x = 0$ is undefined.

$\therefore$ gradient of normal at $x = 0$ is 0 .

$\therefore$ equation of normal at $(0, -2)$ is $y = -2$.

Question 19 **E**

Let x_1 be the first x -intercept, x_2 the second x -intercept, so therefore $f'(x_1) = 0$ and $f'(x_2) = 0$.

$\therefore$ On the graph of $y = f(x)$ there will stationary points at x_1 and x_2 .

To the left of x_1 , the gradient is negative. To the right of x_1 , the gradient is negative.

$\therefore$ At x_1 , there is a point of inflexion.

To the left of x_2 , the gradient is negative. To the right of x_2 , the gradient is positive.

$\therefore$ At x_2 , there is a local minimum.

E is the only graph which has these two features.

Question 20 **C**

$$\text{Given } \int_0^a f(x) dx = -2$$

$$\therefore \int_0^{\frac{a}{2}} f(2x) dx = -1 \text{ as } f(2x) \text{ is a dilation of } \frac{1}{2} \text{ parallel to the } x\text{-axis.}$$

$$\begin{aligned} \text{Also, } \int_0^{\frac{a}{2}} 2f(2x) dx &= 2 \int_0^{\frac{a}{2}} f(2x) dx \\ &= 2 \times -1 \\ &= -2 \end{aligned}$$

Question 21 D

As A and B are independent, so therefore are A' and B' .

$$\therefore \Pr(A'|B') = \Pr(A') = 1 - p$$

OR

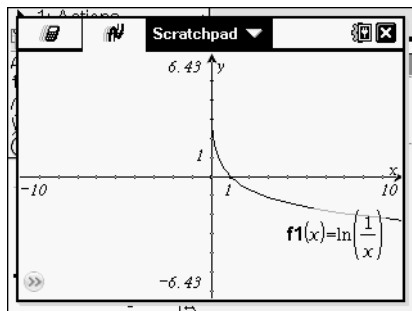
$\Pr(A \cap B) = pq$ this gives the following probability table:

$\cap$	A	A'	
B	pq	$q - pq$	q
B'	$p - pq$	$1 + pq - p - q$	$1 - q$
	p	$1 - p$	1

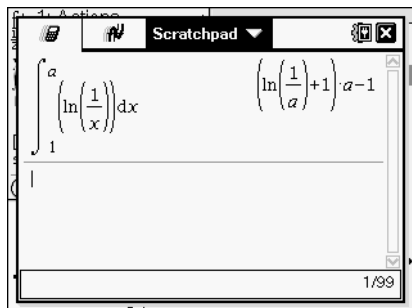
$$\begin{aligned} \Pr(A|B') &= \frac{\Pr(A' \cap B')}{\Pr(B')} = \frac{1 + pq - p - q}{1 - q} \\ &= \frac{(1 - p)(1 - q)}{(1 - q)} \\ &= 1 - p \end{aligned}$$

Question 22 E

A graph of $y = \log_e\left(\frac{1}{x}\right)$ shows that as $a > 1$, the area required is below the x -axis.



Using a CAS calculator:



$$\begin{aligned} \text{Therefore area required is } & -\left(a\left(\log_e\left(\frac{1}{a}\right) + 1\right) - 1\right) = 1 - a\left(\log_e\left(\frac{1}{a}\right) + 1\right) \\ & = 1 - a + a\log_e(a) \end{aligned}$$

SECTION 2

Question 1

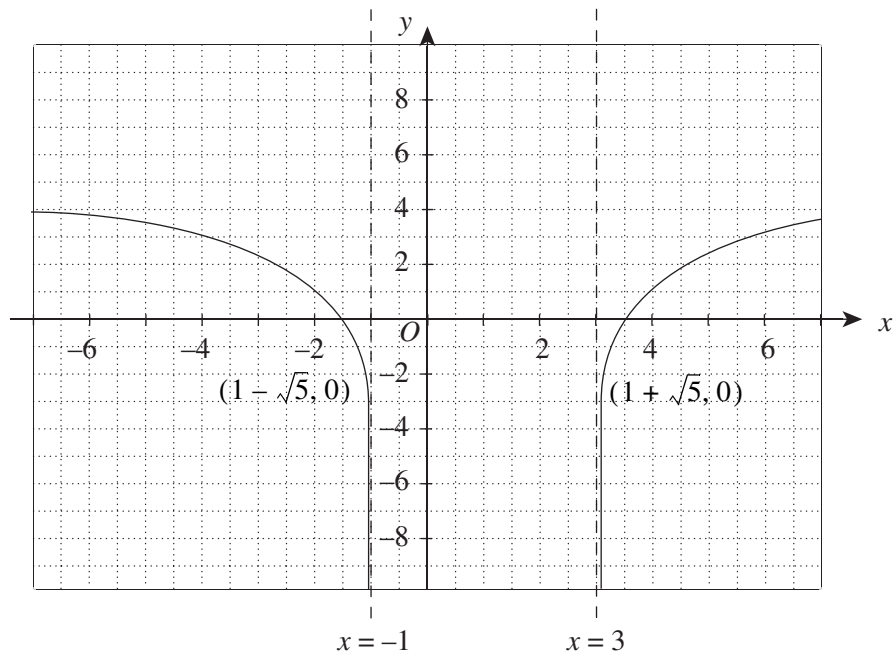
- a. For f to be defined, $((x-1)^2 - m) > 0 \Rightarrow (x-1)^2 > m$. M1

$$\text{Thus } x-1 < -\sqrt{m} \text{ or } x-1 > \sqrt{m}$$

$$\Rightarrow x < 1 - \sqrt{m} \text{ or } x > 1 + \sqrt{m}$$

Hence the domain of f is $(-\infty, 1 - \sqrt{m}) \cup (1 + \sqrt{m}, \infty)$. A1

b.



x -intercepts at $(1 + \sqrt{5}, 0)$, $(1 - \sqrt{5}, 0)$ A1

Asymptotes with equations $x = -1$ and $x = 3$ A1

Graph shape and location A1

- c. Given that $x < -2$, $1 - \sqrt{m} = -2 \Rightarrow m = 9$. M1 A1

- d. Given that $g : (-\infty, -2) \rightarrow \mathbb{R}$, $g(x) = \log_e((x-1)^2 - m)$, the inverse is given by:

$$x = \log_e((y-1)^2 - m) \quad \text{M1}$$

$$e^x = ((y-1)^2 - m)$$

$$y-1 = \pm \sqrt{e^x + m}$$

$$y = 1 \pm \sqrt{e^x + m}$$

As $\text{dom}(g) = \text{ran}(g^{-1}) = (-\infty, -2)$, $g^{-1}(x) = 1 - \sqrt{e^x + m}$. A1

Domain = $(\log_e(9 - m), \infty)$, Range = $(-\infty, -2)$. A1

e. i. $f(x) = \log_e((x-1)^2 - m) \Rightarrow f'(x) = \frac{2(x-1)}{((x-1)^2 - m)}$ A1

A turning point occurs if $f'(x) = 0 \Rightarrow x = 1$.

In addition, $(x-1)^2 - m > 0$ for the function to be continuous and have a turning point.

Hence $m < 0$ and $x = 1$. A1

ii. Starting with $f(x) = \log_e((x-1)^2 + 1)$, we get:

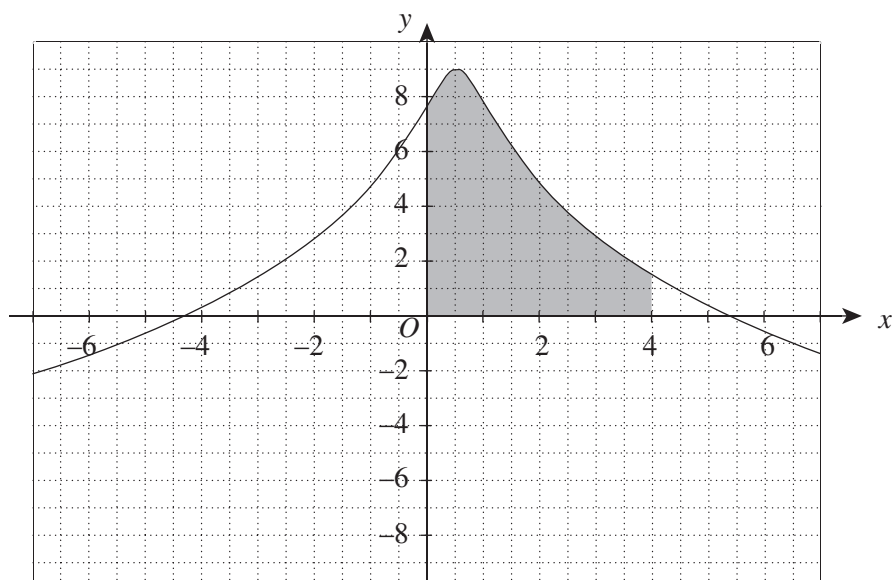
$$y = -\log_e((x-1)^2 + 1) \text{ (reflection in the } x\text{-axis)}$$

which becomes: $y = -\log_e((2x-1)^2 + 1)$ (dilation from the y-axis, factor $\frac{1}{2}$) M1

which becomes: $y = -2\log_e((2x-1)^2 + 1)$ (dilation from the x-axis, factor 2) A1

iii. If we translate $y = -2\log_e((2x-1)^2 + 1)$ A units up, we get:

$$y = -2\log_e((2x-1)^2 + 1) + A$$



Solve on CAS: $\int_0^4 (-2\log_e((2x-1)^2 + 1) + A) dx = 23.5$ (or equivalent) M1

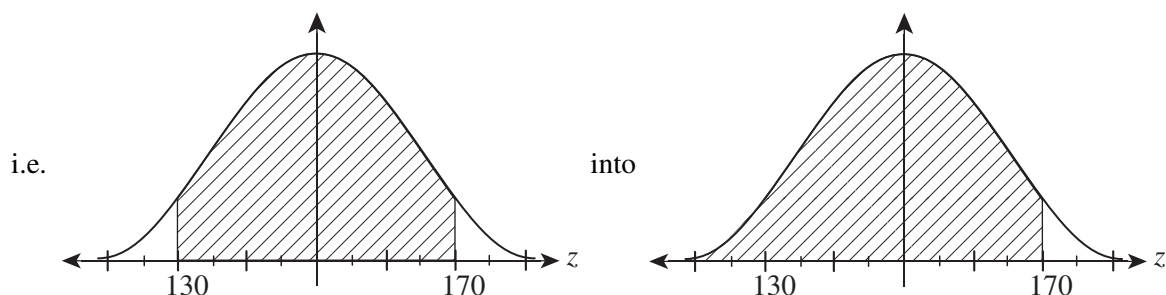
Gives $A = 10.0015$, so $A = 10$, correct to the nearest integer A1

Question 2

a. $\Pr(130 < X < 170) = 0.5763 = 57.63\%$

A1

b. $\Pr(130 < X < 170) \geq 0.8$



$$\therefore \Pr(X < 170) \geq 0.9$$

A1

$$\Pr(Z < z) \geq 0.9$$

$$Z_1 = \text{invnorm}(0.9)$$

$$\frac{170 - 150}{\sigma} \geq 1.28$$

$$\therefore \sigma \leq \frac{20}{1.28} = 15.625$$

M1

To achieve this, $\sigma = 15$, not 16, to the nearest gram.

A1

c. $(0.6)^5 = 0.07776$

A1

d. $\binom{5}{3}(0.6)^3(0.4)^2 = 0.3456$

A1

e. $\Pr(AB \text{ or } BA) = 0.6 \times 0.2 + 0.4 \times 0.6$
 $= 0.36$

M1

A1

f. $p = 0.8$

$$T = \begin{bmatrix} A \setminus A & A \setminus B \\ B \setminus A & B \setminus B \end{bmatrix} = \begin{bmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{bmatrix}$$

$$\therefore \Pr(X_4) = T^3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.744 \\ 0.256 \end{bmatrix}$$

M1

$$\therefore 0.256$$

A1

g. For a steady state, choose a large value of n , e.g. $n = 50$.

$$T^{50} = \begin{bmatrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{bmatrix}$$

$$\therefore 0.75 \text{ or } 75\%$$

A1

$$\text{h.} \quad \therefore \begin{bmatrix} p & (1.4-p) \\ (1-p) & (p-0.4) \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \text{ or nspire, solve } \left(\begin{bmatrix} p & (1.4-p) \\ (1-p) & (p-0.4) \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}, p \right) \quad \text{M1}$$

$$\begin{bmatrix} 4p^3 - 7.6p^2 + 5.16p - 0.56 \\ -4(p-1)(p^2 - 0.9p + 0.39) \end{bmatrix} = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \quad \text{A1}$$

Using CAS: solving $4p^3 - 7.6p^2 + 5.16p - 0.56 = 0.8$

$$p = 0.8591 \quad \text{A1}$$

Question 3

a. The maximum height is given by the maximum value of:

$$f(x) = b \left[\frac{1}{2} + \frac{1}{2} \sin \left(\frac{\pi(4x-d)}{2d} \right) \right]$$

$$f(x)_{\max} = b \left[\frac{1}{2} + \frac{1}{2} \right] = b \quad \text{A1}$$

b. i. Using **part a.** and the graph, $b = 0.1$. The width of the hump is 0.5, so $d = 0.5$. A2

ii. Using $b = 0.1$ and $d = 0.5$, we have

$$f(x) = 0.1 \left[\frac{1}{2} + \frac{1}{2} \sin \left(\frac{\pi(4x-0.5)}{2 \times 0.5} \right) \right] = \frac{1}{20} \left[1 + \sin \left(4\pi x - \frac{\pi}{2} \right) \right] \quad \text{M1}$$

$$\sin \left(\theta - \frac{\pi}{2} \right) = \sin \left(- \left(\frac{\pi}{2} - \theta \right) \right) = -\sin \left(\frac{\pi}{2} - \theta \right) = -\cos(\theta)$$

$$\text{As } f(x) = \frac{1}{20} \left[1 + \sin \left(4\pi x - \frac{\pi}{2} \right) \right] = \frac{1}{20} (1 - \cos(4\pi x)) \quad \text{A1}$$

iii. As $y = \frac{1}{20} (1 - \cos(4\pi x))$ describes this hump, CAS gives $\frac{dy}{dx} = \frac{\pi}{5} \sin(4\pi x)$ M1

This means the magnitude of the maximum gradient is $\frac{\pi}{5}$. A1

Thus the maximum angle of the hump is $\tan^{-1} \left(\frac{\pi}{5} \right) = 32.14^\circ$, which means the speed hump is

not of the described design. A1

iv. $f(x) = b \left[\frac{1}{2} + \frac{1}{2} \sin \left(\frac{\pi(4x-d)}{2d} \right) \right]$ can be re-expressed on CAS as $f(x) = \frac{1}{2} b [1 - \cos(4\pi x)]$ as the value of d remains the same, i.e. $d = 0.5$.

$$\text{This gives } f'(x) = 2\pi b \sin(4\pi x). \quad \text{M1}$$

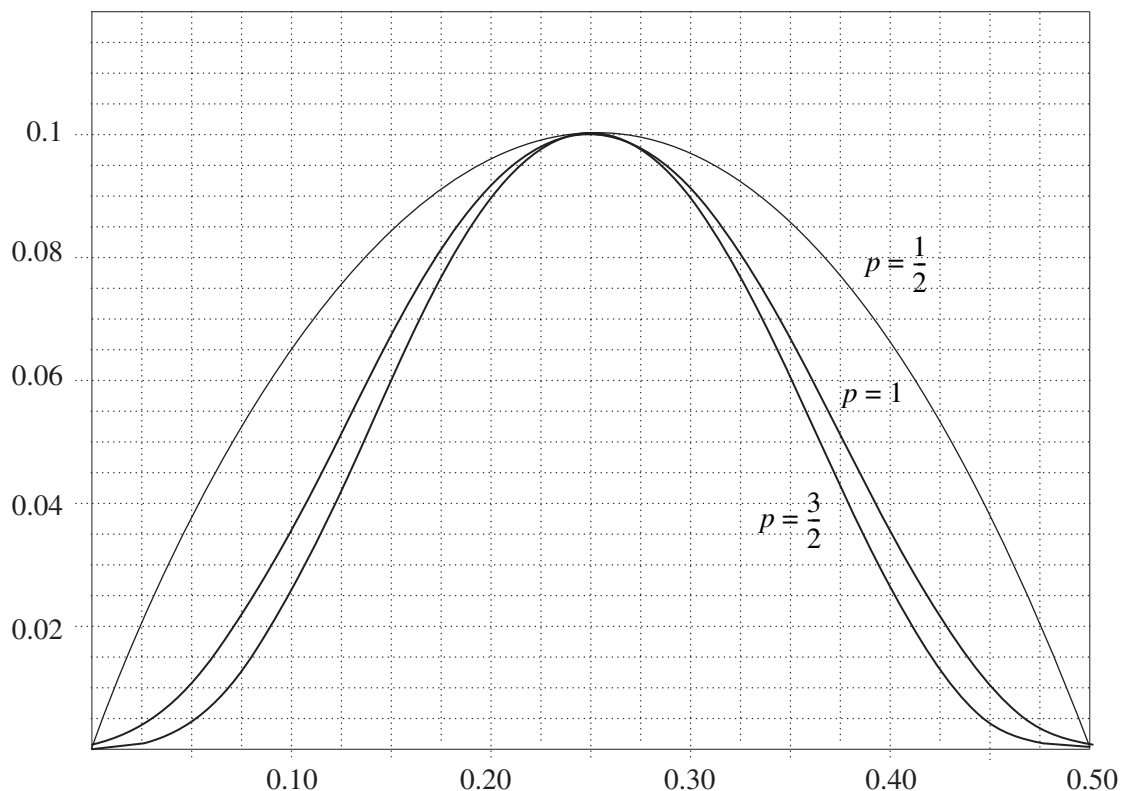
Thus the maximum value for the gradient is $2\pi b$.

$$\text{So } 2\pi b = \tan(15^\circ) = 2 - \sqrt{3} \Rightarrow b = \frac{2 - \sqrt{3}}{2\pi} \quad \text{M1}$$

$$\text{Our original value for } b \text{ was } \frac{1}{10}, \text{ hence } \frac{1}{10} k = \frac{2 - \sqrt{3}}{2\pi} \Rightarrow k = \frac{5(2 - \sqrt{3})}{\pi}.$$

So a dilation of factor $\frac{5(2 - \sqrt{3})}{\pi}$ from the x -axis is required. A1

c. i.



One mark per graph A2

ii. Too steep at the start and end of the hump.

A1

d. i. Now the area between each curve is given by:

$$A = \int_0^{0.5} 0.1[0.5 + 0.5 \sin(\pi(4x - 0.5))] - 0.1[0.5 + 0.5 \sin(\pi(4x - 0.5))]^{1.5} dx \quad \text{A1}$$

The volume difference will be given by:

$$V = 3 \int_0^{0.5} 0.1[0.5 + 0.5 \sin(\pi(4x - 0.5))] - 0.1[0.5 + 0.5 \sin(\pi(4x - 0.5))]^{1.5} dx \quad \text{A1}$$

ii. Calculating by CAS, we get 0.011338 cubic metres. As there are 5 humps, we have 0.057 cubic metres.

A1

Question 4a. $f'(x) = -(x-3)^2(x-2)(5x-12) = 0$ for stationary points.

M1

$$\therefore x = 3 \text{ or } 2 \text{ or } \frac{12}{5}$$

At $x = 2$, a local minimum

A1

 $x = \frac{12}{5}$, a local maximum

A1

 $x = 3$, a stationary point of inflexion

A1

- b.** If $a = b$. A1
- i.e. $g(x) = (a - x)^5$, its nature will be a point of inflexion. A1
- c.** $g(x) = (a - x)^2(b - x)^3$
- $g'(x) = -(x - b)^2(x - a)(5x - 3a - 2b) = 0$ for stationary point M1
- $x = a$ or $x = b$ or $x = \frac{3a + 2b}{5}$ A1
- d.** Using CAS:
- Define $h(x)$.
- Determine $h'(x) = (a - x)^{m-1}(b - x)^{n-1}((m + n)x - (mb + na))$ M1 A1
- Solve $h'(x) = 0$
- Answer $x = a$ or b or $\frac{an + bm}{m + n}$
- e.** For any point to be equidistant from $x = a$ and $x = b$, $\therefore x = \frac{a + b}{2}$.
- $\therefore$ from **Question 4 part d.**, $\frac{an + bm}{m + n} = \frac{a + b}{2}$.
- Solving for either m or n gives $m = n$. A1