

The Mathematical Association of Victoria
Trial Examination 2011
Maths Methods CAS Examination 1 - SOLUTIONS

Question 1

a. $\frac{d}{dx}(x \tan(x))$

Using the product rule

$= \tan(x) + x \sec^2(x)$

1M**1A**

b. i. $\frac{d}{dx}(e^{2x} + 2x)$

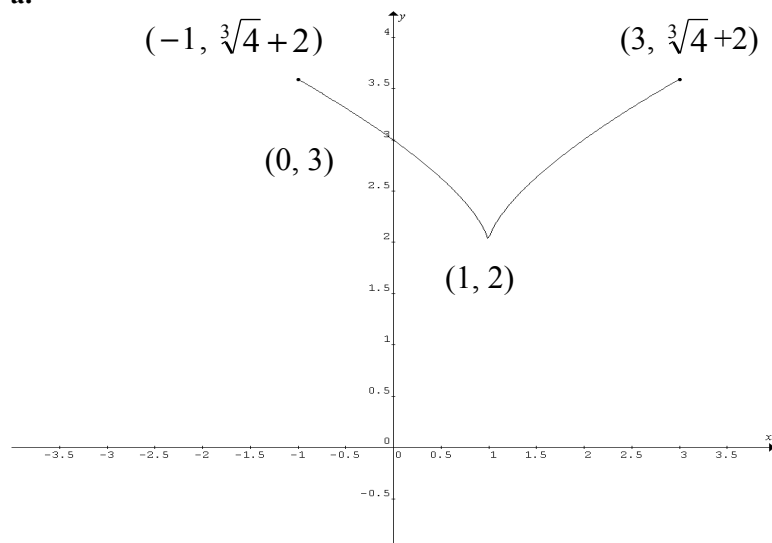
$= 2e^{2x} + 2$

1A

ii. $\int \left(\frac{4(e^{2x} + 1)}{e^{2x} + 2x} \right) dx$

$= 2 \int \left(\frac{2e^{2x} + 2}{e^{2x} + 2x} \right) dx$

$= 2 \log_e(e^{2x} + 2x)$

1A**Question 2****a.**

Shape and closed circles for endpoints

1A

All coordinates correct

1A

b. Average value $= \frac{1}{2-0} \int_0^2 \left((x-1)^{\frac{2}{3}} + 2 \right) dx$

1A

$= \int_0^1 \left((x-1)^{\frac{2}{3}} + 2 \right) dx$ due to symmetry

$= \left[\frac{3}{5} (x-1)^{\frac{5}{3}} + 2x \right]_0^1$

1A

$= (0+2) - \left(-\frac{3}{5} - 0 \right)$

$= 2.6$

1A

Question 3

$$kx + 2y = 6$$

$$3x + (k-1)y = 6$$

For no solution the lines are parallel

$$\begin{vmatrix} k & 2 \\ 3 & k-1 \end{vmatrix} = 0 \text{ or using ratios } \frac{3}{k} = \frac{k-1}{2} \text{ or using gradients } \frac{-k}{2} = \frac{-3}{k-1} \quad \mathbf{1M}$$

$$k(k-1) - 6 = 0$$

$$k^2 - k - 6 = 0$$

$$(k-3)(k+2) = 0$$

$$k = 3 \text{ or } k = -2$$

1A

Check to make sure they are not the same line

$$\text{Using ratios } \frac{3}{k} \neq \frac{6}{6} \text{ or } \frac{k-1}{2} \neq \frac{6}{6} \text{ or using the } y\text{-intercept } \frac{6}{2} \neq \frac{6}{k-1}$$

$$\text{When } k = 3, \frac{3}{3} = \frac{6}{6} = 1, \text{ hence the same line}$$

$$\text{When } k = -2, \frac{3}{-2} \neq \frac{6}{6}, \text{ hence parallel lines, no solution} \quad \mathbf{1A}$$

Question 4

$$2\log_2(x-1) + \log_2(x+1) = 0$$

$$\log_2(x-1)^2 + \log_2(x+1) = 0$$

$$\log_2(x-1)^2(x+1) = 0 \quad \mathbf{1M}$$

$$(x-1)^2(x+1) = 1 \quad \mathbf{1A}$$

$$(x^2 - 2x + 1)(x+1) - 1 = 0$$

$$x^3 - x^2 - x = 0 \quad \mathbf{1A}$$

$$x(x^2 - x - 1) = 0$$

$$x \neq 0, x \neq \frac{1-\sqrt{5}}{2}$$

$$x = \frac{1+\sqrt{5}}{2} \quad \mathbf{1A}$$

Question 5

a. $f(x) = 1 - e^{-x}$

Let $y = 1 - e^{-x}$

Inverse: swap x and y

$$x = 1 - e^{-y} \quad \mathbf{1M}$$

$$e^{-y} = 1 - x$$

$$y = -\log_e(1-x) \quad \mathbf{1A}$$

The domain is $(-\infty, 1)$ **1A**

OR **OR**

$$f^{-1} : (-\infty, 1) \rightarrow \mathbb{R}, \text{ where } f^{-1}(x) = -\log_e(1-x) \quad \mathbf{2A}$$

b. $(0, 0)$ **1A**

Question 6

$$h(x) = g(f(x)) = |x^3 + 3 - 1| = |x^3 + 2| \quad 1A$$

$$h(x) = \begin{cases} 2 + x^3, & x \geq \sqrt[3]{-2} \\ -2 - x^3, & x < \sqrt[3]{-2} \end{cases} \quad 1A$$

$$h'(x) = \begin{cases} 3x^2, & x > \sqrt[3]{-2} \\ -3x^2, & x < \sqrt[3]{-2} \end{cases} \quad 1A$$

Question 7

$$\text{a. } \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$x' = -2(x+1), \quad y' = 3(y-1) \quad 1M$$

$$x = -\frac{x'}{2} - 1, \quad y = \frac{y'}{3} + 1$$

$$\text{Substitute into } y = \frac{2}{x+1} - 1$$

$$\frac{y'}{3} + 1 = -\frac{4}{x'} - 1 \quad 1H$$

$$y' = -\frac{12}{x'} - 6$$

$$\text{The equation of the image is } y = -\frac{12}{x} - 6 \quad 1A$$

b. Translation one unit to the right 1A

Reflection in the y -axis 1A

Dilation by a factor of six from the x -axis 1A

OR OR

There are other solutions such as

Reflect in the x -axis 1A

Dilate by a factor of 6 from the x -axis 1A

Translate by 1 unit in the positive x direction 1A both translations correct

Translate by 12 units in the negative y -direction

The order must be correct

Question 8

a. Using sum of probabilities is 1 1M

$$p + 3p + q + 0.03 + 0.01 = 1$$

$$4p + q = 0.96$$

Since

$$q = 2p \Rightarrow 4p + 2p = 0.96$$

$$6p = 0.96$$

$$p = 0.16 \quad 1A$$

b.

$$\Pr(X < 2 | X < 3) = \frac{\Pr(X < 2 \cap X < 3)}{\Pr(X < 3)}$$

$$= \frac{\Pr(X < 2)}{\Pr(X < 3)} = \frac{0.64}{0.96} = \frac{2}{3} \quad 1A$$

Question 9**a.**

$$\frac{k}{2} \int_0^3 \left(\cos\left(\frac{\pi}{3}t\right) + 1 \right) dt = 1 \quad 1M$$

$$\frac{k}{2} \left[\frac{3}{\pi} \sin\left(\frac{\pi}{3}t\right) + t \right]_0^3 = 1$$

$$\frac{k}{2} \left\{ \left[\frac{3}{\pi} \sin(\pi) + 3 \right] - \left[\frac{3}{\pi} \sin(0) + 0 \right] \right\} = 1$$

$$\frac{k}{2} \times 3 = 1 \Rightarrow k = \frac{2}{3} \quad 1A$$

b.

$$\begin{aligned} \Pr(T > 2) &= \frac{1}{3} \int_2^3 f(t) dt \\ &= \frac{1}{3} \left\{ \left[\frac{3}{\pi} \sin(\pi) + 3 \right] - \left[\frac{3}{\pi} \sin\left(\frac{2\pi}{3}\right) + 2 \right] \right\} \quad 1A \end{aligned}$$

$$= \frac{1}{3} \left\{ 3 - \frac{3}{\pi} \times \frac{\sqrt{3}}{2} - 2 \right\}$$

$$= \frac{1}{3} \left(1 - \frac{3\sqrt{3}}{2\pi} \right) \quad \text{OR} \quad \frac{1}{3} - \frac{\sqrt{3}}{2\pi} \quad 1A$$

Question 10

$$\frac{d}{dx} \sqrt{1 + \cos^3(2x)} = 0$$

Using the chain rule

$$\frac{-6\cos^2(2x)\sin(2x)}{2\sqrt{1 + \cos^3(2x)}} = 0 \quad 1M$$

The denominator cannot equal zero

$$\text{Hence solve } \cos^2(2x)\sin(2x) = 0 \quad 1M$$

$$\cos(2x) = 0 \text{ or } \sin(2x) = 0 \quad 1M$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2} \dots \text{ or } 2x = 0, \pi, 2\pi \dots$$

$$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi \text{ but } x \neq \frac{\pi}{2} \text{ because } \cos^3(2x) \neq -1$$

$$x = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi \quad 1A$$

END OF SOLUTIONS