

Year 2011

VCE

Mathematical Methods

CAS

Trial Examination 1



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**Victorian Certificate of Education
2011**

STUDENT NUMBER

Figures	<table border="1"><tr><td></td><td></td></tr></table>			<table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>								Letter	
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MATHEMATICAL METHOD CAS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 2

Let $f(x) = \frac{1}{1-2x}$ be a function defined on its maximal domain. Find the inverse function f^{-1} stating its domain and range.

3 marks

Question 3

a. Differentiate $x \sin(2x)$ with respect to x .

1 mark

b. Hence find $\int x \cos(2x) dx$

2 marks

- c. The function $f(x) = \begin{cases} 2\cos(2x) & \text{for } 0 \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$

is a probability density function for the continuous random variable X .

- i. Show that $b = \frac{\pi}{4}$.

1 mark

- ii. Find the mean of $f(x)$.

2 marks

- iii. Find the median of $f(x)$.

2 marks

Question 4

If $\frac{d}{dx} \left(\log_e \left(\frac{3x^2 + 4}{4x^2 + 3} \right) \right) = \frac{bx}{g(x)}$, find the value of b and the function $g(x)$.

3 marks

Question 5

The speed v , in metres per second, of an object moving in a straight line is given by a function of time t , in seconds, where $v(t) = \frac{72}{(3t + 2)^2}$ where $t \geq 0$.

Find the distance travelled in metres by the object in the first 2 seconds.

3 marks

Question 6

Let $f : R \rightarrow R$ where $f(x) = x^3 - k^2x$, where k is a positive real constant.

- a. Find in terms of k , the subset of R , for which the graph of the function $f(x)$ is strictly increasing.

2 marks

- b. If the area bounded by the graph of $f(x)$, the x -axis and $x = 0$ and $x = k$ is equal to 64 units, find the value of k .

2 marks

Question 7

Given the functions $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$ where $f(x) = \tan(x)$ and

$g : \mathbb{R}^+ \rightarrow \mathbb{R}$ where $g(x) = \cos(x)$.

- a.** If the two functions intersect at $x = \alpha$, show that $\sin(\alpha) = \frac{1}{2}(\sqrt{5} - 1)$

3 marks

- b.** Show that the functions intersect at right angles.

3 marks

Question 8

A binomial distribution of the random variable X , with six independent trials, is such that $\Pr(X = 3) = \Pr(X = 4)$. If p is the probability of a success on any trial, find the value of p .

3 marks

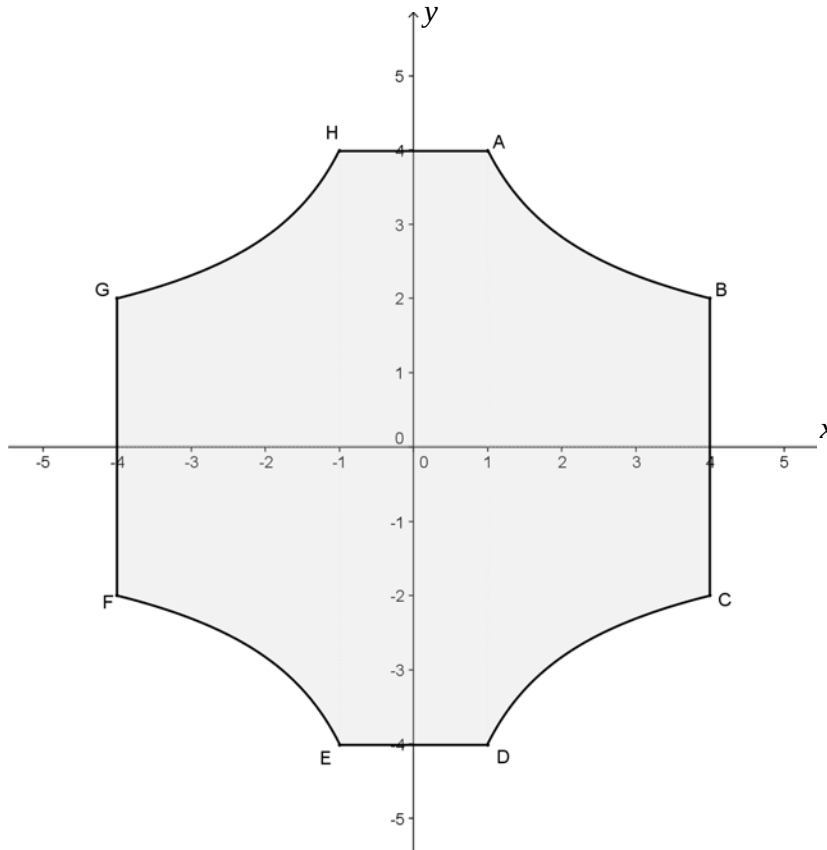
Question 9

a. Evaluate $\int_1^4 \frac{4}{\sqrt{x}} dx$

1 mark

- b. The diagram below shows a shield ABCDEFGH, the shield is symmetrical, with the curve CD being the reflection of AB in the x -axis, and the curve EF the reflection of CD in the y -axis. The function describing the curve AB is given by

$$f : [1, 4] \rightarrow \mathbb{R} \text{ where } f(x) = \frac{4}{\sqrt{x}}.$$



- i. Write down the **function** which describes the curve EF.

1 mark

- ii. Find the total area of the shield.

1 mark

MATHEMATICAL METHODS CAS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods CAS Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

volume of a pyramid: $\frac{1}{3}Ah$

curved surface area of a cylinder: $2\pi rh$

volume of a sphere: $\frac{4}{3}\pi r^3$

volume of a cylinder: $\pi r^2 h$

area of triangle: $\frac{1}{2}bc \sin(A)$

volume of a cone: $\frac{1}{3}\pi r^2 h$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + h f'(x)$

Probability

$$\Pr(A) = 1 - \Pr(A')$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Transition Matrices $S_n = T^n \times S_0$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$