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Mathematical Methods

2011

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

a. Let $a = x - 2y$ and $b = z - 2x$.

Express the following system of simultaneous linear equations in terms of a and b .

2 marks

$$-3x - 2y + 2z = 3$$

$$4x - 4y - z = 1$$

b. Hence, or otherwise, solve the above system of simultaneous linear equations for x and y in terms of z .

2 marks

Question 2

Let $f(x) = |2 - 2x|$ and $g(x) = -\frac{1}{2}f\left(1 - \frac{x}{2}\right) + 1$.

a. Express $g(x)$ in the form $\begin{cases} ax + b, & x < c \\ dx + e, & x \geq f \end{cases}$, where $a, b, c, d, e, f \in \mathbb{R}$.

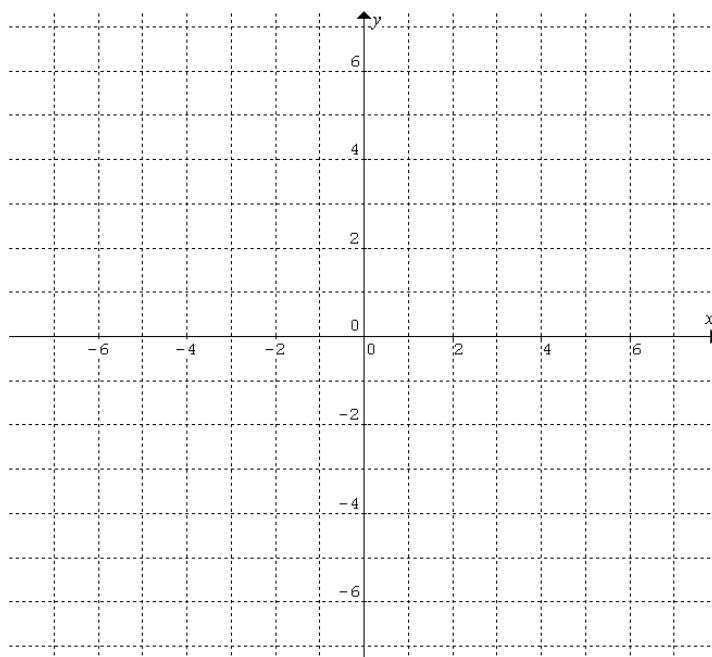
2 marks

- b.** Described the relationship between the graph of the transformed function $g(x)$ and the graph of the original function $f(x)$ in terms of dilations, reflections and translations.

2 marks

- c.** Sketch the graph of $y = g(x)$, label the x and y intercepts with coordinates.

2 marks



Question 3

Let $f(x) = 1 + \frac{1}{e^x}$.

- a.** Show that $f(x) + f(-x) = f(x) \times f(-x)$.

2 marks

- b.** Find the value of $f'(x) \times f'(-x)$.

1 mark

Question 4

Let $P(x) = x^4 + 2x^3$.

a. Show that (i) $x + 2$ is a factor of $P(x)$, and (ii) the remainder is 3 when $P(x)$ is divided by $x - 1$.

2 marks

b. (i) Given $P(x) = (x - 1)(x + 2)Q(x) + ax + b$, where $a, b \in R$. Find the values of a and b . 2 marks

(ii) Hence find the remainder when $P(x)$ is divided by $x^2 + x - 2$. 1 mark

c. Find polynomial $Q(x)$. 2 marks

Question 5

Let $x^a = y^b$ and $y^b = \left(\frac{y}{x}\right)^c$, where $x, y \neq 1$. Express c in terms of a and b .

3 marks

Question 6

If $\int_a^b f(x)dx = ab$ where $a, b \in \mathbb{R}$ and $b > a$, find $\int_a^b f(a+b-x)dx$ in terms of a and b .

3 marks

Question 7

Find $\{x : e^{2\sin 2x} + e^{\sin 2x+1} - e^{\sin 2x} - e = 0, x \in [0, \pi]\}$.

2 marks

Question 8

Consider $f(x) = (\cos 2x)^{-1}$ where $x \in \left[0, \frac{\pi}{4}\right)$.

a. Find $f'(x)$.

1 mark

b. Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{8}} \sec 2x \tan 2x dx$.

2 marks

Question 9

Two identical biased coins are tossed together. Let random variable X be the number of heads showing.

- a. Complete the following probability distribution table for random variable X .

2 marks

X	0	1	2
$\Pr(X = x)$			0.16

- b. Find the value of $E(X)$.

1 mark

- c. Find the exact value of $\Pr(X > 0 | X < 2)$.

1 mark

Question 10

Probability density function $f(x) = \begin{cases} 0 & x < \pi \\ a \sin^2 x & \pi \leq x \leq 2\pi \\ 0 & x > 2\pi \end{cases}$ specifies the probability distribution for random variable X .

- a. Find the exact value of a . (Hint: $\cos 2x = 1 - 2\sin^2 x$)

2 marks

- b. Write down the value of $\overline{X}$.

1 mark

- c. Hence show that $2b + \sin 2b = 1$ if $\int_{\overline{X}-b}^{\overline{X}+b} f(x) dx = \frac{1}{\pi}$.

2 marks

End of exam 1