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GROUP**

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**MATHS METHODS (CAS) 3 & 4
TRIAL EXAMINATION 1
SOLUTIONS
2011**

Question 1

a. $y = \sqrt{2x^2 - 1}$
 $= (2x^2 - 1)^{\frac{1}{2}}$

Method 1

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} (2x^2 - 1)^{-\frac{1}{2}} \times 4x & (1 \text{ mark}) \\ &= \frac{2x}{(2x^2 - 1)^{\frac{1}{2}}} \\ &= \frac{2x}{\sqrt{2x^2 - 1}}\end{aligned}$$

(1 mark)

Method 2

$$\begin{aligned}y &= (2x^2 - 1)^{\frac{1}{2}} & \text{let } u = 2x^2 - 1 \\ y &= u^{\frac{1}{2}} & \frac{du}{dx} = 4x\end{aligned}$$

$$\begin{aligned}\frac{dy}{du} &= \frac{1}{2} u^{-\frac{1}{2}} & (1 \text{ mark}) \\ &= \frac{1}{2u^{\frac{1}{2}}} \\ &= \frac{1}{2\sqrt{u}}\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \quad (\text{chain rule}) \\ &= \frac{1}{2\sqrt{u}} \times 4x \\ &= \frac{4x}{2\sqrt{2x^2 - 1}} \\ &= \frac{2x}{\sqrt{2x^2 - 1}} & (1 \text{ mark})\end{aligned}$$

b. $f(x) = \frac{x}{e^{3x}}$

$$f'(x) = \frac{e^{3x} \times 1 - 3e^{3x} \times x}{(e^{3x})^2} \quad (\text{quotient rule})$$

$$= \frac{e^{3x} - 3xe^{3x}}{e^{6x}} \quad (1 \text{ mark})$$

$$f'(1) = \frac{e^3 - 3e^3}{e^6}$$

$$= \frac{-2e^3}{e^6}$$

$$= \frac{-2}{e^3}$$

(1 mark)

Question 2

$$\log_e(3) + 2\log_e(x) = \log_e(4x)$$

$$\log_e(3) + \log_e(x^2) = \log_e(4x)$$

$$\log_e(3x^2) = \log_e(4x)$$

$$3x^2 = 4x$$

$$3x^2 - 4x = 0$$

$$x(3x - 4) = 0$$

$$x = 0 \text{ or } x = \frac{4}{3}$$

(1 mark)

(1 mark)

but $\log_e(x)$ is not defined for $x = 0$ so $x = \frac{4}{3}$

(1 mark)

Question 3

a. $g(x) = 3\log_e(x-2)$

Let $y = 3\log_e(x-2)$

Swap x and y for inverse.

$$x = 3\log_e(y-2)$$

$$\frac{x}{3} = \log_e(y-2)$$

$$e^{\frac{x}{3}} = y-2$$

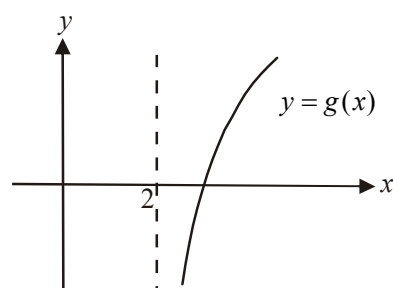
$$y = 2 + e^{\frac{x}{3}}$$

$$d_g = (2, \infty) \quad r_g = R$$

So $d_{g^{-1}} = R \quad r_{g^{-1}} = (2, \infty)$

So $g^{-1}: R \rightarrow R, \quad g^{-1}(x) = 2 + e^{\frac{x}{3}}$

(1 mark)



(1 mark) – correct rule

(1 mark) – correct domain

b. i. $h(x) = g^{-1}(g(x))$

$$= x$$

(1 mark)

ii. $d_h = d_g$ (since h is a composite function)

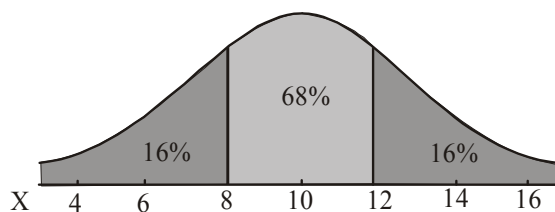
$$= (2, \infty)$$

(1 mark)

Question 4

- a. Note that since variance = 4, standard deviation = $\sqrt{4} = 2$.

$$\Pr(X > 12) = 0.16$$

**(1 mark)**

- b. $\Pr(X > 12 | X > 10)$

$$= \frac{\Pr(X > 12 \cap X > 10)}{\Pr(X > 10)}$$

(conditional probability)

(1 mark)

$$= \frac{\Pr(X > 12)}{\Pr(X > 10)}$$

$$= \frac{0.16}{0.5}$$

$$= 0.32$$

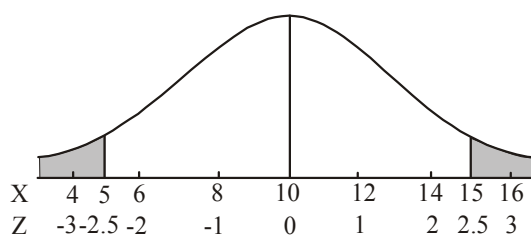
(1 mark)

- c.

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{5 - 10}{2}$$

$$z = -2.5$$

(1 mark)

Because of the symmetry of the normal curve,

$$\Pr(Z > 2.5) = \Pr(X < 5)$$

So $a = 2.5$

(1 mark)

Question 5

$$\sin\left(\frac{x}{2}\right) + \frac{1}{\sqrt{3}} \cos\left(\frac{x}{2}\right) = 0$$

$$\sin\left(\frac{x}{2}\right) = -\frac{1}{\sqrt{3}} \cos\left(\frac{x}{2}\right)$$

$$\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = -\frac{1}{\sqrt{3}}$$

$$\tan\left(\frac{x}{2}\right) = -\frac{1}{\sqrt{3}}$$

$$\frac{x}{2} = \frac{5\pi}{6} + n\pi, \quad n \in \mathbb{Z}$$

$$x = 2\left(\frac{5\pi}{6} + n\pi\right), \quad n \in \mathbb{Z}$$

$$x = \frac{5\pi}{3} + 2n\pi, \quad n \in \mathbb{Z}$$

(alternative answer $\frac{x}{2} = \frac{-\pi}{6} + n\pi, \quad n \in \mathbb{Z}$ so $x = \frac{-\pi}{3} + 2n\pi, \quad n \in \mathbb{Z}$)

**(1 mark)****(1 mark)****(1 mark)****Question 6**

- a. We have a binominal distribution where $n = 3$ and $p = 0.6$.

Method 1

$$\begin{aligned} \Pr(X \geq 2) &= \Pr(X = 2) + \Pr(X = 3) \\ &= {}^3C_2 (0.6)^2 (0.4)^1 + {}^3C_3 (0.6)^3 (0.4)^0 \\ &= 3 \times 0.36 \times 0.4 + 0.6^3 \\ &= 0.432 + 0.216 \\ &= 0.648 \end{aligned}$$

(1 mark) – recognition of binominal distribution**(1 mark)** – correct answerMethod 2

$$\begin{aligned} \Pr(X \geq 2) &= 1 - \Pr(X < 2) \\ &= 1 - \{\Pr(X = 0) + \Pr(X = 1)\} \\ &= 1 - \left\{ {}^3C_0 (0.6)^0 (0.4)^3 + {}^3C_1 (0.6)^1 (0.4)^2 \right\} \\ &= 1 - (0.4^3 + 3 \times 0.6 \times 0.16) \\ &= 1 - (0.064 + 0.288) \\ &= 1 - 0.352 \\ &= 0.648 \end{aligned}$$

(1 mark) – recognition of binominal distribution**(1 mark)** – correct answer

- b. Let the number of orders placed at the drive-through be n .

$$\begin{aligned}\Pr(X \geq 1) &= 0.84 \\ 1 - \Pr(X = 0) &= 0.84\end{aligned}\quad (1 \text{ mark})$$

$$1 - {}^nC_0(0.6)^0(0.4)^n = 0.84 \quad (1 \text{ mark})$$

$$\begin{aligned}1 - 1 \times 1 \times (0.4)^n &= 0.84 \\ -(0.4)^n &= -0.16 \\ (0.4)^n &= 0.16 \\ n &= 2\end{aligned}$$

(1 mark)

Two orders need to be placed.

Question 7

We are looking for $\frac{dr}{dt}$, the rate at which the radius of the balloon is changing.

$$\text{Now, } \frac{dr}{dt} = \frac{dr}{dV} \cdot \frac{dV}{dt} \quad (\text{chain rule}) \quad (1 \text{ mark})$$

$$\text{Now, } V = \frac{4}{3}\pi r^3 \quad (\text{formula sheet})$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dr}{dV} = \frac{1}{4\pi r^2}$$

$$\text{Also } \frac{dV}{dt} = 2 \quad (\text{given})$$

$$\text{So } \frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

$$\text{becomes } \frac{dr}{dt} = \frac{1}{4\pi r^2} \times 2$$

$$= \frac{1}{2\pi r^2}$$

$$\text{When } r = 4, \quad \frac{dr}{dt} = \frac{1}{32\pi} \quad (1 \text{ mark})$$

The radius of the balloon is increasing at the rate of $\frac{1}{32\pi}$ cm/sec.

(1 mark)

Question 8

$$g: R \setminus \{0\} \rightarrow R, \quad g(x) = 1 + \frac{1}{x}$$

To Show: $4g(2u) - g(-u) = 3g(u)$

$$\begin{aligned}
 LHS &= 4g(2u) - g(-u) \\
 &= 4\left(1 + \frac{1}{2u}\right) - \left(1 - \frac{1}{u}\right) && \text{(1 mark)} \\
 &= 4 + \frac{4}{2u} - 1 + \frac{1}{u} \\
 &= 3 + \frac{2}{u} + \frac{1}{u} \\
 &= 3 + \frac{3}{u} \\
 &= 3\left(1 + \frac{1}{u}\right) \\
 &= 3g(u) \\
 &= RHS \\
 &\text{as required.}
 \end{aligned}$$

(1 mark)

Question 9

$$f(x+h) \approx f(x) + h f'(x)$$

$$\begin{aligned}
 f(x) &= \sqrt{x} \\
 &= x^{\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 f'(x) &= \frac{1}{2} x^{-\frac{1}{2}} \\
 &= \frac{1}{2\sqrt{x}} && \text{(1 mark)}
 \end{aligned}$$

$$h = 0.03 \quad \text{(1 mark)}$$

So $f(x+h) \approx f(x) + h f'(x)$

becomes $f(x+0.03) \approx \sqrt{x} + \frac{0.03}{2\sqrt{x}}$

$$\begin{aligned}
 f(9+0.03) &\approx \sqrt{9} + \frac{0.03}{2 \times \sqrt{9}} \\
 &= 3 + \frac{0.03}{6} \\
 &= 3 + 0.005 \\
 &= 3.005 && \text{(1 mark)}
 \end{aligned}$$

Question 10

The period of the graph of $y = a \sin(2x)$ is $\frac{2\pi}{2} = \pi$ so the graph intersects the x -axis at the right end of the shaded region at $x = \frac{\pi}{2}$.

(1 mark)

So $\int_0^{\frac{\pi}{2}} a \sin(2x) dx = 4$ **(1 mark)**

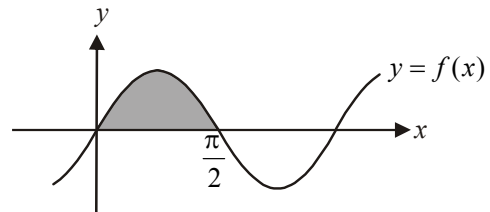
$$a \left[-\frac{1}{2} \cos(2x) \right]_0^{\frac{\pi}{2}} = 4$$
 (1 mark)

$$-\frac{a}{2} (\cos(\pi) - \cos(0)) = 4$$

$$-\frac{a}{2} (-1 - 1) = 4$$

$$-\frac{a}{2} \times -2 = 4$$

$$a = 4$$

**(1 mark)****Question 11**

- a. Since the graph of $y = f(x)$ is not smooth at the point where $x = 0$, then $d_{f'} = \mathbb{R} \setminus \{0\}$.

(1 mark)

b. $f(x) = 2|x| - 3x^4 + 1$

Method 1

$$f(x) = \begin{cases} 2x - 3x^4 + 1 & \text{if } x \geq 0 \\ -2x - 3x^4 + 1 & \text{if } x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 2 - 12x^3 & \text{if } x > 0 \\ -2 - 12x^3 & \text{if } x < 0 \end{cases}$$

(1 mark)**(1 mark)**Method 2

$$f'(x) = \frac{2|x|}{x} - 12x^3 \quad \text{for } x \in \mathbb{R} \setminus \{0\}$$

(1 mark) – first term
(1 mark) – second term