

2010 Trial Examination

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	4	4	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 19 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 – Multiple-choice questions**Instructions for Section 1**

Answer all questions on the answer sheet provided for multiple choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Question 1

Consider the following system of linear equations:

$$2x + 8z = 26$$

$$4x - 4y - 14z = -38$$

$$8x - 4y + 2z = 14$$

The set of solutions in terms of the parameter p , where $p \in \mathbb{R}$, is:

- A. $x = 13 + 4p$, $y = \frac{5}{2}(3 - t)$, $z = p$
- B. $x = 13 - 4p$, $y = \frac{45-15p}{2}$, $z = p$
- C. $x = 13 - 4p$, $y = \frac{45-15p}{2}$, $z = 0$
- D. $x = 13 - 4p$
- E. $z = p$

Question 2

Which of the following is a true statement for the graph of $y = \frac{-2}{(x-1)^2} + 3$

- A. The equation of the horizontal asymptote is $y = -3$.
- B. As $x \rightarrow \infty$, $y \rightarrow 0$.
- C. The y – intercept has coordinates $(0, 1)$
- D. The equation of the vertical asymptote is $x = -1$
- E. The x – intercept has coordinates $(0.22, 0)$

Question 3

Which of the following is not a true statement about the graph of $y = -(x - 1)^2 - k^2$

- A. There are no x – intercepts.
- B. The equation of the axis of symmetry is $x = 1$
- C. The turning point has coordinates $(1, -k^2)$
- D. The y – intercept has coordinates $(0, -1 - k^2)$
- E. For all real values of x , $y \leq -1 - k^2$

Question 4

Consider the function $f: R \rightarrow R$, $f(x) = |2e^x - 2|$. Which of the following statements is true?

- A. $f'(x) = \begin{cases} 2e^x & x > 0 \\ -2e^x & x < 0 \end{cases}$
- B. $f'(x) = |2e^x + 1|$, $x \in R$
- C. $f'(x) = |2e^x|$, $x \in R \setminus \{1\}$
- D. $f'(x) = \begin{cases} e^x & x > 0 \\ -e^x & x < 0 \end{cases}$
- E. $f'(x) = \begin{cases} -2e^x & x > 0 \\ 2e^x & x < 0 \end{cases}$

Question 5

Grain is poured in a large silo. The volume, V cubic metres, of grain in the silo at time t hours is given by $V = \frac{2}{35}(t^2 - 0.14t + 3.7)$. The average rate of change of volume over the first 15 minutes, in cubic metres per hour, is closest to:

- A. 12.8543
- B. 0.0629
- C. 0.1285
- D. 0.0063
- E. 6.2860

Question 6

If $\Pr(Z > c) = 0.25$, where Z is a standard normal random variable, then c is closest to:

- A. 0.6745
- B. -0.6745
- C. 0.5987
- D. 0.7734
- E. -0.7734

TURN OVER

Question 7

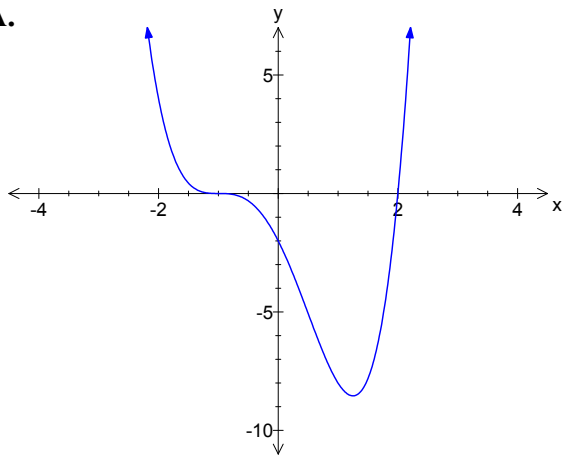
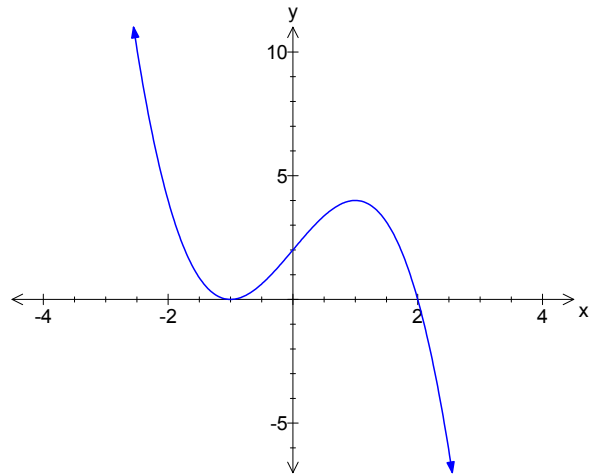
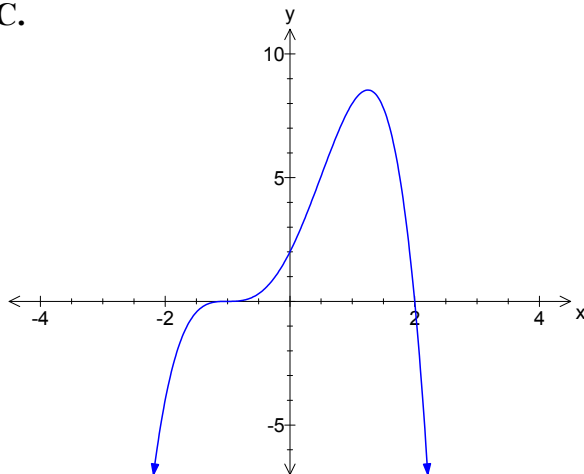
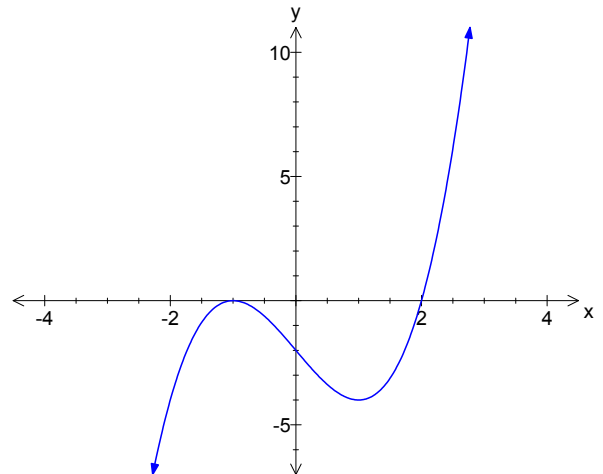
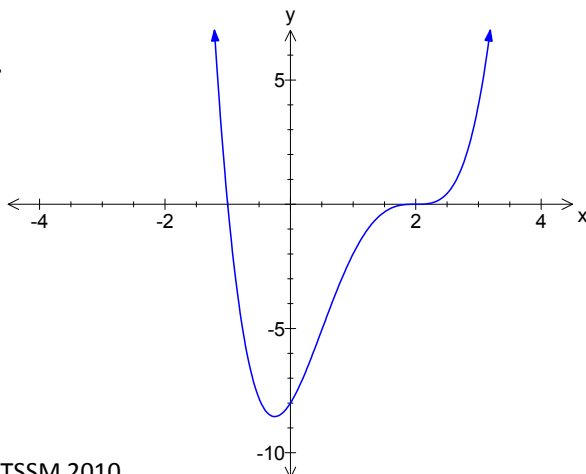
A polynomial function f has derivative f' with the following properties:

$$f'(x) = 0 \text{ for } x = -1 \text{ and } x = 1.25$$

$$f'(x) > 0 \text{ for } x > 1.25$$

$$f'(x) < 0 \text{ for all other real values of } x$$

Which of the following could be the graph of $y = f(x)$?

A.**B.****C.****D.****E.**

Question 8

If $f'(x) = (2x - 1)(-x + 4)$, then $f(x)$ could be:

- A. $\frac{x(4x^2 - 27x + 24)}{6}$
- B. $\frac{2x^2}{3} - \frac{9x}{2} + 4x$
- C. $-2x^3 + 9x^2 - 4$
- D. $\frac{-2x^2}{3} + \frac{9x}{2} + 4x$
- E. $\frac{-2x^3}{3} + \frac{9x^2}{2} - 4x$

Question 9

The following table represents a discrete probability distribution for a random variable, X .

x	1	2	3	4
$\Pr(X = x)$	$2m$	m	$4m$	$3m$

The expected value of X is:

- A. $\frac{1}{10}$
- B. $\frac{14}{10}$
- C. $\frac{14}{5}$
- D. 2
- E. $\frac{8}{5}$

Question 10

If $Z \sim N(0, 1)$, then $\Pr(Z > 2.416)$ is:

- A. 0.9922
- B. 0.9241
- C. 0.0784
- D. 0.9925
- E. 0.0078

TURN OVER

Question 11

The graph of a function lies above the x – axis for $0 \leq x \leq 4$ and $\int_0^4 f(x) dx = 9$. The graph of f is dilated by a factor of 2 from the x – axis, then translated 4 units down. The resulting function is $g(x)$. Find $\int_0^4 g(x) dx$.

- A. 0.5
- B. 0
- C. 2
- D. 2.5
- E. 28

Question 12

The gradient of a curve at any point is given by $f'(x) = \sin\left(\frac{\pi x}{6}\right) + d$, where d is a constant. If the curve has a stationary point at $(3, 2)$, then its equation is:

- A. $f(x) = \frac{6}{\pi} \cos\left(\frac{3\pi}{6}\right) + 5$
- B. $f(x) = \frac{-6}{\pi} \cos\left(\frac{\pi x}{6}\right) - x + 5$
- C. $f(x) = \frac{-6}{\pi} \sin\left(\frac{3\pi}{6}\right) + x - 5$
- D. $f(x) = \frac{-6}{\pi} \cos\left(\frac{\pi x}{6}\right) + x + 5$
- E. $f(x) = \frac{6}{\pi} \sin\left(\frac{\pi x}{6}\right) - x$

Question 13

It is found that 3 out of 15 balloons are defective. Thirty balloons are selected at random. The probability that 2 are defective is:

- A. 0.0084
- B. 0.0007
- C. 1
- D. 0.0337
- E. 0.5

Question 14

If $X \sim N(10, 4)$ and $\Pr(X > x) = 0.47$, then x is closest to:

- A. 9.8495
- B. -10.1505
- C. 10.1505
- D. -9.8495
- E. 0.5

Question 15

The life span of light bulbs is normally distributed with an expected value of 36 months and a standard deviation of 3 months. The probability that a light bulb last for less than 27 months is closest to:

- A. 0.0135
- B. 0.0014
- C. 0.9987
- D. 0.9865
- E. -0.9987

Question 16

Let $f(x) = \sqrt{2x}$, $x \geq 0$ and $g(x) = (x + 2)(x - 1)$, $x \in D$. Then the largest domain D such that $f \circ g$ exists is:

- A. $\mathbb{R} \setminus (-2, 1)$
- B. $\mathbb{R}^+$
- C. $x > -0.5$
- D. $\mathbb{R} \setminus [-2, 1]$
- E. $x < -0.5$

Question 17

An antiderivative of $3x\sqrt{x}$ with respect to x is:

- A. $\frac{6\sqrt[5]{x^2}}{5}$
- B. $\frac{6\sqrt{x^5}}{5}$
- C. $\frac{6}{5\sqrt{x^5}}$
- D. $\frac{5\sqrt{x^5}}{6}$
- E. $5x^{\frac{5}{2}}$

TURN OVER

Question 18

If X is a random variable with a probability density function defined by

$$f(x) = \begin{cases} \frac{1}{k}(x^3 - 4) & 2 \leq x \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

The value of k is:

- A. $\frac{4}{561}$
- B. 63
- C. $\frac{1}{63}$
- D. $\frac{561}{4}$
- E. 25

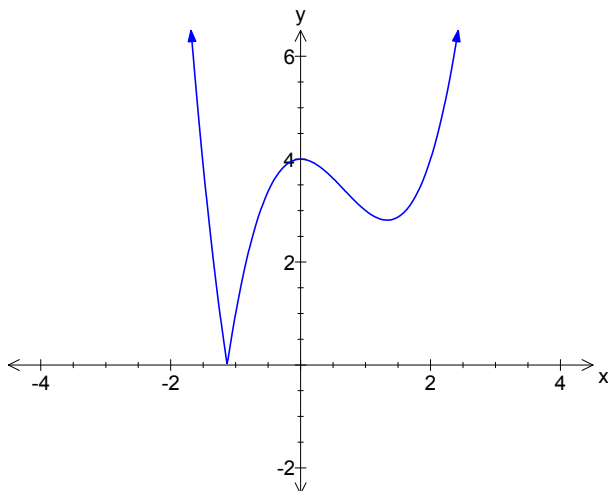
Question 19

The left rectangle approximation using rectangles of width 1 to the area of the region enclosed by the curve with equation $y = \log_e(-x + 1)$, the x -axis and the line $x = -4$ is:

- A. $10\log_e 12$
- B. $2\log_e 2 + \log_e 5$
- C. $2\log_e 2 + \log_e 3$
- D. $\log_e 14$
- E. $\log_e 120$

Question 20

The equation of the graph shown below is best describe by



- A. $|x^3 - 2x^2 + 4|$
- B. $|x^3 - 2x^2| + 4$
- C. $x^3 - 2x^2 + 4$
- D. $-|x^3 - 2x^2 + 4|$
- E. $-x^3 - 2x^2 + 4$

Question 21

If $y = \frac{1}{3} \log_e(5x + 2) - 1$, then the equation of the inverse is:

- A. $\frac{1}{5}(e^{3x+1} - 2)$
- B. $\frac{1}{5}(e^{3x+3} + 2)$
- C. $\frac{1}{5}(e^{3x+3} - 2)$
- D. $\frac{1}{5}(e^{3x-3} - 2)$
- E. $5(e^{3x+3} - 2)$

Question 22

The function $: R \rightarrow R, f(x) = -5 \sin\left(\frac{\pi x}{4}\right) + 1$, has period and range respectively:

- A. $\frac{\pi}{2}, (-4, 6)$
- B. $\frac{\pi}{2}, [-4, 6]$
- C. $8, [-2, 6]$
- D. $2\pi, (-2, 6)$
- E. $8, [-4, 6]$

**END OF SECTION 1
TURN OVER**

SECTION 2**Instructions for Section 2**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

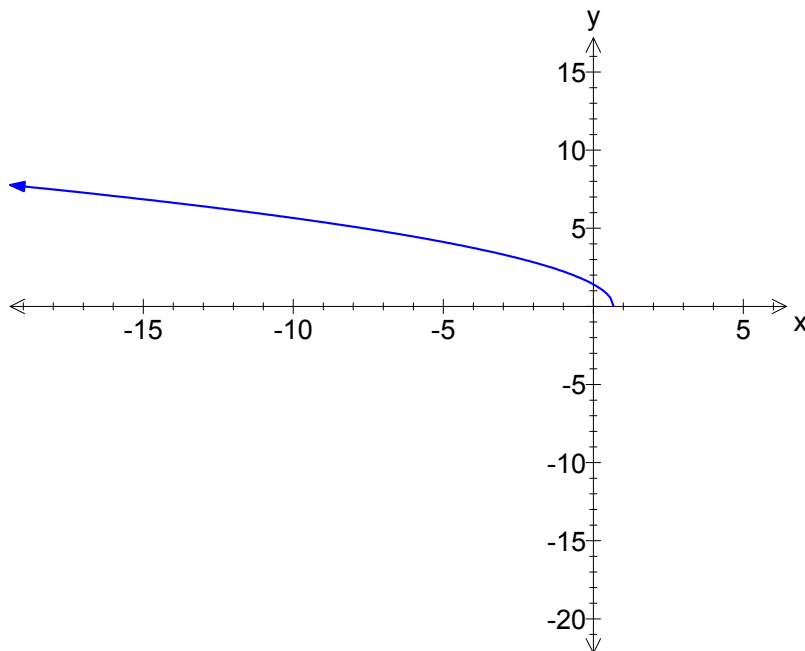
In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The diagram below shows the graph of the function $f: \left(-\infty, \frac{2}{3}\right] \rightarrow \mathbb{R}, f(x) = \sqrt{2-3x}$.



- a. Use calculus to show that the gradient of the tangent to the function $f(x)$ in terms of x , is $\frac{-3}{2\sqrt{2-3x}}$.

2 marks

- b. Find the equation, $g(x)$, of the tangent at $x = \frac{119}{192}$. Give exact values.

2 marks

- c. Show that $f \circ g$ does not exist.

2 marks

- d. Find the combined function $g \circ f$.

3 marks

- e. Sketch the graph of $g(f(x))$ on the axes above, showing all key features.

2 marks

- f. Sketch the graph of $h(x) = g(f(x)) + f(x)$ on the axes above, showing all key features.

2 marks

Total 13 marks

TURN OVER

Question 2

A manufacturer makes candles with lengths that are normally distributed. It is known that 40% of all the candles are at least 49.62 mm and three-quarters of the candles are less than 51.37 mm.

- a. Find the average length and the standard deviation of the candles made, correct to two decimal places.

3 marks

- b. The manufacturer would like each candle produced to have length between 47.15 and 49.87 mm. What is the probability, correct to four decimal places, that the length of a randomly selected candle is in this range?

2 marks

The candles are packed and sold in boxes that each holds 25 candles. A candle fits into the box if its length is less than 51.42 mm.

- c. What is the probability, correct to four decimal places, that a randomly selected candle produced by the manufacturer fits into a box?

2 marks

- d. What is the probability, correct to four decimal places, that the length of a candle which fits into a box is between 47.15 and 49.87 mm.

3 marks

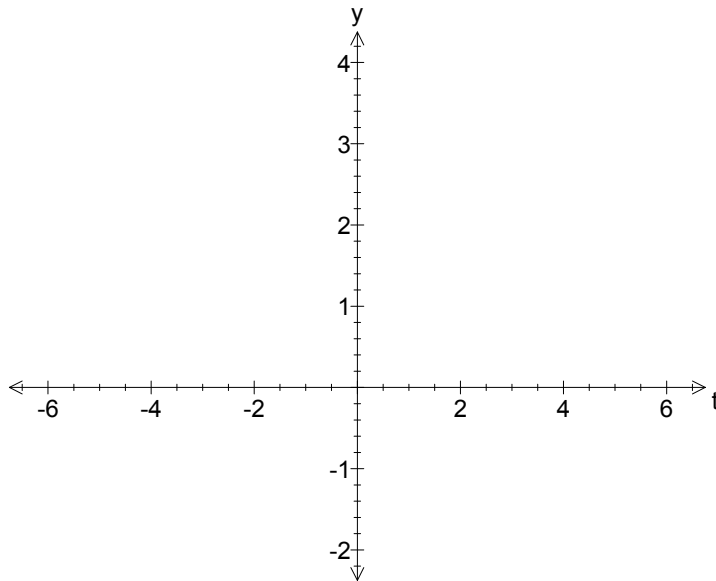
The time in hours that it takes to manufacture a large number of candles on a particular day is a continuous random variable with probability density function given by

$$f(t) = \begin{cases} -0.5t + 2 & 2 \leq t \leq a \\ 0 & \text{elsewhere} \end{cases}$$

- e. Find the value of a .

1 mark

- f. Sketch the probability density function, and label the maximum with its coordinates.



2 marks

TURN OVER

- g.** What is the probability, correct to four decimal places, that the manufacturer will take more than 140 minutes to manufacture the large amount of candles?

2 marks

- h.** What is the average time taken to manufacture the large amount of candles, correct to 4 decimal places?

2 marks

- i.** What is the probability, correct to four decimal places, that the time to manufacture the candles is within two standard deviations of the mean?

3 marks

Total 20 marks

Question 3

- a. The gradient of a curve at any point is given by $\frac{dh}{dt} = \sin\left(\frac{\pi t}{6}\right)$. The depth of water in a tidal pool can be modelled by h , where h metres is the depth of the water and t is the number of hours after 6 a.m. on a given day. Find an expression for h in terms of t , if there is no water in the pool at 6 a.m.

3 marks

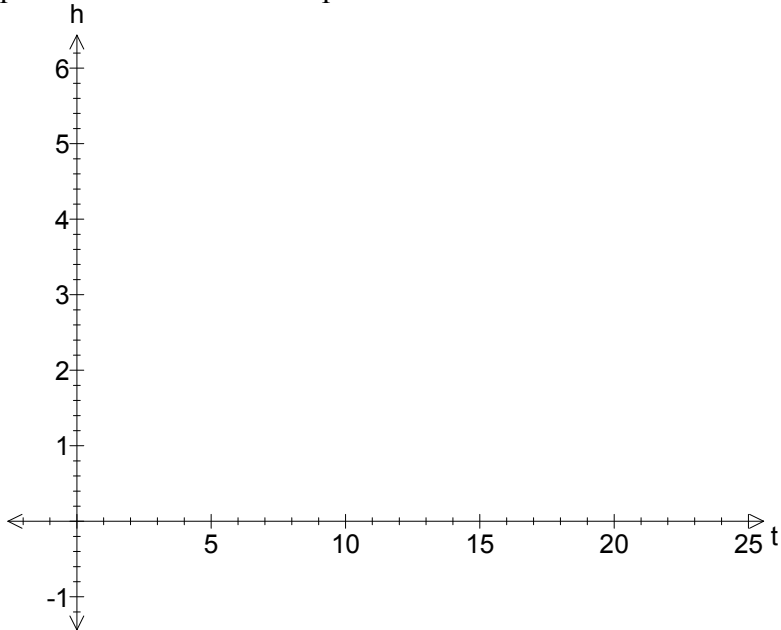
- b. It is only safe to swim in the tidal pool when the water level is at the most two metres. Between what hours, over a 24-hour period, is it safe to swim? Give the time to the nearest minute.

2 marks

- c. What is the exact average depth of water in the tidal pool over the 24-hour period?

2 marks

- d. Sketch the graph of h and construct a rectangle whose area is the same as the area under the graph of h over the 24-hour period.



3 marks

- e. Describe the transformations required to transform the graph of $y = \cos(t)$ into the graph of h .

2 marks

- f. Find the general solution to the equation $0 = \frac{-6}{\pi} \cos\left(\frac{\pi t}{6}\right) + \frac{6}{\pi}$.

1 mark

- g.** Consider the functional equation $k(t + \pi) = k(t)$. Show that the function with rule $k(t) = \cos(2t)$ satisfies this functional equation. Give reasons for your answer.

2 marks

Total 15 marks

Question 4

The total surface area of a rectangular box with a square base and open at the top, is 675 cm^2 .

- a.** Express h in term of x .

2 marks

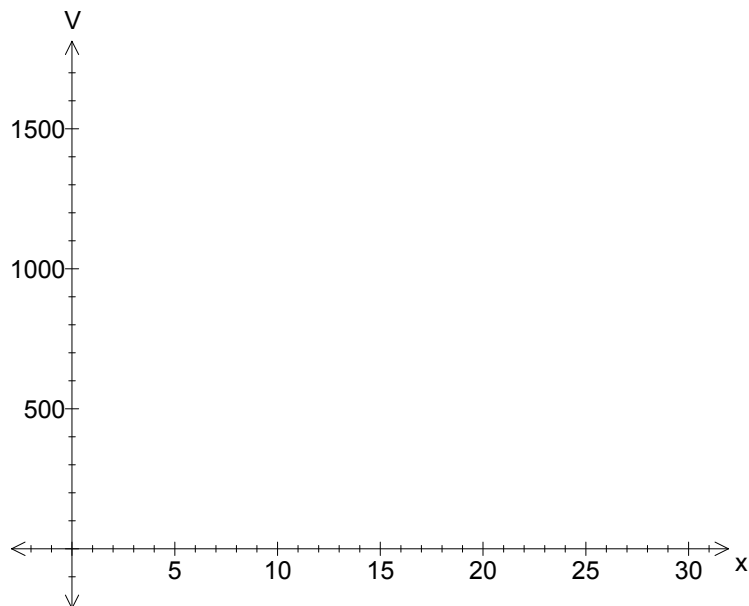
- b. i.** Find the dimensions of the box so that its volume is a maximum, and hence state the volume of the box.

4 marks

- ii. State the domain for this volume, correct to two decimal places.

2 marks

- c. Sketch the graph of V against x for the domain found in Q4bii. Show key features correct to 2 decimal places.



2 marks

Total 10 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name: _____

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E