

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1



2010 Trial Examination

SOLUTIONS

Question 1

a. $\frac{dy}{dx} = -2e^{-2x} \sin(3x - 1) + 3e^{-2x} \cos(3x - 1)$

M1+A1
2 marks

b. $f'(x) = \frac{3xe^{3x} - e^{3x}}{x^2} = \frac{e^{3x}(3x-1)}{x^2}$
 $f'(1) = 2e^3$

M2+A1
3 marks

Question 2

a. $\int e^{3\pi x-1} dx = \frac{e^{3\pi x-1}}{3\pi} + c$

M1+A1
2 marks

b. $\left[-\frac{3}{2}\log_e(9-2x)\right]_m^4 = 1$
 $-\frac{3}{2}\log_e(9-8) + \frac{3}{2}\log_e(9-2m) = 1$
 $\log_e(9-2m) = \frac{2}{3}$
 $e^{\frac{2}{3}} = 9-2m$
 $2m = 9 - e^{\frac{2}{3}}$
 $m = \frac{9 - e^{\frac{2}{3}}}{2}$

M2+A1
3 marks

Question 3

- a. The transformation matrix is $\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$, which represent a dilation by a factor of 2 away from the y -axis, a dilation by a factor of 3 away from the x -axis and a reflection in the x -axis.

M1+A1

2 marks

- b. $y = \cos(x)$

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ -3y \end{bmatrix}$$

$$u = 2x \text{ and } v = -3y$$

$$x = \frac{u}{2} \text{ and } y = \frac{v}{-3}$$

$$\frac{v}{-3} = \cos\left(\frac{u}{2}\right)$$

$$y' = -3\cos\left(\frac{x'}{2}\right)$$

M1+A1

2 marks

- c. $\cos\left(\frac{x'}{2}\right) = 0, -\frac{\pi}{2} \leq \frac{x'}{2} \leq \frac{\pi}{2}$.

$$\frac{x'}{2} = \frac{\pi}{2} \quad \text{or} \quad \frac{x'}{2} = \frac{3\pi}{2} \Rightarrow \text{is not in the domain} \therefore \text{not part of the solution}$$

$$x' = \pi$$

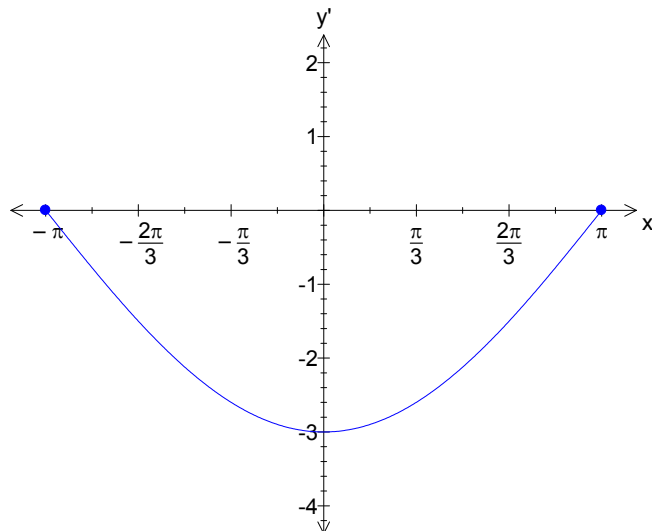
$$\frac{x'}{2} = \frac{3\pi}{2} - 2\pi = \frac{-\pi}{2}$$

$$x' = -\pi$$

M2+A1

3 marks

- d.



A2

2 marks

Question 4

$$\frac{dV}{dt} = 8 \text{ m}^3/\text{minute}, r = \frac{h}{3} \text{ and } \frac{dV}{dh} = \frac{\pi h^2}{9}$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$8 = \frac{\pi h^2}{9} \times \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{8 \times 9}{4\pi} = \frac{18}{\pi} \text{ m/min}$$

M2+A1
3 marks

Question 5

a. $x \in (1, \infty)$

A1
1 mark

b. $x = \frac{1}{(y-1)^2} + 2$

$$x - 2 = \frac{1}{(y-1)^2}$$

$$(y-1)^2 = \frac{1}{x-2}$$

$$y = \frac{1}{\sqrt{x-2}} + 1$$

$$f^{-1}(x) = \frac{1}{\sqrt{x-2}} + 1, x > 2$$

M2+A1
3 marks

Question 6

$$f(x) = \sqrt[4]{x} = x^{\frac{1}{4}}, \quad f'(x) = \frac{1}{4}x^{-\frac{3}{4}}, \quad x = 16, \quad h = -0.08$$

$$f(x+h) \approx f(x) + hf'(x)$$

$$\sqrt[4]{15.92} \approx \sqrt[4]{16} - 0.08 \times \frac{1}{4}(16)^{-\frac{3}{4}}$$

$$\sqrt[4]{15.92} \approx 2 - 0.08 \times \frac{1}{32}$$

$$\sqrt[4]{15.92} \approx 2 - \frac{1}{400} \approx 1 \frac{399}{400} \text{ or } 2 - 0.0025 \approx 1.9975$$

M3+A1
4 marks

Question 7

a. $\int_0^2 -x + 4 - x(x-1)^2 dx = \int_0^2 -x^3 + 2x^2 - 2x + 4 dx$
 $\left[-\frac{x^4}{4} + \frac{2x^3}{3} - x^2 + 4x \right]_0^2 = -4 + \frac{16}{3} - 4 + 8 = \frac{16}{3} \text{ units}^2$

M2+A1

3 marks

b. $h'(x) = 3x^2 - 4x + 1$
 $h'(0) = 1$
 $m_{normal} = -1 \text{ and at } (0,0)$
 $y - 0 = -1(x - 0)$
 $y = -x$

M1+A1

2 marks

Question 8

a. $0 \leq p(x) \leq 1$
 $p(-1) = \frac{1}{24}(7 - 2 \times -1) = \frac{9}{24}$
 $p(0) = \frac{1}{24}(7 - 2 \times 0) = \frac{7}{24}$
 $p(1) = \frac{1}{24}(7 - 2 \times 1) = \frac{5}{24}$
 $p(2) = \frac{1}{24}(7 - 2 \times 2) = \frac{3}{24}$
 $\sum p(x) = \frac{9+7+5+3}{24} = 1$

M1+A1

2 marks

b.

i. $\text{mean} = E(X) = \sum x \times p(x) = -1 \times \frac{9}{24} + 1 \times \frac{5}{24} + 2 \times \frac{3}{24} = \frac{2}{24} = \frac{1}{12}$

A1

1 mark

ii. $P(X > -1 | X \leq 1) = \frac{(X > -1 \cap X \leq 1)}{X \leq 1} = \frac{7+5}{24} \times \frac{24}{21} = \frac{12}{21} = \frac{4}{7}$

M1+A1

2 marks