

Trial Examination 2010

VCE Mathematical Methods (CAS) Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

Question and answer booklet of 9 pages, with a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.

Write your **name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2010 VCE Mathematical Methods (CAS) Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Let $f : [0, 2\pi] \rightarrow \mathbb{R}$, where $f(x) = \sin^2\left(2x - \frac{\pi}{2}\right) - 1$.

- a. Find the y-intercept of the graph of $y = f(x)$.

1 mark

- b. Find the x-intercepts of the graph of $y = f(x)$.

3 marks

Question 3

Let $f : (-\infty, a) \rightarrow \mathbb{R}$, where $f(x) = (x - 2)^2 + 4$.

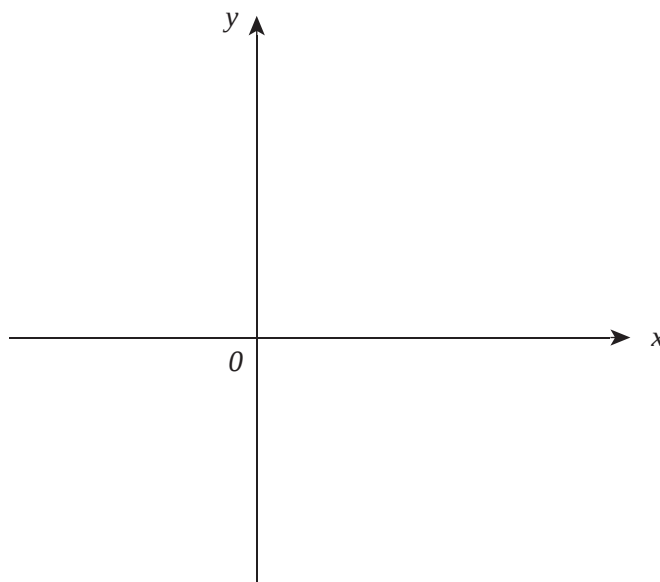
- a. State the largest value of a such that the graph of $y = f(x)$ is one-to-one.

_____ 1 mark

- b. Find the rule of the inverse function f^{-1} .

2 marks

- c. Sketch the graph of $y = f^{-1}(f(x))$ on the axes below.



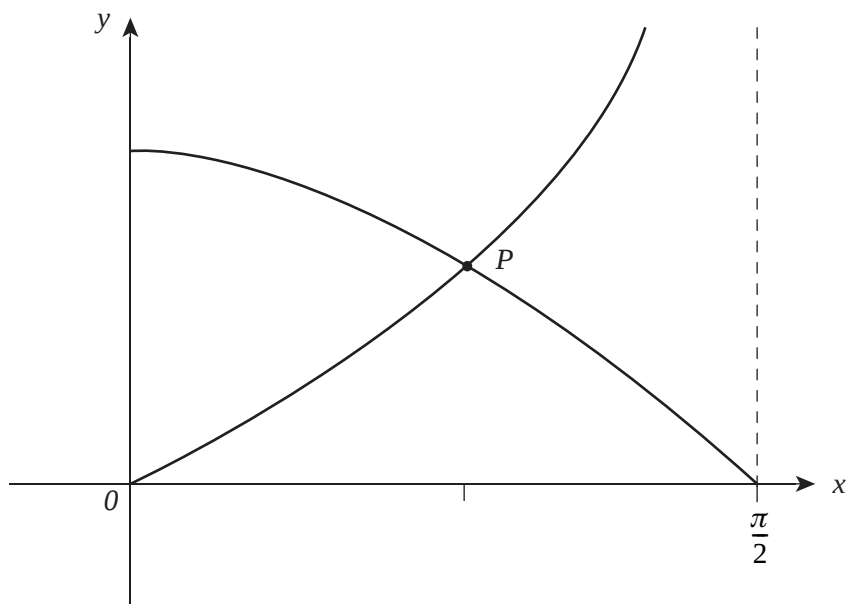
1 mark

Question 4

Consider the two functions f and g defined by the rules $f(x) = \tan(x)$ and $g(x) = \sqrt{2}\cos(x)$.

- a. On the axes below, the graphs of f and g are shown over the domain $\left[0, \frac{\pi}{2}\right)$.

Label each graph as f or g and write down the coordinates of P .



2 marks

- b. Differentiate $\log_e(\cos(x))$ and hence find an antiderivative of $\tan(x)$.

2 marks

- c. Find the area enclosed by the y -axis and the graphs of f and g .

3 marks

Question 5

Consider the independent events A and B where $\Pr(A|B) = \frac{3}{4}$ and $\Pr(B) = \Pr(A' \cap B')$.

Find $\Pr(B)$.

3 marks

Question 6

Solve the equation $e^x - 4 = 5e^{-x}$ for x .

2 marks

Question 7

The time in hours, T , that a randomly chosen flight from a particular airport is delayed has a probability density function given by:

$$f(t) = \begin{cases} \frac{-t}{50} + \frac{1}{5} & \text{for } 0 \leq t \leq 10 \\ 0 & \text{for } t < 0 \text{ or } t > 10 \end{cases}$$

The probability that a randomly chosen flight is delayed by 3 hours, at most, equals $\frac{51}{100}$.

A person has already had their flight delayed so they have been waiting in the departure lounge for 3 hours.

What is the probability that this person will be delayed by up to a further 3 hours before departure?

3 marks

Question 8

Let $f: R \rightarrow R$, $f(x) = x^2 + k$, where k is a real number.

Determine the equation of the tangent to the graph of f at the point where $x = a$, and hence show that k cannot be negative if this tangent is to pass through the origin.

3 marks

Question 9

A transformation is described by the equation:

$$TX + B = X' \text{ where } T = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -4 \\ 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } X' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- a. Find the image of point (2, 3).

1 mark

- b. Find the image of the line with equation $y = 2x - 4$ under this transformation.

3 marks

- c. Write in words, in the correct order, the transformations described by the equation $TX + B = X'$.

2 marks

Question 10

- a. Use the relationship $f(x + h) \approx f(x) + hf'(x)$ for small positive values of h , to find the approximate change in the volume of a cuboid with volume $V \text{ cm}^3$, given by $V = 6x^3$, if x changes from 3 cm to 3.02 cm.

2 marks

- b. Explain whether this approximation is either greater or less than the exact change.

1 mark

END OF QUESTION AND ANSWER BOOKLET