

Q1a The product rule: $\frac{d}{dx}(x^3 e^{2x}) = x^3(2e^{2x}) + (3x^2)e^{2x}$
 $= x^2(2x+3)e^{2x}$

Q1b $f(x) = \log_e(x^2 + 1)$, $f'(x) = \frac{2x}{x^2 + 1}$, $f'(2) = \frac{4}{5}$

Q2a An antiderivative is $\int \cos(2x+1)dx = \frac{1}{2}\sin(2x+1)$

Q2b $\int_2^3 \frac{1}{1-x} dx = [-\log_e|1-x|]_2^3 = -\log_e(2) = \log_e\left(\frac{1}{2}\right)$, $\therefore p = \frac{1}{2}$

Q3a $f(x) = \frac{1}{x^2}$, $g(x) = f(f(x)) = \frac{1}{\left(\frac{1}{x^2}\right)^2} = x^4$, $x \in \mathbb{R}^+$

Q3b $g^{-1}(x) = x^{\frac{1}{4}}$, $x \in \mathbb{R}^+$, $g^{-1}(16) = 16^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2$

Q4a amplitude = 4, period = $\frac{2\pi}{\frac{1}{3}} = 6\pi$

Q4b $\frac{\sin(x)}{\cos(x)} = \frac{1}{\sqrt{3}}$, $\tan(x) = \frac{1}{\sqrt{3}}$, $x = -\frac{5\pi}{6}$, $\frac{\pi}{6} \in [-\pi, \pi]$

Q5a $\Pr(X > \mu) = 0.5$

Q5b $\Pr(X > 7) = \Pr(X < 3) = \Pr\left(Z < \frac{3-5}{3}\right) = \Pr\left(Z < -\frac{2}{3}\right)$

$\therefore b = -\frac{2}{3}$



Q6
 $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3x-1 \\ 2y+4 \end{bmatrix}$

$\therefore x = \frac{x'+1}{3}$ and $y = \frac{y'-4}{2}$

Substitute into $y = 2x^2 + 1$, simplify and remove the 's,

$y = \frac{4}{9}x'^2 + \frac{8}{9}x' + \frac{58}{9}$

$\therefore a = \frac{4}{9}$, $b = \frac{8}{9}$, $c = \frac{58}{9}$

Q7a $\int_0^5 ax(5-x)dx = 1$, $\int_0^5 (5ax - ax^2)dx = 1$, $\left[\frac{5ax^2}{2} - \frac{ax^3}{3}\right]_0^5 = 1$

$\frac{125a}{2} - \frac{125a}{3} = 1$, $a = \frac{6}{125}$

Q7b $\Pr(X < 3) = \int_0^3 \frac{6}{125}x(5-x)dx$

Q8 $p^2 + p^2 + \frac{p}{4} + \frac{4p+1}{8} = 1$

Simplify: $16p^2 + 6p - 7 = 0$

Factorise: $(2p-1)(8p+7) = 0$ where $p > 0 \therefore p = \frac{1}{2}$

Q9a $\frac{d}{dx}(x^2 \log_e(x)) = x^2\left(\frac{1}{x}\right) + (2x)\log_e(x)$ where $x > 0$

$\therefore \frac{d}{dx}(x^2 \log_e(x)) = x + 2x \log_e(x)$

For use in Q9b, $x \log_e(x) = \frac{1}{2}\left(\frac{d}{dx}(x^2 \log_e(x)) - x\right)$

Q9b Area = $\int_1^3 x \log_e(x) dx$

$= \int_1^3 \frac{1}{2}\left(\frac{d}{dx}(x^2 \log_e(x)) - x\right) dx = \frac{1}{2}\left[x^2 \log_e(x) - \frac{x^2}{2}\right]_1^3$

$= \frac{9}{2}\log_e(3) - 2 \therefore a = \frac{9}{2}$, $b = 3$, $c = -2$

Q10 $y = x^{\frac{1}{2}} + d$, $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

At $(9, c)$, $\frac{dy}{dx} = a$, the gradient of the tangent $y = ax - 1$.

$\therefore \frac{1}{2\sqrt{9}} = a$, $a = \frac{1}{6}$

$\therefore$ The tangent is $y = \frac{1}{6}x - 1$, and $(9, c)$ is on the tangent

$\therefore c = \frac{1}{6} \times 9 - 1 = \frac{1}{2}$

$\left(9, \frac{1}{2}\right)$ is on the curve $y = x^{\frac{1}{2}} + d$, $\therefore \frac{1}{2} = 9^{\frac{1}{2}} + d$, $d = -\frac{5}{2}$

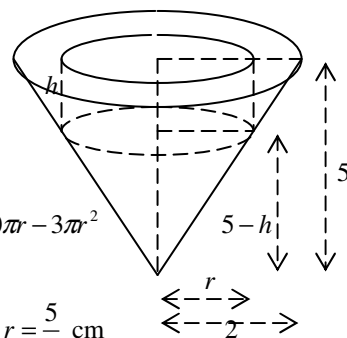
Q11a Similar triangles:

$\frac{5-h}{r} = \frac{5}{2} \therefore h = 5\left(1 - \frac{r}{2}\right)$

Q11b

$S = 2\pi r \times 5\left(1 - \frac{r}{2}\right) + 2\pi r^2 = 10\pi r - 3\pi r^2$

Q11c $\frac{dS}{dr} = 10\pi - 6\pi r = 0 \therefore r = \frac{5}{3}$ cm



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