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Mathematical Methods

2010

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Solve the following system of simultaneous linear equations for x and y .

2 marks

$$-0.5x + y = -1.5$$

$$2x + y = 1$$

$$x - 2y = 3$$

Question 2

Given $f(x) = x^2 - 2x$ and $g(x) = -\frac{1}{2}f(1 - 2x) + \frac{3}{2}$,

a. express $g(x)$ in the form $a(x - b)(x - c)$, where $a, b, c \in R$.

2marks

b. Described the relation between the graph of the transformed function $g(x)$ and the graph of the original function $f(x)$ in terms of dilations, reflections and translations.

2 marks

Question 3

Given $f(x) = 1 + \log_e x$,

a. show that $f(xy) + f\left(\frac{x}{y}\right)$ is a function of x only.

2 marks

b. Find the solutions to the equation $f(xy) + f\left(\frac{x}{y}\right) = 0$, $x, y \in \mathbb{R}$.

2 marks

Question 4

Find the exact value of $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} (\cos^2 x - \sin^2 x) dx$.

3 marks

Question 5

Consider $f(x) = x^2 + 4x - 5$ and $g(x) = e^x$.

a. State the domain of $f(g(x))$. 1 mark

b. Determine the range of $f(g(x))$. 2 marks

c. Solve $f(g(x)) = 8(e^x - 1)$ for x . Write the solution(s) in exact form. 2 marks

Question 6

Consider $h: \left[0, \frac{1}{2}\right] \rightarrow \mathbb{R}$, $h(x) = \frac{1}{2} \cos\left(\pi x - \frac{\pi}{2}\right) - 1$.

a. Determine the range of h . 2 marks

b. Determine the inverse of h including its domain. Express your answer in simplest form. 3 marks

c. Solve $h(x) = -\frac{3}{4}$ for x . 2 marks

Question 7

a. Factorise $f(x) = 8x^3 - 12x^2 + 6x - 1$ completely.

2 marks

b. Hence or otherwise find a *factorised* anti-derivative of $f(x)$.

1 mark

Question 8

Water is poured into a container of irregular shape to a depth of 1.20 metres in 2 minutes at 25 litres per minute. Let the volume of water in the container be V and the depth of water be h at time t .

a. Find $\frac{dV}{dt}$ in cm^3 per minute.

1 mark

b. Find $\frac{\Delta h}{\Delta t}$ in metres per minute.

1 mark

c. Find $\frac{\Delta h}{\Delta V}$ in cm^{-2} .

2 marks

Question 9

Two fair dice are rolled together three times and the sum of the two uppermost numbers in each roll is recorded.

- a.** Complete the following probability distribution of the random variable X , the sum of the two uppermost numbers in rolling two fair dice together.

2 marks

X	2	3	4	5	6	7	8	9	10	11	12
$\Pr(X = x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$		$\frac{1}{6}$					

- b.** Let random variable Y be the number of times that $X = 7$ when the two fair dice are rolled together three times. Find $\Pr(Y \geq 1)$.

2 marks

Question 10

A train arrives at a station at 7:30 am, and departs the station between 7:35 am and 7:40 am. If you arrive at the station *after* 7:33 am, determine the probability that you catch the train.

(Hint: Construct a probability density function which describes the distribution of probability in missing the train if you arrive at the station after time t .)

4 marks

End of exam 1