

Q1

$$-0.5x + y = -1.5 \dots (1)$$

$$2x + y = 1 \dots (2)$$

$$x - 2y = 3 \dots (3)$$

Equations (1) and (3) represent two superimposed straight lines, and equation (2) represents a straight line cutting across (1) and (3). The coordinates of the intersecting point is:

$$2 \times \text{eq}(2) + \text{eq}(3), 5x = 5, x = 1 \text{ and } y = -1.$$

Q2a $f(x) = x^2 - 2x$ and

$$g(x) = -\frac{1}{2}f(1-2x) + \frac{3}{2} = -\frac{1}{2}[(1-2x)^2 - 2(1-2x)] + \frac{3}{2}$$

$$= -\frac{1}{2}[4x^2 - 1] + \frac{3}{2} = -2x^2 + 2 = -2(x-1)(x+1)$$

Q2b $g(x)$ is the result of the following sequential transformations of $f(x)$.

Vertical dilation by a factor of $1/2$

Horizontal dilation by a factor of $1/2$

Reflection in the x -axis

Reflection in the y -axis

Translation to the right by $1/2$ of a unit

Translation upwards by $3/2$ units

Q3a Given $f(x) = 1 + \log_e x$

$$f(xy) + f\left(\frac{x}{y}\right) = 1 + \log_e(xy) + 1 + \log_e\left(\frac{x}{y}\right)$$

$$= 1 + \log_e x + \log_e y + 1 + \log_e x - \log_e y$$

$$= 2(1 + \log_e x) = 2f(x)$$

Q3b $f(xy) + f\left(\frac{x}{y}\right) = 0, \therefore 2(1 + \log_e x) = 0$

$$\therefore \log_e x = -1, x = e^{-1}.$$

Solution set is $\{(x, y) : x = e^{-1} \text{ and } y \in R\}$.

Q4 $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} (\cos^2 x - \sin^2 x) dx = \int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cos(2x) dx = \left[\frac{\sin(2x)}{2} \right]_{\frac{\pi}{8}}^{\frac{\pi}{6}}$

$$= \frac{\sin \frac{\pi}{3}}{2} - \frac{\sin \frac{\pi}{4}}{2} = \frac{\sqrt{3} - \sqrt{2}}{4}$$

Q5a Domain of $f(g(x))$ is R .

Q5b Since $g(x) = e^x > 0, \therefore f(g(x)) > -5$

The range of $f(g(x))$ is $(-5, \infty)$.

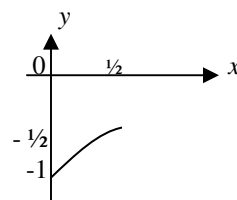
Q5c $(e^x)^2 + 4e^x - 5 = 8e^x - 8$

$$(e^x)^2 - 4e^x + 3 = 0, (e^x - 3)(e^x - 1) = 0$$

$$\therefore e^x = 3 \text{ or } e^x = 1 \therefore x = \log_e 3 \text{ or } 0$$

Q6a $h: \left[0, \frac{1}{2}\right] \rightarrow R, h(x) = \frac{1}{2} \cos\left(\pi x - \frac{\pi}{2}\right) - 1$ can be simplified

to $h(x) = \frac{1}{2} \sin(\pi x) - 1$.



The range of h is $\left[-1, -\frac{1}{2}\right]$.

Q6b The inverse of $y = \frac{1}{2} \sin(\pi x) - 1$ is $x = \frac{1}{2} \sin(\pi y) - 1$,

$$\therefore 2(x+1) = \sin(\pi y), \therefore y = \frac{1}{\pi} \sin^{-1}[2(x+1)].$$

Hence $h^{-1}: \left[-1, -\frac{1}{2}\right] \rightarrow R, h^{-1}(x) = \frac{1}{\pi} \sin^{-1}[2(x+1)]$.

Q6c $h(x) = -\frac{3}{4}, \frac{1}{2} \sin(\pi x) - 1 = -\frac{3}{4}, \sin(\pi x) = \frac{1}{2},$

$$\pi x = \frac{\pi}{6}, x = \frac{1}{6} \in \left[0, \frac{1}{2}\right].$$

Q7a $f(x) = 8x^3 - 12x^2 + 6x - 1 = (8x^3 - 1) - (12x^2 + 6x)$

$$= ((2x)^3 - 1) - (12x^2 + 6x) = (2x - 1)(4x^2 + 2x + 1) - 6x(2x - 1)$$

$$= (2x - 1)(4x^2 - 4x + 1) = (2x - 1)(2x - 1)^2$$

$$= (2x - 1)^3$$

Q7b $\int f(x) dx = \int (2x - 1)^3 dx = \frac{(2x - 1)^4}{8}$

$$\text{Q8a } \frac{dV}{dt} = 25 \text{ litres per minute} = 25000 \text{ cm}^3 \text{ per minute}$$

$$\text{Q8b } \frac{\Delta h}{\Delta t} = \frac{1.20}{2} = 0.60 \text{ metres per minute or } 60 \text{ cm per minute}$$

$$\text{Q8c } \text{Since } \frac{dV}{dt} \text{ is constant, } \frac{\Delta V}{\Delta t} = \frac{dV}{dt} = 25000 \text{ cm}^3 \text{ per minute}$$

$$\frac{\Delta h}{\Delta V} = \frac{\frac{\Delta h}{\Delta t}}{\frac{\Delta V}{\Delta t}} = \frac{60}{25000} = 0.0024 \text{ cm}^{-2}$$

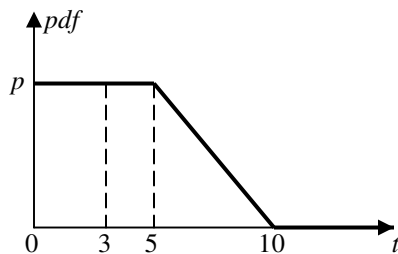
Q9a

5	6	7	8	9	10	11	12
$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

$$\text{Q9b } \Pr(Y \geq 1) = 1 - \Pr(Y = 0) = 1 - \left(\frac{5}{6}\right)^3 = \frac{91}{216} \approx 0.4213$$

Q10 Let 7:30 am be $t = 0$, 7:35 am be $t = 5$ and 7:40 am be $t = 10$.

A probability density function which describes the distribution of probability in missing the train if you arrive at the station after time t is shown below.



$$\text{Total area under graph} = \frac{1}{2}(5+10)p = 1, \therefore p = \frac{2}{15}.$$

If you arrive after 7:33 am ($t = 3$),

$$\Pr(\text{miss the train}) = \text{area under graph (0 to 3)} = \frac{2}{15} \times 3 = 0.4$$

$$\Pr(\text{catch the train}) = 1 - 0.4 = 0.6$$

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