



Victorian Certificate of Education 2009

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures

Words

Letter

--

MATHEMATICAL METHODS (CAS)

Written examination 1

Friday 6 November 2009

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 9 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

This page is blank

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Differentiate $x \log_e(x)$ with respect to x .

2 marks

- b. For $f(x) = \frac{\cos(x)}{2x+2}$ find $f'(\pi)$.

3 marks

TURN OVER

Question 2

- a. Find an anti-derivative of $\frac{1}{1-2x}$ with respect to x .

2 marks

- b. Evaluate $\int_1^4 (\sqrt{x} + 1) dx$.

3 marks

Question 3

Let $f: R \setminus \{0\} \rightarrow R$ where $f(x) = \frac{3}{x} - 4$. Find f^{-1} , the inverse function of f .

3 marks

Question 4

Solve the equation $\tan(2x) = \sqrt{3}$ for $x \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$.

3 marks

Question 5

Four identical balls are numbered 1, 2, 3 and 4 and put into a box. A ball is randomly drawn from the box, and not returned to the box. A second ball is then randomly drawn from the box.

- a. What is the probability that the first ball drawn is numbered 4 and the second ball drawn is numbered 1?

1 mark

- b. What is the probability that the sum of the numbers on the two balls is 5?

1 mark

- c. Given that the sum of the numbers on the two balls is 5, what is the probability that the second ball drawn is numbered 1?

2 marks

TURN OVER

Question 6

Oil is leaking at a constant rate to form a circular puddle on the floor. The oil is being added to the puddle at the rate of 10 mm^3 per minute causing the puddle to spread out evenly, with constant depth of 2 mm.

When the radius of the puddle is r mm, the volume, $V \text{ mm}^3$, of oil in the puddle is given by $V = 2\pi r^2$.

Find the rate of change of the radius of the puddle when the radius is 30 mm. Give an exact answer, with units of mm per minute.

3 marks

Question 7

The random variable X has this probability distribution.

X	0	1	2	3	4
$\Pr(X = x)$	0.1	0.2	0.4	0.2	0.1

Find

a. $\Pr(X > 1 | X \leq 3)$

2 marks

b. $\text{Var}(X)$, the variance of X .

3 marks

Question 8

Let $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = e^x + k$, where k is a real number. The tangent to the graph of f at the point where $x = a$ passes through the point $(0, 0)$. Find the value of k in terms of a .

3 marks

Question 9

Solve the equation $2 \log_e(x) - \log_e(x+3) = \log_e\left(\frac{1}{2}\right)$ for x .

4 marks

Question 10

- a. Use the relationship $f(x+h) \approx f(x) + hf'(x)$ for a small positive value of h , to find an approximate value for $\sqrt[3]{8.06}$.

4 marks

- b. Explain why this approximate value is greater than the exact value for $\sqrt[3]{8.06}$.

1 mark

MATHEMATICAL METHODS AND MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

This page is blank

Mathematical Methods and Mathematical Methods (CAS)

Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$	volume of a pyramid:	$\frac{1}{3}Ah$
curved surface area of a cylinder:	$2\pi rh$	volume of a sphere:	$\frac{4}{3}\pi r^3$
volume of a cylinder:	$\pi r^2 h$	area of a triangle:	$\frac{1}{2}bc \sin A$
volume of a cone:	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e x + c$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

mean: $\mu = E(X)$

variance: $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

probability distribution		mean	variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$