MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

2009 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: C

Explanation: x=0, y intercept is 6 and C is the only one that gives this result
(or do long division and other features also become clear)

Question 2

Answer: B

Explanation: product rule: $\ln(x^2 - x) + \dfrac{x(2x-1)}{x(x-1)}$

$$\ln(x^2 - x) + \frac{(2x-1)}{(x-1)}$$

Question 3

Answer: A

Explanation : when $x=e, y=-2$, when $x=5, y=-2\ln(5)$, the range is $[-2\ln(5),-2)$

Question 4

Answer: D

Explanation: swap x and y for inverse

$x = (y-2)^2 + 3$

$\pm\sqrt{x-3} = y-2$, due to domain only negative

$y = -\sqrt{x-3} + 2$, domain of $f(x)$ is the range of the inverse function

$f^{-1} : [3, \infty) \to R, f^{-1}(x) = -\sqrt{x-3} + 2$

Question 5

Answer: B

Explanation: $y = g\left(-\frac{1}{2}(x-2)\right)$

$= g\left(-\frac{x}{2} + 1\right) \Rightarrow g\left(1 - \frac{x}{2}\right)$

Question 6

Answer: C

Explanation: Let $e^x = a, \Rightarrow a^2 - a - 6 = 0$

$(a-3)(a+2) = 0$

$e^x = 3, e^x = -2, \Rightarrow x = \ln(3)$, only

Question 7

Answer: D

Explanation: The graph of $y = |\log_e x|$ graph has been translated 2 units left, reflected in the x-axis then translated 1 unit down

Question 8

Answer: C

Explanation: Product of the given functions is a cubic function and C is the only possible option. Also, key points are x = 0,1,-1.

Question 9

Answer: C

Explanation: $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2, r = 1$

$\frac{dV}{dr} = 4\pi$

Question 10

Answer: A

Explanation: $\left[2x^2 - 5x\right]_0^k = 3$

$2k^2 - 5k - 3 = 0$

$(2k+1)(k-3) = 0$

$k = -\frac{1}{2}, 3$

Question 11

Answer: B

Explanation: $0.3 = \frac{\Pr(A \cap B)}{0.4}$

$\Pr(A \cap B) = 0.12$

$0.6 = \Pr(A) + 0.4 - 0.12$

$\Pr(A) = 0.32$

Question 12

Answer: E

Explanation: $0.7 + 6k = 1$

$k = 0.05$

$E(X) = 0 + 0.1 + 0.9 + 0.6 = 1.6$

Question 13

Answer: E

Explanation: 1-binomcdf(30,0.85,26)=0.3217

Question 14

Answer: A

Explanation: turning point at (2,2), point of inflection at (5,7) makes it a quartic curve

Question 15

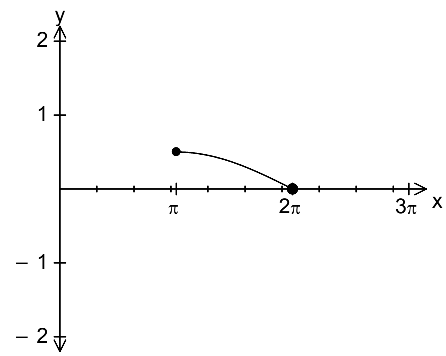
Answer: B

Explanation: $\dfrac{\Pr(1.6 < X < 1.8)}{\Pr(X < 1.8)} = \dfrac{normalcdf(1.6,1.8,1.5,0.25)}{normalcdf(-10\text{^}(99),1.8,1.5,0.25)} = 0.2594$

Question 16

Answer: C

Explanation: mode is the highest point, $x = \pi$



Question 17

Answer: E

Explanation: Let $y = 0$

$$0 = 4\sin\left(\frac{x}{2}\right) + 2, x \in [-2\pi, 2\pi]$$

$$-\frac{1}{2} = \sin\left(\frac{x}{2}\right)$$

$$\frac{x}{2} = -\frac{\pi}{6}, -\frac{5\pi}{6} \Rightarrow x = -\frac{\pi}{3}, -\frac{5\pi}{3}$$

Question 18

Answer: A

Explanation: $\int_{-1}^{3}(5)dx - \frac{1}{2}\int_{-1}^{3}(g(x))dx$

$[5x]_{-1}^{3} - \frac{1}{2}(-6)$

$15 + 5 + 3 = 23$

Question 19

Answer: A

Explanation: $\int\left(\frac{4}{3-2(x-1)}\right)dx$

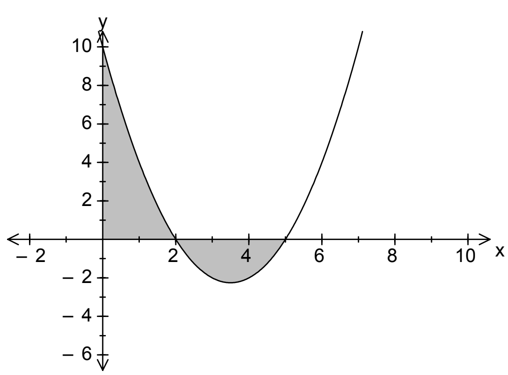
$\int\left(\frac{4}{1-2x}\right)dx$

$-\frac{4}{2}\ln|1-2x| + c \Rightarrow -2\ln|1-2x| + c$

Question 20

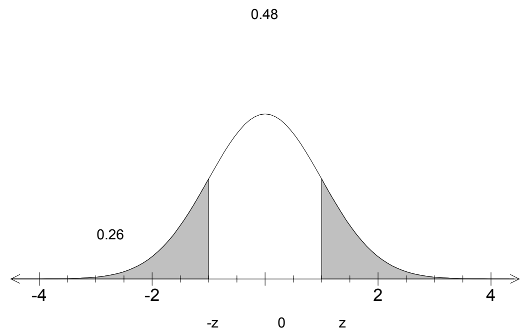
Answer: A

Explanation: $\int_{0}^{2}(x^2 - 7x + 10)dx - \int_{2}^{5}(x^2 - 7x + 10)dx = 13\frac{1}{6}\, sq.units$



$\int_0^5 x^2 - 7x + 10\, dx = 4.167$ (*Area* = 13.167)

Question 21



Answer: A

Explanation: $invNorm(0.26,0,1) = 0.6433$

Question 22

Answer: D

Explanation: $x = 5, y = 26\ln(16)$, and $x = 3, y = 26\ln(10)$

$$\text{average rate} = \frac{26\ln(16) - 26\ln(10)}{5-3}$$

$$= \frac{26\left(\ln\left(\frac{16}{10}\right)\right)}{2}$$

$$= 13\ln\left(\frac{8}{5}\right)$$

SECTION 2: Analysis Questions

Question 1

a. Turning point at $x = 1 \therefore b = 1$, x intercept at $x = 6 \therefore c = 6$,

At back door (0,3) sub into equation $3 = a(-1)^2(-6)$

$$3 = -6a$$

$$a = -\frac{1}{2}, b = 1, c = 6$$

M1+A1
2 marks

b. expand (or use product rule)

$$y = -\frac{1}{2}x^3 + 4x^2 - \frac{13}{2}x + 3$$

$$\frac{dy}{dx} = -\frac{3}{2}x^2 + 8x - \frac{13}{2}, \frac{dy}{dx} = 0$$

$$0 = -3x^2 + 16x - 13$$

$$\therefore TP\ \ is \left(4\frac{1}{3}, 9\frac{7}{27}\right)$$

M2+A1
3 marks

c. $(0,3)(4,9) \Rightarrow m = \frac{9-3}{4-0} = \frac{3}{2}$

$$y - 3 = \frac{3}{2}(x - 0)$$

$$y = \frac{3}{2}x + 3$$

M1+A1
2 marks

d. Find point on curve $Area = \int_0^4 \frac{3x + 6 + x^3 - 8x^2 + 13x - 6}{2} dx$

$$= \int_0^4 (8x + \frac{x^3}{2} - 4x^2) dx$$

$$= \left[4x^2 + \frac{1}{8}x^4 - \frac{4}{3}x^3\right]_0^4$$

$$= 10\frac{2}{3} sq.units$$

M1+A1
2 marks

e. Let $x=2, y=2$ $(f(2)=2)$

$$\frac{dy}{dx}=-\frac{3}{2}x^2+8x-\frac{13}{2}, x=2$$

$$m_t=3.5, \Rightarrow m_n=-\frac{2}{7}$$

$$y-2=-\frac{2}{7}(x-2)$$

$$\therefore g(x)=-\frac{2x}{7}+2\frac{4}{7}$$

M2+A1
3 marks

f. Substituting: $y=0$ into the equation of the normal gives $x=9$. As the side gate is at (6, 0), the fence will not fit into the backyard before the side gate.

A1
1 mark

Question 2

a. $$a^x=\frac{b}{b^x}$$

$$a^x b^x=b$$

$$(ab)^x=b$$

$$x=\frac{\ln(b)}{\ln(ab)}$$

$$\frac{\ln(b)}{\ln(a)+\ln(b)}$$

M1+A1
2 marks

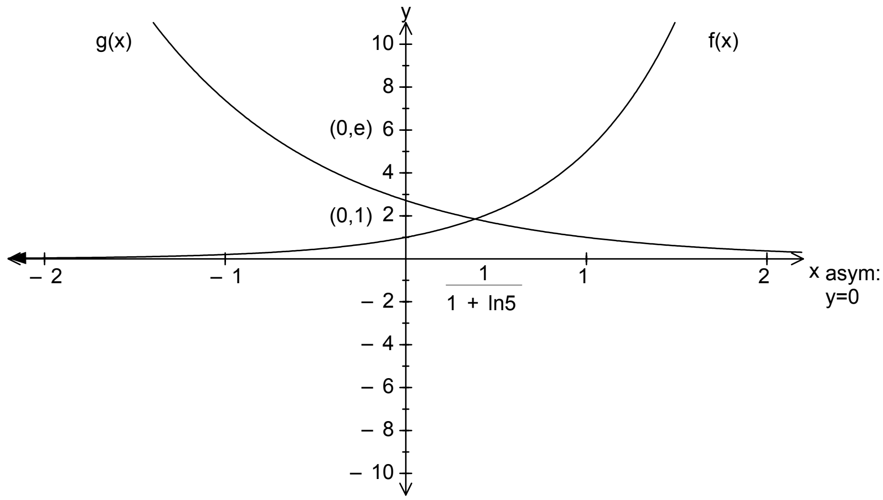
b. $$\frac{\ln(4)}{\ln(9)+\ln(4)}$$

$$\frac{2\ln(2)}{2\ln(3)+2\ln(2)}$$

$$\frac{\ln(2)}{\ln(6)}$$

M1+A1
2 marks

c. Correct shape, show important points, (0,e), (0,1)



A2
2 marks

d. point of intersection at $x = \dfrac{1}{1+\ln(5)}$

$5^x = e^{1-x}$

$\ln(5)^x = 1 - x$

$\ln(5) = \dfrac{1-x}{x}$

$\ln(5) + 1 = \dfrac{1}{x}$

$x = \dfrac{1}{1+\ln(5)}$

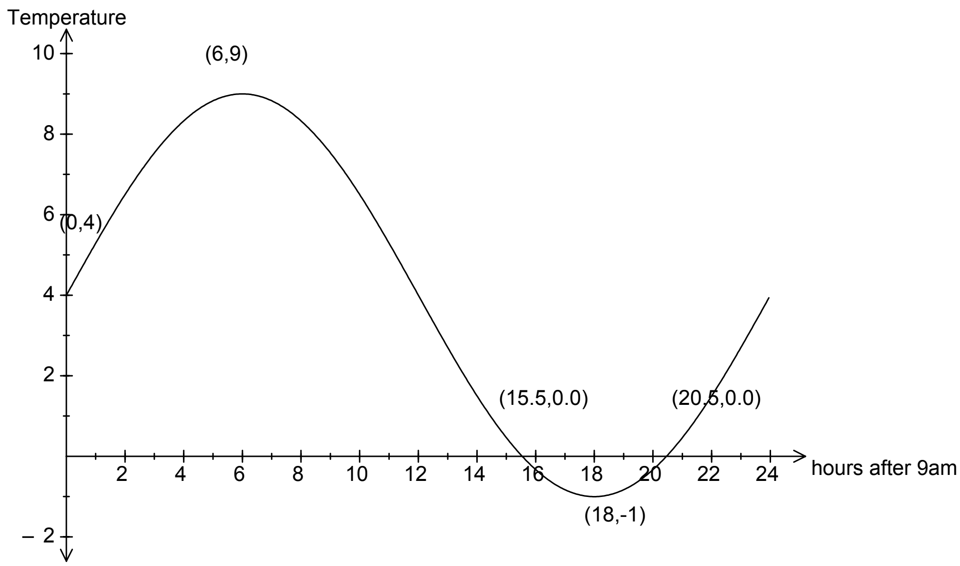
M1+A1
2 marks

Question 3

a. Temperature range is [-1,9]

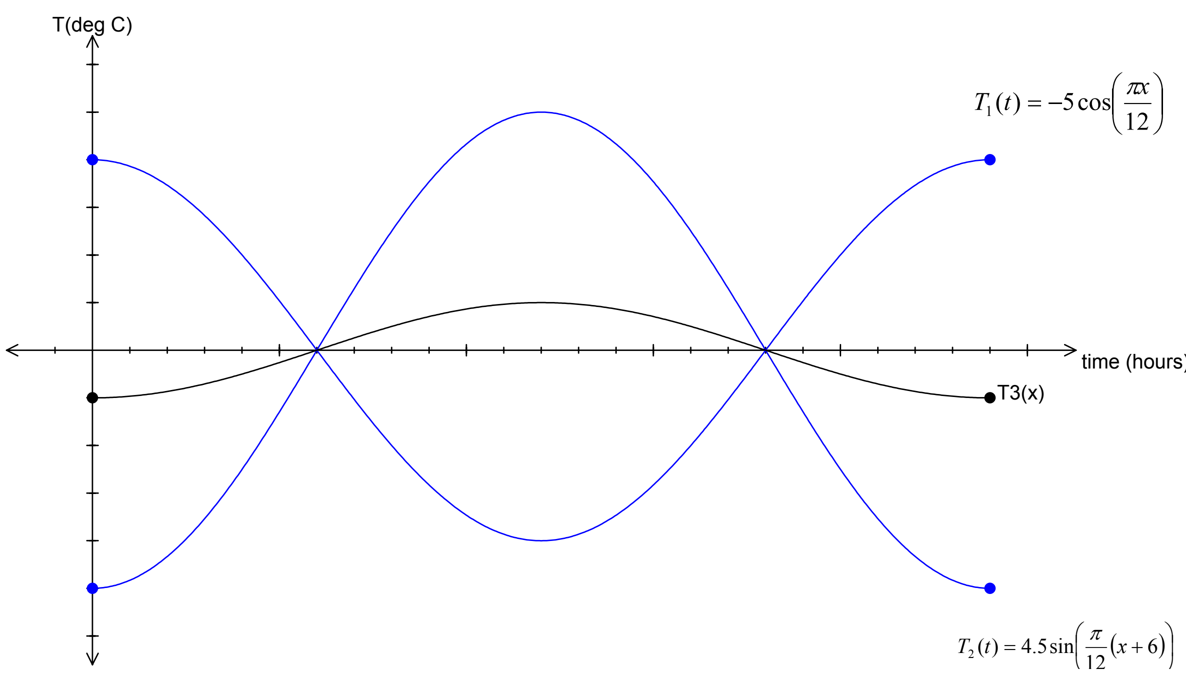
A1
1 mark

b.



A3
3 marks

c.



A1
1 mark

d. [-1,1]

A1
1 mark

Question 4

a. $V = \pi\left(\frac{x}{2}\right)^2 l$

$100 = \pi\left(\frac{x}{2}\right)^2 l$

$l = \frac{400}{\pi x^2}$

M1+A1
2 marks

b. $V = a^2 l$

$= \frac{x^2}{2} \times \frac{400}{\pi x^2}$

$= \frac{200}{\pi} cm^3$

M1+A1
2 marks

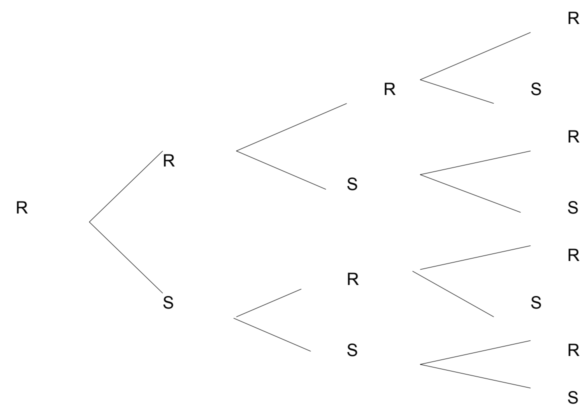
c. $\Pr(X \le 2) =$ binomcdf(100,.03,2)=0.4198

M1+A1
2 marks

d. $\Pr(X \ge 9) = 1 - \text{binomcdf}(10, 0.4198, 8) = 0.0025$

M1+A1
2 marks

e.



Pr (round in April|round in Jan)

$= (0.65)^3 + 0.65 \times 0.35 \times 0.25 + 0.35 \times 0.25 \times 0.65 + 0.35 \times 0.75 \times 0.25$

$= 0.4540$

M2+A1
3 marks

f. Pr (square on at least 2 of next 3 | round in Jan)

$= 0.65 \times 0.35 \times 0.75 + 0.35 \times 0.25 \times 0.35 + 0.35 \times 0.75 \times 0.25 + 0.35 \times (0.75)^2$

$= 0.4638$

M1+A1
2 marks

g.

i. $\dfrac{6}{5}\displaystyle\int_1^m \left(x^2 - x\right)dx = 0.5$

$$\frac{6}{5}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^m = 0.5$$

$$\frac{6}{5}\left[\frac{m^3}{3} - \frac{m^2}{2} - \frac{1}{3} + \frac{1}{2}\right] = 0.5$$

By using calculator, m=1.75. Therefore median cost is \$1.75

M2+A1
3 marks

ii. $\dfrac{6}{5}\displaystyle\int_1^{1.6} \left(x^2 - x\right)dx = 0.3024$

M1+A1
2 marks

h.

i. $\Pr(Z < z) = 0.11 \Rightarrow z = invNorm(0.11,0,1) = -1.2265$

$$\sigma = \frac{107-108}{-1.2265} = 0.82$$

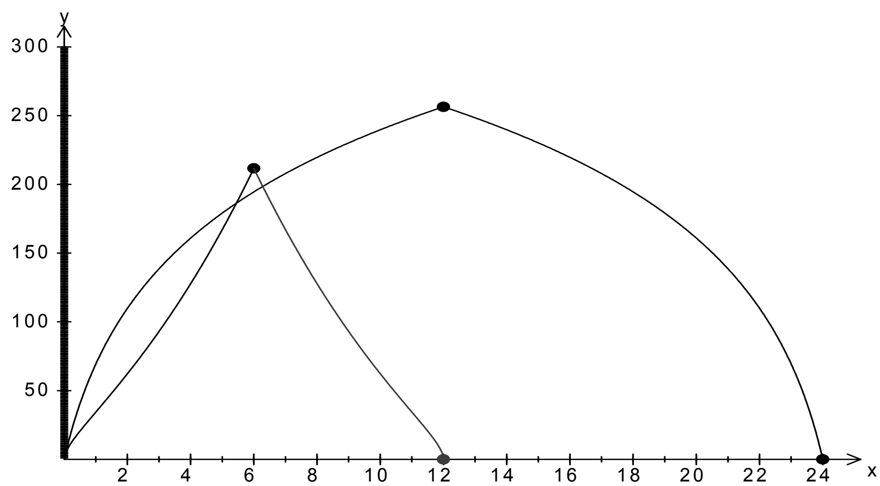
M2+A1
3 marks

ii. $\mu \pm 2\sigma \Rightarrow [106.37,109.63]g$

A1
1 mark

Question 5

a.



A1
1 mark

b. **i.** reflected in *y* axis, then translated 24 units right

ii. reflected in *y* axis then translated 12 units right

A2
2 marks

c. **i.** On calculator A = 256.49 *g*

ii. B=211.65 *g*

A2
2 marks

d. $198.54025 \times 2 = 397.08g$ (2 x *y* part of intersection of A_1 *and* B_2)

A1
1 mark

e. 6.28 hours = 6 hours 17min (*x* part of intersection of A_1 *and* B_2)

A1
1 mark

f. *X* parts of intersection of B_1 and $y=125$ *and* B_2 and $y=125$ times are 3.9244 and 8.0756 hours giving 4.1512 hours or 4 hours 9 min.

M1+A1
2 marks